In this section, we will share your points of view on teaching and learning mathematics and your responses to anything contained in this journal. We appreciate your interest and value the views of those who write.

Erratum

The previous issue of *delta-K* (Volume 38, Number 1, December 2000) failed to include one of the authors in Comments on Contributors. The following acknowledgment statement should have been included:

> Werner W. Liedtke is a professor in the Faculty of Education, Department of Elementary Education, at the University of Victoria, British Columbia. He also teaches distance education for the Knowledge Network. In addition, he supervises student teachers and delivers inservice courses to parents of preschool children and to teachers, locally and throughout the province.

My apologies for this omission and any inconvenience it may have caused for the author and the readers.

Finding Your Inner Mathematician

Keith Devlin

Many people assume that it takes a special kind of brain to be able to do mathematics—that unless you were born with some kind of "math gene," you simply are not going to be able to get math, no matter how hard you try. As someone who struggled hard with math in school until I was 15, and then got it all at once, I never believed the math-gene theory. What made the difference for me was that everything suddenly made sense—perfect, simple, elegant sense.

Having taught mathematics for 30 years, I am convinced that everyone has the capacity to do mathematics, at least through high school algebra and geometry. In fact, all you really need to do math are nine basic mental abilities that our ancestors developed thousands of years ago to survive in a hostile world:

1. *Number sense.* This is not the same as being able to count. It's much more basic than that and includes the ability to recognize the difference between one object, a collection of two objects and a collection of three objects—and to recognize that a collection of three objects has more members than a collection of two. Number sense is not something we learn. Child psychologists have demonstrated conclusively during the past 20 years that we are born with number sense.
2. *Numerical ability.* This does involve learning—both to count and to understand numbers as abstract entities. Early methods of counting, such as making notches in sticks or bones, go back at least 30,000 years. The Sumerians are the first people we know of who used abstract numbers; between 8000 and 3000 B.C., they inscribed numerical symbols on clay tablets.
3. *Spatial-reasoning ability.* This includes the ability to recognize shapes and to judge distances accurately, both of which have obvious survival value. In addition to forming the basis for geometry, this ability is important for a lot of mathematical thinking that is not, on the face of it, visual or geometric.
4. *A sense of cause and effect.* Much of mathematics depends on "if this, then that" reasoning, an abstract form of thinking about causes and their effects.
5. *The ability to construct and follow a causal chain of facts or events.* A mathematical proof of a theorem is a highly abstract version of a causal chain of facts.

6. *Algorithmic ability.* An algorithm is a step-by-step procedure for performing a certain mathematical task—the mathematician's equivalent of a recipe for baking a cake. In elementary school, we are taught algorithms for adding, subtracting, multiplying and dividing whole numbers and fractions. Secondary school algebra requires that we learn algorithms to solve equations. Algorithmic ability is an abstract version of the fifth ability on this list.
7. *The ability to understand abstraction.* Humans developed the capacity to think about abstract notions, along with acquiring language, 75,000 to 200,000 years ago.
8. *Logical-reasoning ability.* The ability to construct and follow a step-by-step logical argument is fundamental to mathematics. It is another abstract version of the fifth ability.
9. *Relational-reasoning ability.* This involves recognizing how things and people are related to each other, and being able to reason about those relationships. Much of mathematics deals with relationships between abstract objects.

The human brain acquired those nine abilities at least 75,000 years ago. They are basic mental attributes crucial to our daily lives. The question is: What does it take to put those abilities together and do math?

The key is the ability to handle abstraction—the seventh ability on the list. We can all use our brains to reason about physical objects we are familiar with, and we can carry out the same kinds of reasoning about imaginary variants of those objects—for example, the characters in a Harry Potter book or on *Star Trek*. Mathematical thinking involves one more step: reasoning about purely abstract objects. The trick is to make those abstract objects seem real—to fool the brain into thinking that it's dealing with real objects. Once you have taken that step into the world of the abstract, the rest is comparatively easy. After all, the mind is then performing tasks that it finds natural and instinctive.

Although making the abstract seem real sounds hard, we all do much the same thing whenever we read a novel or watch a movie. So am I saying that to do mathematics you have to treat it like reading a novel or watching a movie?

In fact, I'm going a step further. When you start reading a novel, or you watch a movie for the first time, you have to familiarize yourself with the characters and the situation in which they find themselves. In the case of mathematics, the characters never change, only their situations. You have to familiarize yourself with the characters just once, and from then on everything amounts to finding out new things about them.

What does that remind you of? It reminds me of a television soap opera, like the long-running *As the World Turns.* That isn't a joke. The secret to being able to do mathematics is to think of math as a soap opera.

I'm not talking about the love lives of mathematicians here—it's math itself that constitutes the soap opera. The characters are not fictitious people but mathematical objects: numbers, geometric figures, topological spaces and so on. The facts and relationships of interest are not births, deaths, marriages, love affairs and business deals, but mathematical facts and relationships like: Are objects A and B equal? What object has property P? What is the relationship between objects X and Y? Do all objects of type X have property P? How many objects of type Z are there?

Mathematicians think about mathematical objects and the relationships among them using the same mental abilities that most people use to think about physical space or about other people.

Mathematicians don't have a different kind of brain. They have learned to use a standard-issue brain in a slightly different way. What distinguishes a great mathematician from a high school student struggling in a geometry class is the degree to which each one can cope with abstraction. The mathematician learns to create and hold an abstract world in her mind, and then reason about that world as if it were real.

The importance of abstraction and the brain's difficulty in handling abstract objects have three clear implications for mathematics teaching. First, we should start with what is familiar and concrete, and move gradually into the abstract. Second, we must realize that the key—the real challenge—is for the student to come to view the abstract objects of mathematics as real. Third, we need to accept the fact that a period of repetitive training is unavoidable—because repeated use is the only way to make abstract objects seem sufficiently real for the brain to process them.

Much of the current debate about mathematics teaching is focused on whether rote learning of basic math skills is still important in an age of electronic calculators and computers. That debate misses the point. The real value of learning basic math skills today is not that you will need to use those skills per se; chances are you won't. Rather, the benefit is to make the abstract objects of mathematics become so familiar—and seem so real—that you can reason about them using the same mental capacities you use to reason about everyday things. Unless you can get to that stage, you'll never be able to master the more sophisticated kinds of mathematics that today are part of the jobs of stockbrokers, architects, scientists, builders, Olympic coaches, physicians and many other people.

Of course, not everybody will use those forms of math in their daily lives. But mastering mathematical abstraction, like learning a foreign language, is much easier when you are young. Good, effective instruction in math should be part of everyone's education, so that no one is shut out of such an important area of modern life.

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Evaluate: $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}}$

Two Fishermen

Ron: Paul, if you gave me one of your fish, I would have twice as many as you.

Paul: But if you gave me one, we would have the same number of fish.

How many fish does each fisherman have?