Mathematics as communication is an important curriculum standard; hence, the mathematics curriculum emphasizes the continued development of language and symbolism to communicate mathematical ideas. Communication includes regular opportunities to discuss mathematical ideas and to explain strategies and solutions using words, mathematical symbols, diagrams and graphs. While all students need extensive experience in expressing mathematical ideas orally and in writing, some students may have the desire or should be encouraged by teachers to publish their work in journals.

delta-K *invites students to share their work with others beyond their classroom. Submissions could include papers on a particular mathematical topic, a mathematics project, an elegant solution to a mathematical problem, an interesting problem, an interesting discovery, a mathematical proof, a mathematical challenge, an alternative solution to a familiar problem, poetry about mathematics, a poster or anything deemed to be of mathematical interest.*

Teachers are encouraged to review students' work prior to submission. Please attach a dated statement that permission is granted to the Mathematics Council of the Alberta Teachers' Association to publish the work in delta-K. *The student author (or the parents if the student is under 18 years of age) must sign this statement, indicate the student's grade level, and provide an address and a telephone number.*

This issue's submissions are high school mathematics projects, completed as course requirements.

Two Pure Mathematics 30 projects were submitted. Leah Hemphill, a Grade 12 student at Ecole Secondaire Beaumont Composite High School in Beaumont, submitted "The Power of Exponential Growth." Her teacher is Corlene Balding. Ekua Yorke, a Grade 12 student at Holy Trinity High School in Edmonton, also submitted "The Power of Exponential Growth." Her teacher is Len Bonifacio.

Three students submitted Applied Mathematics 30 projects. Because the projects dealt with the same topic and produced similar solutions and conclusions, "Medical Research" is published only once, with credit given to all three students. Chris Kjemtrup (Grade 12), Malcolm Fowler (Grade 12) and Krystal Becker (Grade 11) are students at Parkland Composite High School in Edson. Their teacher is Joanna James.

Two Mathematics 31 projects were submitted. Laura Coleman and Jennifer Dodsworth, Grade 12 students at Parkland Composite High School in Edson, submitted "Binkity Visits the Egg Factory." Their teacher is Gordon Booth. Michael Buttrey, a Grade 12 student at Parkland Composite High School in Edson, submitted "Accurate 3-D Visualization of an Object." His teacher is Gordon Booth.

The Power of Exponential Growth

Leah Hemphill

This investigation covers exponential growth and binary (and other) numerical sequences.

Exponential Growth of *Escherichia Coli*

This section describes the exponential growth of the bacterium *Escherichia coli.*

Table 1 shows the growth of a single cell of *E. coli* over a period of two hours.

Table 1
Growth of an *E. Coli* Cell

Time in Minutes	0	20	40	60	80	100	120
Number of *E. Coli* Cells	1	2	4	8	16	32	64

To describe this growth, we can use the general equation $f(t) = Ab^{t/k}$, where $f(t)$ is the number of cells after a given time, A is the original number of cells, b is the growth rate of the bacteria, t is the total time (in minutes) and k is the time (in minutes) for one growth period. In the case of *E. coli*, $A = 1$, $b = 2$ and $k = 20$. Thus, the equation used to calculate the number of *E. coli* cells present after a given time t is $f(t) = (1)(2)^{t/20} = 2^{t/20}$.

We can then make three lists (which are combined to form Table 2). List 1 (L_1) represents the time period, with 1 being the initial stage (0 minutes). List 2 (L_2) is the number of cells present at the beginning of each time period. List 3 (L_3) is the logarithm of L_2 to the base 2 (that is, $\log_2 L_2$).

We can then use a graphing calculator to graph L_1 versus L_2 (see Graph 1).

The equation of Graph 1 can be found by using the exponential regression function on a graphing calculator: $y = 0.5(2)^x$.

The equation for *E. coli* growth found earlier was $f(t) = (1)(2)^{t/20}$. Both this equation and the exponential regression equation, $y = 0.5(2)^x$, show that the number of *E. coli* bacteria is doubling. However, the values are different. The two equations differ because of the A values of 1 and 0.5. This difference is a result of how the data was organized. In Table 1, which was used to formulate the equation $f(t) = (1)(2)^{t/20}$, the initial time was 0, at which point the number of bacteria was 1. L_1 represents this point as time period 1. The calculator formulates a different equation than the one manually determined.

It is estimated that after a population of *E. coli* has divided 30 times, approximately 1.5 percent of the cells will be mutants. We can estimate the number of bacteria present after 30 doubling periods by using Graph 1. The information from Graph 1 can be transferred to a table (see Table 3) so it is easier to read.

Using Table 3, we can easily find the number of mutant *E. coli* cells present after 30 doubling periods by multiplying the total number of bacteria by 1.5 percent:

$536{,}870{,}903 \times 0.015 = 8{,}053{,}063.545$

Thus, approximately 8,053,064 *E. coli* cells after 30 doubling periods will be mutants.

We can graph L_1 versus L_3 (see Graph 2) to produce a linear graph.

To find the relationship between the equation for L_3 and the exponential regression equation, consider that the exponential regression equation can be written as $y = (0.5)(2)^x$ or as $L_2 = (0.5)(2)^{L1}$. Given that $L_3 = \log_2 L_2$ and $0.5 = 2^{-1}$, the following calculations can be made:

$$
\begin{aligned}
L_3 &= \log_2 L_2 \\
&= \log_2 [(0.5)(2)^{L1}] \\
&= \log_2 (2^{-1})(2^{L1}) \\
&= \log_2 (2^{L1-1}) \\
2^{L3} &= 2^{L1-1} \\
L_3 &= L_1 - 1
\end{aligned}
$$

Since $L_1 = x$ in the exponential regression equation, if x is substituted into the place of L_1, the equation for L_3 becomes $L_3 = x - 1$.

Binary Sequences

The number 13 can be written as the sum of binary numbers (base 2): $13 = 2^0 + 2^2 + 2^3 = 1 + 4 + 8$. We can represent this as a series of switches in a circuit. The digit 1 is used to indicate that a switch is "on," and the digit 0 is used to indicate that a switch is "off."

Binary Numeral 13

16	8	4	2	1
0	1	1	0	1

Thus, the binary representation of the number 13 is 01101.

Graph 1

L_1 versus L_2

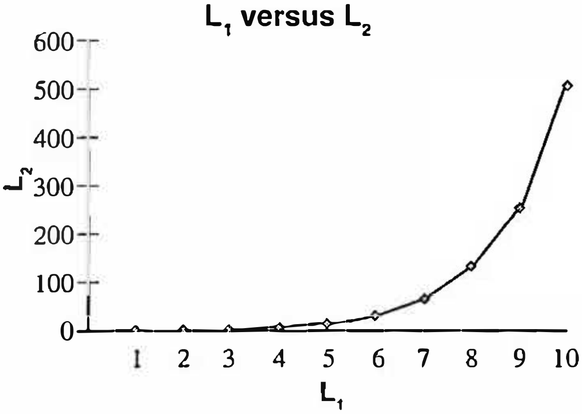


Table 2

Three Lists

Time Period (L_1)	Number of Cells (L_2)	$\log_2 L_2$ (L_3)
1	1	0
2	2	1
3	4	2
4	8	3
5	16	4
6	32	5
7	64	6
8	128	7
9	256	8
10	512	9

If a circuit board consists of five switches, the number of different codes that can be represented if at least one switch is on can be found by calculating all possible codes then subtracting unwanted outcomes. In this case, there is only one unwanted outcome (all switches off).

$2^5 - 1 = 32 - 1 = 31$

Thus, 31 different codes can be represented.

The largest number that can be represented by the five switches (meaning that all the switches are on) is also 31, as the following representation shows.

16	8	4	2	1
1	1	1	1	1

$16 + 8 + 4 + 2 + 1 = 31$

A number can be converted to a different base. For example, to convert 14 to a base 3 number, construct the following table:

243	81	27	9	3	1
0	0	0	1	1	2

The top row contains the numerical values for base 3, in decreasing exponential order (3^5, 3^4, 3^3, 3^2, 3^1 and 3^0). The bottom row indicates the number of times the value can fit into the number being converted (in this case, 14). For this example, only values of 0–2 can be placed in the bottom row. No groups of 243 can fit into 14, and the same is true for 81 and 27; therefore, a 0 is placed under these numbers. Continuing moving to the right, a 1 is placed under 9, because one group of 9 fits into 14. The process is continued until the table is complete. Thus, the numerical representation of 14 as a base 3 number is 000112.

We can use the same process to find the numerical representation of 92 as a base 4 number.

1,024	256	64	16	4	1
0	0	1	1	3	0

Table 3

Number of Bacteria Present in Each Doubling Period

Doubling Period, x	Number of Bacteria, y
29	268,435,451
30	536,870,903
31	1,073,741,805

Reproduction of the House Mosquito

This part of the project deals with a real-life phenomenon that can be described using exponential growth patterns: the reproduction of the house mosquito.

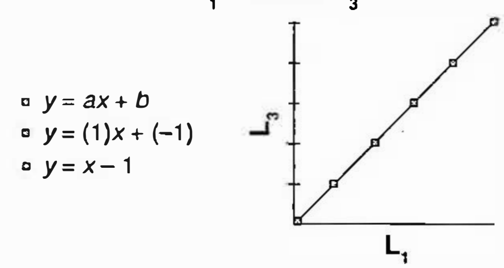
The female house mosquito lays 1–100 eggs in her life span of approximately seven days. For this example, I will assume that 100 eggs will be laid; therefore, an initial population of two mosquitoes (one male and one female) will increase 50 times over a period of seven days. Assuming, first, that an equal number of males and females are hatched in each growth period, and, second, that all females lay 100 eggs each, we can represent the increase in the mosquito population with the equation $f(t) = (2)(50)^{t/7}$, where t is the time in days.

When the equation is graphed (see Graph 3), it shows a pattern of exponential growth.

The next graph (Graph 4) shows the continuous growth of the population over a period of 20 days.

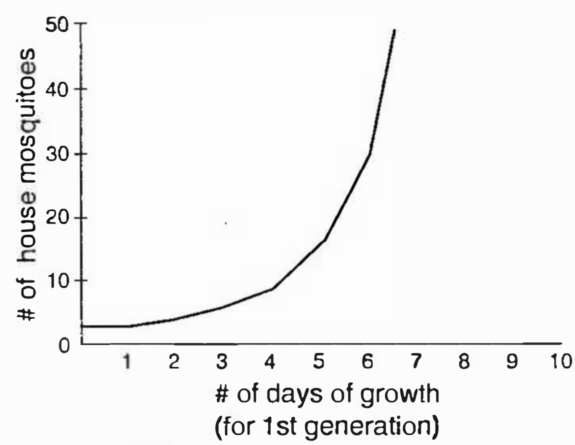
Graph 2

L_1 versus L_3



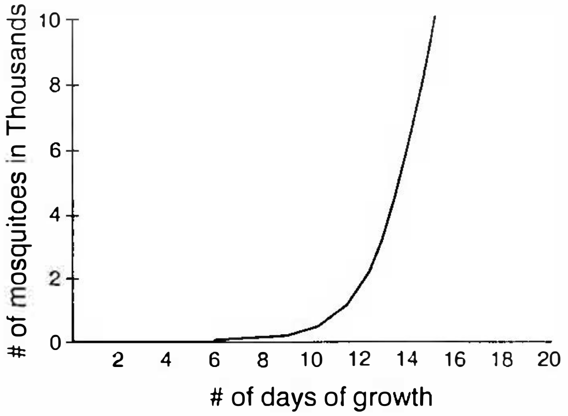
Graph 3

Growth of the Mosquito Population over Seven Days



Graph 4

Growth of the Mosquito Population over 20 Days



As this graph shows, after approximately 15 days, the population of mosquitoes has increased drastically from 2 to 10,000. Although this example assumes that no predators or diseases exist in the environment, it is still an excellent example of the power of exponential growth.

Acknowledgments

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The mathematician's best work is art, a high perfect art, as daring as the most secret dreams of imagination, clear and limpid. Mathematical genius and artistic genius touch one another.

Gösta Mittag-Leffler

Imagination is more important than knowledge.

Albert Einstein