Communication is an important process standard in school mathematics; hence, the mathematics curriculum emphasizes the continued development of language and symbolism to communicate mathematical ideas. Communication includes regular opportunities to discuss mathematical ideas and to explain strategies and solutions using words, mathematical symbols, diagrams and graphs. While all students need extensive experience in expressing mathematical ideas orally and in writing, some students may have the desire or should be encouraged by teachers to publish their work in journals.

delta-K *invites students to share their work with others beyond their classroom. Submissions could include papers on a particular mathematical topic, a mathematics project, an elegant solution to a mathematical problem, an interesting problem, an interesting discovery, a mathematical proof, a mathematical challenge, an alternative solution to a familiar problem, poetry about mathematics, a poster or anything deemed to be of mathematical interest.*

Teachers are encouraged to review students' work prior to submission. Please attach a dated statement that permission is granted to the Mathematics Council of the Alberta Teachers' Association to publish the work in delta-K. *The student author (or the parents if the student is under 18 years of age), must sign this statement, indicate the student's grade level and provide an address and telephone number.*

This issue's submissions are high school mathematics projects, completed as course requirements.

One Pure Mathematics 30 project was selected for publication. Shari Monner, a Grade 12 student at Holy Family CyberHigh School in High Prairie, submitted "Pure Mathematics 30 Student Project: Sunrise and Sunset." Her teacher is Sheryl Heikel.

One Applied Mathematics 30 project was selected for publication. Mark Fredrick, a Grade 12 student at Barrhead Composite High School in Barrhead, submitted "Applied Mathematics 30 Student Project: Medical Research: Huntington's Disease." His teacher is Leahan Schaffrick.

Pure Mathematics 30 Student Project: Sunrise and Sunset

Shari Monner

The topic to be investigated relates to the theme of sunrise and sunset.

An almanac on the Internet may be used to generate source data for times of sunrise and sunset for various dates equally dispersed throughout the year. To study the effect of latitude on sunrise and sunset, an additional set of data should be obtained for a second location, at a different latitude. For example, Edmonton is located at N53°36' W113°30' and Mexico City is located at N19°24' W99°12'.

Part A

1. **Use the data for sunrise and sunset times in Edmonton to create lists in a graphing calculator or spreadsheet program. Note that times are written in decimal form, so 18:24 is shown as 18.40 h.**

   L1, L2 and L3 for Edmonton are shown in Appendix A.

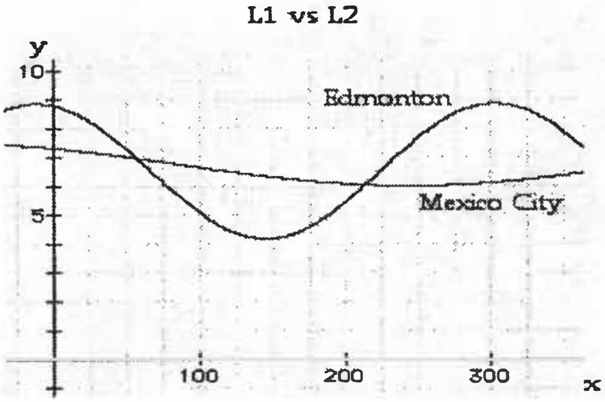
2. **Use the data in L2 and L3 to create data for a new list, L4, that shows the number of hours of daylight as a function of day number *n*. On your graphing calculator, graph: i. L1 versus L2, ii. L1 versus L3, iii. L1 versus L4. Sketch the graphs on the axes given.**

   L4 for Edmonton is shown in Appendix A and the graphs are shown below.

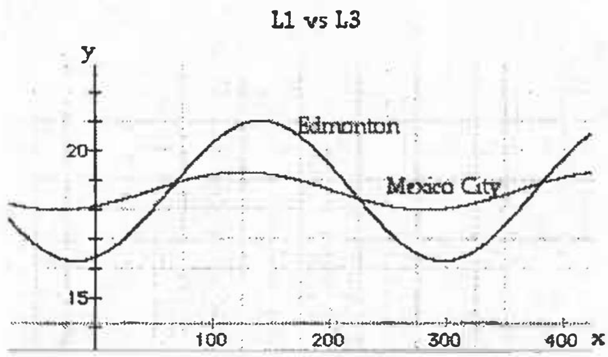
3. **Using the data given for Mexico City, repeat steps 1 and 2 above. Sketch the graphs on the same axes as above.**

L1, L2, L3, and L4 for Mexico City are shown in Appendix A and the graphs are shown below.

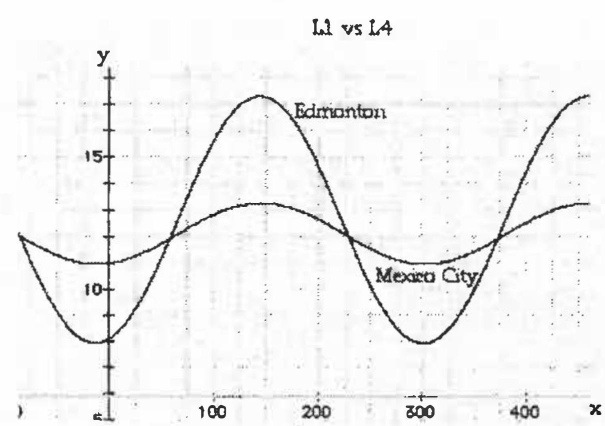
Day number versus time of sunrise



Day number versus time of sunset



Day number versus hours of sunlight



4. Find the equation and explain how a student could obtain a function of the form $f(n) = a \sin b\,(n - c) + d$, correct to four decimals to represent

i. the number of hours of sunlight as a function of the day number, n, for both locations

Edmonton equation:
$y = 4.6767 \sin(0.0168x - 1.3160) + 12.1638$
$f(n) = 4.6767 \sin 0.0168(n - 78.3333) + 12.1638$

Mexico equation:
$y = 1.1544 \sin(0.0169x - 1.3408) + 12.1184$
$f(n) = 1.1544 \sin 0.0169(n - 79.3373) + 12.1184$

ii. time of sunrise as a function of n for Edmonton

Edmonton sunrise equation:
$y = 2.3303 \sin(0.0165x + 1.8040) + 6.5145$
$f(n) = 2.3303 \sin 0.0165(n + 109.3333) + 6.5145$

iii. the time of sunset as a function of n for Mexico City

Mexico sunset equation:
$y = 0.6446 \sin(0.0157x - 0.9300) + 18.6255$
$f(n) = 0.6446 \sin 0.0157(n - 59.2357) + 18.6255$

$y = a \sin(bx - c) + d$ functions are calculated by regression* on a graphing calculator, $f(n) = a \sin b(n - c) + d$ is then calculated from $y = a \sin(bx - c) + d$ using $c = \frac{c_y}{b}$ to put the equation in the correct form.

*Regression steps used to determine equations are listed in Appendix B

5. For Mexico City, consider June 20 as the first day of summer and consider December 22 as the first day of winter. Using the data from these days, determine algebraically a sine equation in the form of $y = a \sin b(n + c) + d$, for sunlight hours. State all the features of this sinusoidal equation that can be inferred from the parameters of $y = a \sin b(n + c) + d$.

June 20		**December 22**	
day	171	day	356
rise	5.98	rise	7.10
set	19.28	set	18.07
hours	13.30	hours	10.97

maximum	13.3 at $x = 171$
minimum	10.97 at $x = 356$
period	365 (number of days in a year)
amplitude	$13.3 - 10.97 = 2.33$ $\frac{2.33}{2} = 1.165$
vertical displacement	$10.97 + 1.165 = 12.135$
horizontal displacement	$\frac{365 - 171}{2} = \frac{185}{2} = 92.5$ $171 - 92.5 = 78.5$

$y = a \sin b(n - c) + d$
$a = 1.165$
$b = \frac{2\pi}{365}$
$c = -78.5$
$d = 12.135$
$y = 1.165 \sin \frac{2\pi}{365} (n - 78.5) + 12.135$

The variable a in the above equation shows how varied the hours of sunlight are in Mexico City. The higher the value of a, the more variation there is during the year in hours of sunlight.

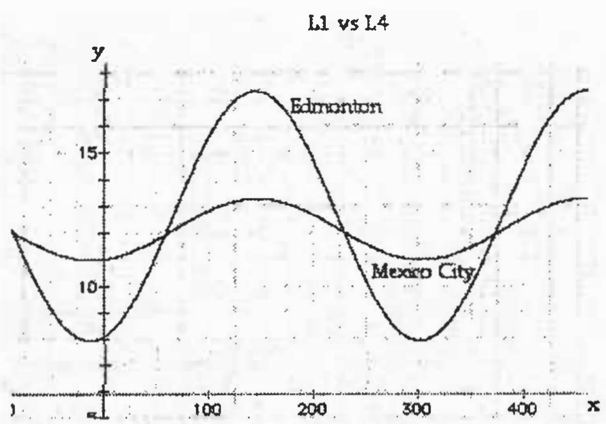
The variable b in the above equation is taken from the period, which is the number of days in a year. The cycle of hours of sunlight per day is repeated annually.

The variable c in the above equation shows the number of days from the beginning of the year to the first day of spring.

The variable d in the above equation is the average hours of sunlight per day in a year in Mexico City.

6. Explain how the latitude of a location affects hours of sunlight.

Hours of Sunlight Per Day



As you can see from the above graph, the latitude of a location drastically affects the variation in hours of sunlight per day. Mexico City has less variation than Edmonton in its hours of sunlight per day because Edmonton is much farther from the equator than Mexico City.

7. One factor that affects a region's growing season is hours of sunlight. If the growing season is made up of days with at least 15 hours of daylight, what are the approximate dates of the start and end of the growing season in Edmonton? Explain your method for determining these dates.

The start date of the growing season in Edmonton is April 28, and the end date is August 14.

Using the regression method shown in Appendix B, I found the equation for the sinusoidal function that corresponds with the data in a graph of day number versus hours of daylight in Edmonton. I graphed this function on the same grid as $y = 15$ and used the calculate–intersect function on my graphing calculator. When the sinusoidal function is above the line $y = 15$, the day numbers corresponding to the y coordinates are days in the growing season.

Part B

1. Determine the number of hours of sunlight to the nearest minute that exist in Edmonton on May 24 by doing the solution algebraically using the equation found in Part A, 4i.

$f(n) = 4.6767 \sin 0.0168(n - 78.3333) + 12.1638$

May 24 = Day 124 = n

$f(n) = 4.676 \sin 0.0168(124 - 78.3333) + 12.1638$

$f(124) = 15.41$

$0.41 \times 60 = 24.6$

On May 24, the number of hours of sunlight was 15:25.

2. Describe how you could check your solution to 1 above using a graphing calculator.

Graph the function, $f(n) = 4.6767 \sin 0.0168 (n - 78.3333) + 12.1638$, trace to $x = 124$ and record the value of y; this value should be equal to 15:25 in decimal form, which is 15.41.

3. What other day would have the same amount of sunlight hours as May 24?

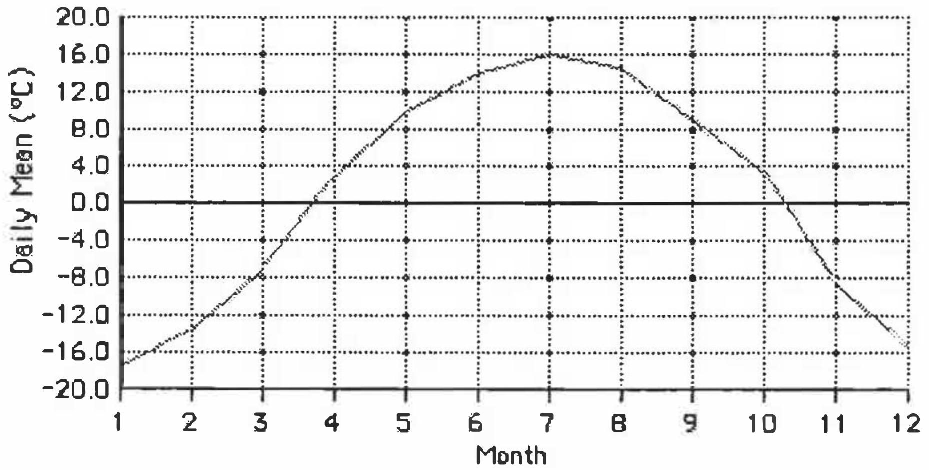
Either August 7 or 8 would have the same number of sunlight hours as May 24. August 7 had 15:27 hours and August 8 had 15:23 hours.

Part C

Many phenomena exhibit a periodic pattern. Locate a source of data other than sunrise and sunset times that appear to be periodic. (Some suggestions are tides, predator–prey populations and the distance of the moon from the Earth.)

1. Using this data, sketch a graph that represents the periodic relationship. See Appendix C for data information.

Daily Mean Temperature of Peace River, Alberta (1944–90)



2. Using an appropriate regression model, determine an equation that will represent the data.

Regression steps used are listed in Appendix B.

$y = 17.9901 \sin(0.4685x - 1.6847) - 1.3507$

3. Analyze the equation for its "fit" with the graph of the data.

It doesn't show the highs and lows appropriately because the amplitude is a bit too high, although the above equation is as close to every point on the graph as possible, the minimum temperature is a bit too low in comparison to the data. It is, however, close enough to give an idea of the likely average temperature in each month in years to come.

Summary

Sinusoidal functions can be used to predict many occurrences based on previously collected data. Sunrise and sunset times, fluctuations in these times and average temperature changes in the year all demonstrate the seasons, which occur year after year. This is yet another example of the predictability of natural occurrences: evidence of a creator.

Acknowledgments

Thanks to my teacher Sheryl Heikel for answering all my questions and providing me with appropriately scaled graphs. Thanks to my father George Monner for helping me find the raw data on temperature changes. Thanks to my classmate Mark Glass for his mathematical consultation.

Appendix A

Edmonton Data

L1	L2	L3	L4
1	8.85	16.42	7.57
6	8.82	16.52	7.70
11	8.77	16.63	7.86
16	8.70	16.77	8.07
21	8.60	16.92	8.32
26	8.48	17.07	8.59
31	8.35	17.23	8.88
36	8.22	17.40	9.18
41	8.05	17.57	9.52
46	7.88	17.73	9.85
51	7.72	17.90	10.18
56	7.53	18.07	10.54
61	7.33	18.22	10.89
66	7.13	18.38	11.25
71	6.93	18.53	11.60
76	6.73	18.68	11.95
81	6.53	18.85	12.32
86	6.33	19.00	12.67
91	6.13	19.15	13.02
96	5.93	19.30	13.37
101	5.73	19.45	13.72
106	5.53	19.62	14.09
111	5.35	19.77	14.42
116	5.17	19.92	14.75
121	4.98	20.07	15.09
126	4.83	20.22	15.39
131	4.67	20.37	15.70
136	4.53	20.50	15.97
141	4.40	20.63	16.23
146	4.30	20.75	16.45
151	4.20	20.87	16.67
156	4.13	20.95	16.82
161	4.08	21.03	16.95
166	4.07	21.08	17.01
171	4.07	21.12	17.05
176	4.08	21.13	17.05
181	4.13	21.12	16.99

L1	L2	L3	L4
186	4.20	21.08	16.88
191	4.28	21.02	16.74
196	4.38	20.93	16.55
201	4.50	20.83	16.33
206	4.63	20.70	16.07
211	4.77	20.57	15.80
216	4.90	20.42	15.52
221	5.03	20.25	15.22
226	5.18	20.08	14.90
231	5.33	19.90	14.57
236	5.48	19.72	14.24
241	5.62	19.52	13.90
246	5.77	19.33	13.56
251	5.92	19.12	13.20
256	6.07	18.92	12.85
261	6.20	18.72	12.52
266	6.35	18.52	12.17
271	6.50	18.30	11.80
276	6.65	18.10	11.45
281	6.80	17.90	11.10
286	6.95	17.72	10.77
291	7.10	17.52	10.42
296	7.27	17.33	10.06
301	7.42	17.15	9.73
306	7.58	16.98	9.40
311	7.75	16.83	9.08
316	7.90	16.70	8.80
321	8.05	16.57	8.52
326	8.20	16.45	8.25
331	8.35	16.37	8.02
336	8.48	16.30	7.82
341	8.60	16.25	7.65
346	8.68	16.23	7.55
351	8.77	16.23	7.46
356	8.82	16.27	7.45
361	8.85	16.32	7.47

L1 = Day number
L2 = Time of sunrise
L3 = Time of sunset
L4 = Hours of sunlight

L1 versus L2

Equation:
$y = 2.33 \sin(0.02x + 1.81) + 6.52$

L1 versus L3

Equation:
$y = 2.38 \sin(0.02x - 1.24) + 18.63$

L1 versus L4
Equation:
$y = 4.68 \sin(0.02x - 1.31) + 12.16$

Mexico City Data

L1	L2	L3	L4
1	7.18	18.17	10.99
6	7.20	18.21	11.01
11	7.22	18.27	11.05
16	7.23	18.33	11.10
21	7.23	18.38	11.15
26	7.22	18.43	11.21
31	7.20	18.48	11.28
36	7.17	18.53	11.36
41	7.13	18.57	11.44
46	7.10	18.62	11.52
51	7.05	18.65	11.60
56	6.98	18.68	11.70
61	6.93	18.70	11.77
66	6.87	18.73	11.86
71	6.80	18.77	11.97
76	6.73	18.78	12.05
81	6.65	18.80	12.15
86	6.58	18.83	12.25
91	6.52	18.85	12.33
96	6.45	18.87	12.42
101	6.38	18.88	12.50
106	6.32	18.92	12.60
111	6.25	18.93	12.68
116	6.20	18.97	12.77
121	6.15	18.98	12.83
126	6.10	19.02	12.92
131	6.05	19.05	13.00
136	6.02	19.08	13.06
141	6.00	19.12	13.12
146	5.98	19.15	13.17
151	5.97	19.18	13.21
156	5.97	19.22	13.25
161	5.97	19.25	13.28
166	5.97	19.27	13.30
171	5.98	19.28	13.30
176	6.00	19.30	13.30
181	6.03	19.32	13.29

L1	L2	L3	L4
186	6.05	19.32	13.27
191	6.08	19.32	13.24
196	6.12	19.30	13.18
201	6.15	19.28	13.13
206	6.18	19.27	13.09
211	6.20	19.23	13.03
216	6.23	19.18	12.95
221	6.27	19.15	12.88
226	6.28	19.10	12.82
231	6.32	19.03	12.71
236	6.33	18.97	12.64
241	6.35	18.90	12.55
246	6.37	18.83	12.46
251	6.38	18.77	12.39
256	6.40	18.68	12.28
261	6.42	18.62	12.20
266	6.43	18.53	12.10
271	6.45	18.47	12.02
276	6.47	18.40	11.93
281	6.48	18.32	11.84
286	6.52	18.25	11.73
291	6.53	18.20	11.67
296	6.57	18.13	11.56
301	6.60	18.08	11.48
306	6.63	18.05	11.42
311	6.67	18.00	11.33
316	6.72	17.98	11.26
321	6.77	17.97	11.20
326	6.82	17.95	11.13
331	6.87	17.95	11.08
336	6.92	17.95	11.03
341	6.97	17.97	11.00
346	7.02	18.00	10.98
351	7.07	18.03	10.96
356	7.10	18.07	10.97
361	7.15	18.12	10.97

L1 = Day number
L2 = Time of sunrise
L3 = Time of sunset
L4 = Hours of sunlight

L1 versus L2

Equation:
$y = 0.74 \sin(0.01x + 2.27) + 6.75$

L1 versus L3

Equation:
$y = 0.64 \sin(0.02x - 0.93) + 18.63$

L1 versus L4

Equation:
$y = 1.15 \sin(0.02x - 1.34) + 12.12$

Appendix B

Regression steps: (Key Strokes in Bold)

1. STAT → 1: Edit (Under EDIT menu) → Enter Lists 1–4
2. STAT → [Right Arrow] to CALC menu → [Up Arrow] to C: SinReg → ENTER → 2nd → 1 (L1) →, → 2nd → 2 (L2) (or 3 for L3 and so on . . .) → ENTER
3. Y= → [Up Arrow] (to Plot1*) → ENTER → [Down Arrow] (to Y1=) → VARS → 5: Statistics. . . → [Right Arrow] (to EQ menu) → 1: RegEQ → GRAPH

*Plot 1 is preset to show L1 versus L2, Plot 2 to show L1 versus L3 and Plot 3 to show L1 versus L4

Appendix C

Temperature Data Acquired from Environment Canada's website www.cmc.ec.gc.ca/climate/normals/ALTAP005.HTM.

Month	Daily Mean (C)
1	–17.5
2	–13.3
3	–7.2
4	3.0
5	9.9
6	14.1
7	15.9
8	14.6
9	9.2
10	3.4
11	-8.5
12	–15.2

Heron of Alexandria (c. 50?)

There were four water fountains. The first fountain filled the cistern in one day, the second filled the cistern in two days, the third in three days and the fourth in four days. How long does it take for all four water fountains to fill the cistern together?