Communicating in the Language of Mathematics

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As the classroom mathematics curriculum expands to encompass the entire range of skills included in the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989), the process by which a student arrives at the answer to a problem becomes as important as the answer itself. Answers alone often fail to reveal the nature of a student's thinking, the strategies used in the problem-solving process or the level of understanding. Additionally, the standards document includes the expectation that students will be able to "relate their everyday language to mathematical language and symbols" (NCTM 1989, 26).

Using oral or written communication as a tool with which students can reflect their understanding of mathematics helps them make connections and personalize mathematical concepts. When students communicate mathematical information, they remember it, understand it, and use it to uncover and find even more information (Perkins 1992).

Teachers need to know how to help students grow into accomplished communicators of mathematics who can describe their thinking processes clearly. Teachers must help students make their thinking visible to others by encouraging them to talk and write about the process they use to solve problems.

The author's past efforts to encourage students to discuss and explain their problem-solving process have focused on (1) journal writing, (2) student-authored story problems, (3) the mathematician's chair, (4) cooperative-learning activities, and (5) parent newsletters. However, two articles in the May 1992 issue of *Educational Leadership*—"Creating Tests Worth Taking" by Grant Wiggins and "Evaluating Problem Solving in Mathematics" by Walter Szetela and Cynthia Nicol—present more ideas to add to the existing student-communication activities occurring in a second-grade classroom. The goal of embedding speaking and writing into the daily mathematical activities of students is being met through the following activities.

Mathematics Journal

Ask students to keep a mathematics journal, which can constitute a major part of the daily curriculum but, when added to a student's portfolio, can also furnish an ongoing record of the student's mathematical growth.

In our classroom, students begin each day by recording statistical data related to the date, weather and various problem-solving activities (Figure 1): (1) day, date, number of school days attended in the current school year and number of school days remaining; (2) at least five numbered sentences that equal the date; (3) the weather report—temperature, precipitation, wind speed, wind direction and cloud type; (4) predictions for the next day's weather and the color, size and shape of the next day's calendar piece; (5) answers to various measurement activities, such as the time shown on a Judy clock, the amount of money in a container, the weight of an object or group of objects, or an estimation of the quantity, weight or length of an object using a standard for comparison; and (6) the solution to a open-ended mathematics problem.

At the end of each day, students use their journal to reflect on the day's mathematical activities. Students are asked to think about how they would answer such questions as "How did you help another person?" and "What did you learn that you did not know before?"

Student-Authored Story Problems

Ask students to create original story problems for someone else—a classmate, a teacher, a student in another classroom or a family member—to solve.

The directions to the student include the following:

- Write a story problem using you imagination or the information in a picture, newspaper advertisement, poster or short story.
- Have other people solve your problem.
- After seeing the solutions to your problem, lead a class discussion about your problem and the solutions.

Mathematician's Chair

Ask students to sit in a chair that has been designated the "mathematician's chair" and to share original problems that they have authored or solutions to a problem written by someone else.

A mathematician's chair is very similar to an author's chair, except that students share mathematics problems and solutions rather than stories or books. Expect students, while in the mathematician's chair, to use effective speaking skills and to communicate their thoughts clearly and completely. Also, expect classmates to use effective listening skills and to give the author useful and usable feedback:

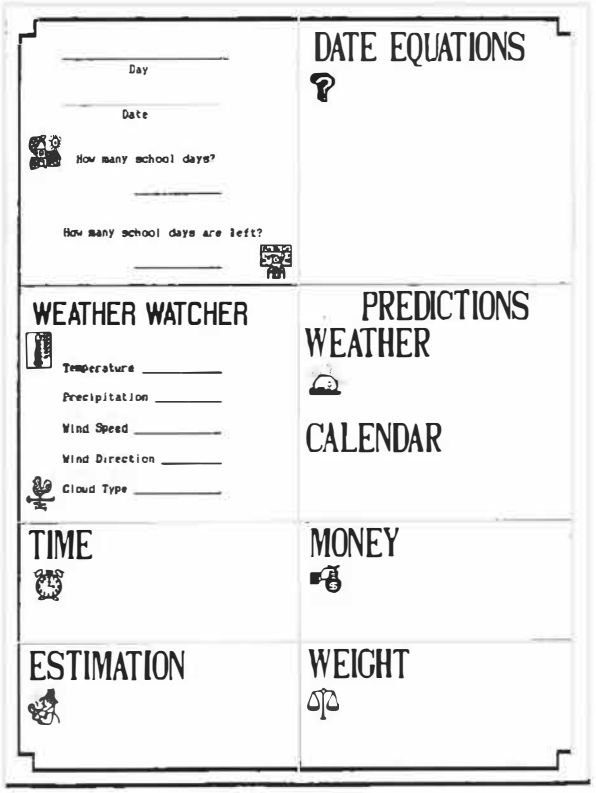
- What did you think about the problem?
- Do you agree or disagree with the solution?
- How could the author improve the problem or solution?
- How could the author change the problem to create a new problem or change the solution to arrive at a new way to solve the problem?

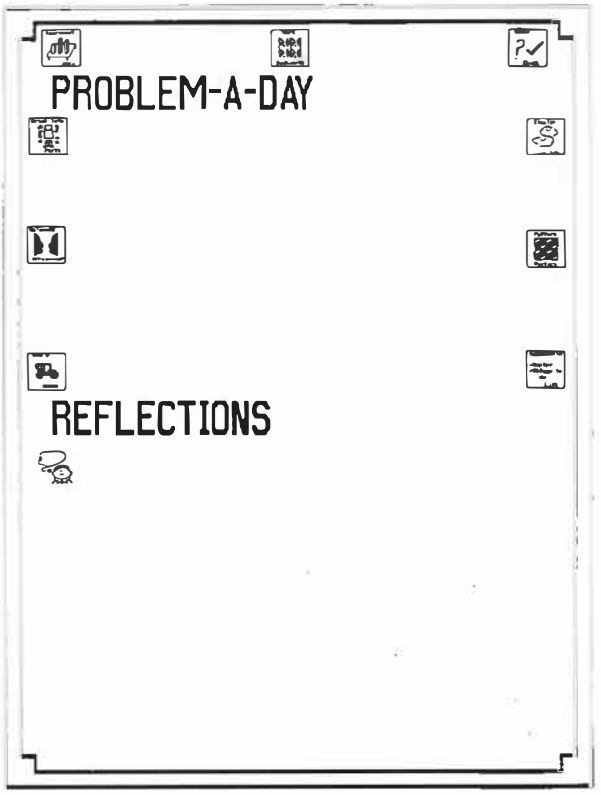
Cooperative Learning

Have students engage in cooperative problem solving by asking them to describe the process that they will use to solve a problem, to work collaboratively on the problem, and to reflect on the effectiveness of the group and the contributions of individual members.

Talking with peers in cooperative-learning groups is especially important for young children. Students become comfortable with new words when they are free to experiment with language in a nonthreatening environment. To communicate their thinking to others more effectively, students must have frequent opportunities to hear and speak mathematics with peers, teachers and parents.

Figure 1
Mathematics journal





Family Newsletter

Ask students to write weekly or monthly "Family Newsletters."

Give the following directions to the students:

- Pretend you are a reporter for a newspaper. Write a story about something that occurred in mathematics class since the last "Family Newsletter." Use your journal and portfolio to help write the story.
- Ask at least two other students to listen to the story so that they can offer ideas for improvement.

Mathematics Communication Structures

"Mathematics communication structures" were created to add variety to students' communication tasks. Each structure listed subsequently was designed to give students a framework that supports and enhances the process of mathematical communication.

Structure 1

Present a problem and the answer arrived at by an imaginary person. Have the student write a letter to this person, explaining agreement or disagreement with the answer.

- Directions to the student: (1) Write a letter to the person who solved this problem. (2) Explain why you agree or disagree with the answer.
- Sample problem: Which number does not belong? Kristina thinks the answer is 6.

6	12
10	13

Structure 2

Present an already solved problem with a significant error. Have the student comment on the error by reacting to a series of questions about the solution.

- Directions to the student: (1) Read the problem and look at how this person solved the problem. (2) Answer each question that follows the problem.
- Sample problem: Chris and Bob have to be home by 9 p.m. It is now 7 p.m. How many hours may they play before they have to go home? Travis solved the problem this way.

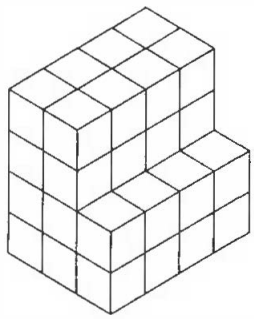
$$\begin{array}{r} 9 \\ +\,7 \\ \hline 16 \end{array}$$ I say 16 hours.

Explain whether Travis's reasoning was correct or incorrect.

Structure 3

Present a problem with all the facts and conditions, but have students write a different question for the problem. Have the students solve the new problem and explain why their new question made the problem more or less difficult to solve.

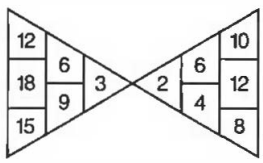
- Directions to the student: (1) Solve the problem as stated. (2) Write a different question but do not change any other part of the problem situation. (3) Solve the new problem and explain why this problem was easier or harder to solve than the original.
- Sample problem: This shape is made by stacking cubes on top of one another. How many cubes are in the bottom layer?



Structure 4

Present a problem and a partial solution. Have the students complete the solution.

- Directions to the student: (1) Finish the solution to this problem. (2) Describe another way to solve the problem.
- Sample problem: List all the ways that you could score 18 points by throwing two darts. Here is Aaron's partial solution:
  One dart hit the 3 and the other dart hit the 15.



Structure 5

Present a problem with facts unrelated to the question. Have the students identify these facts and rewrite the problem, leaving out any irrelevant information.

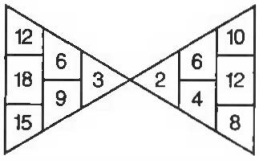
- Directions to the student: (1) Which facts are not needed to solve the problem? (2) Rewrite the problem, leaving out any unneeded information. (3) Solve the problem.

- Sample problem: Farmer Brown has 10 cows and 23 horses. Farmer Smith has 16 cows and 17 horses. If Farmer Brown and Farmer Smith put all their cows in the same barn, how many cows would be in the barn? (Written by Melissa Santoyo, Jefferson Elementary School.)

Structure 6

Present a problem and have the students explain how to solve the problem using only words. Have the students construct and solve a similar problem.

- Directions to the student: (1) Using only words, tell how you would solve this problem. (2) Write a similar problem and describe all the ways that the two problems are alike and different. (3) Solve the problem you wrote.
- Sample problem: Jill threw three darts at the target shown. What is the largest score she could make?



Structure 7

After the students have solved a problem, have them write a new problem with a different context, preserving the original problem structure.

- Directions to the student: (1) Solve the problem. (2) Tell how you solved the problem. (3) Write a new problem that can be solved in the same way. Give your problem to another person and check the solution.
- Sample problem: Tim and Bill are going camping for three days. The guidebook says that four campers need six litres of water for each day. How much water do you think that Tim and Bill should take on their camping trip? Explain your answer.

Structure 8

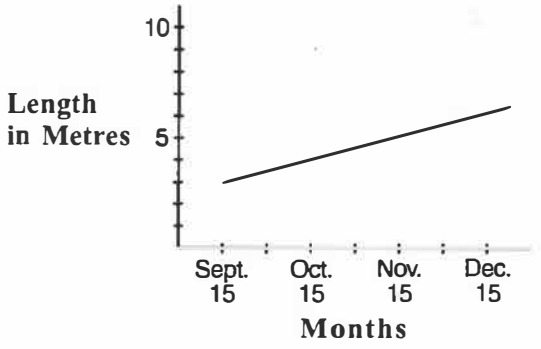
Present a problem without numerals. Have the students estimate the missing numbers, research appropriate numerals and solve the problem. The problem should be based on a real-world situation—the missing information should be available to the student by gathering these data.

- Directions to the student: (1) Estimate the answer. (2) Complete the problem using the data you have gathered. (3) Solve the problem and tell how you found the answer.
- Sample problem: Mr. Buschman drives_____ miles to work each day. How many miles does he drive to work during October?

Structure 9

Present the students with a graph or table and have them write a story that represents the data on the graph or table.

- Directions to the student: Write a story about the data in this graph or table.
- Sample problem: Ben's shadow at the same time each time.



Structure 10

Present the students with trends or sample data. Ask them to make predictions to complete the table and write a story that includes the data in the table.

- Directions to the student: (1) Look carefully at the data. (2) Make predictions using the data and fill in the missing information in the table. (3) Explain why you think your predictions are accurate. (4) Write a story that includes the data in the table.
- Sample problem: Rainfall amounts measured in inches.

	Jan.	Feb.	March	April	May	June
Portland	6.4	6.8		3.4		1.0
Jefferson	5.9	6.2	4.4	3.1		
Baker	4.3		1.7	0.5		0.0
Astoria	10.6	11.3			7.5	4.3

Structure 11

Present the student with a real classroom problem that requires a group of students to share actual objects in the classroom or on the playground. Have the student develop and test a plan for sharing the objects.

- Directions to the student: (1) Make a plan for sharing the objects in the problem. (2) Test your plan to see how it works. (3) Have another student tell why he or she thinks your plan is fair or unfair. (4) Write your teacher a letter in which

you describe your plan and tell how well your plan worked.

- Sample problem: You have 10 students in your group, but your group only has two bags of pattern blocks. You will get to use the pattern blocks for 20 minutes each day this week. Only one student can use a bag of pattern blocks at a time. Develop a plan for how your group can share the two bags of pattern blocks.

Structure 12

Ask the students to write and publish an original story problem in the form of a "letter problem" (Figure 2). Once completed, letter problems are placed in classrooms throughout the school for other students to solve. Solutions are mailed to the problem's author through the in-school mail-delivery system.

- Directions to the student authoring a letter problem: (1) Write five original story problems. Meet with two other students and use consensus building to choose the best problem. (2) Edit and publish your problem. (3) Place several copies of your problem in chosen classrooms. (4) Read all solutions and reply to each person, telling why you agree or disagree with the solution.
- Directions to the student solving a letter problem: (1) Take one letter problem from the display. Solve the problem, write in detail how you found the answer and tell why you think your answer is both correct and complete. (2) Mail your solution to the author of the problem through the in-school mail-delivery system. You will receive a reply in a few days.

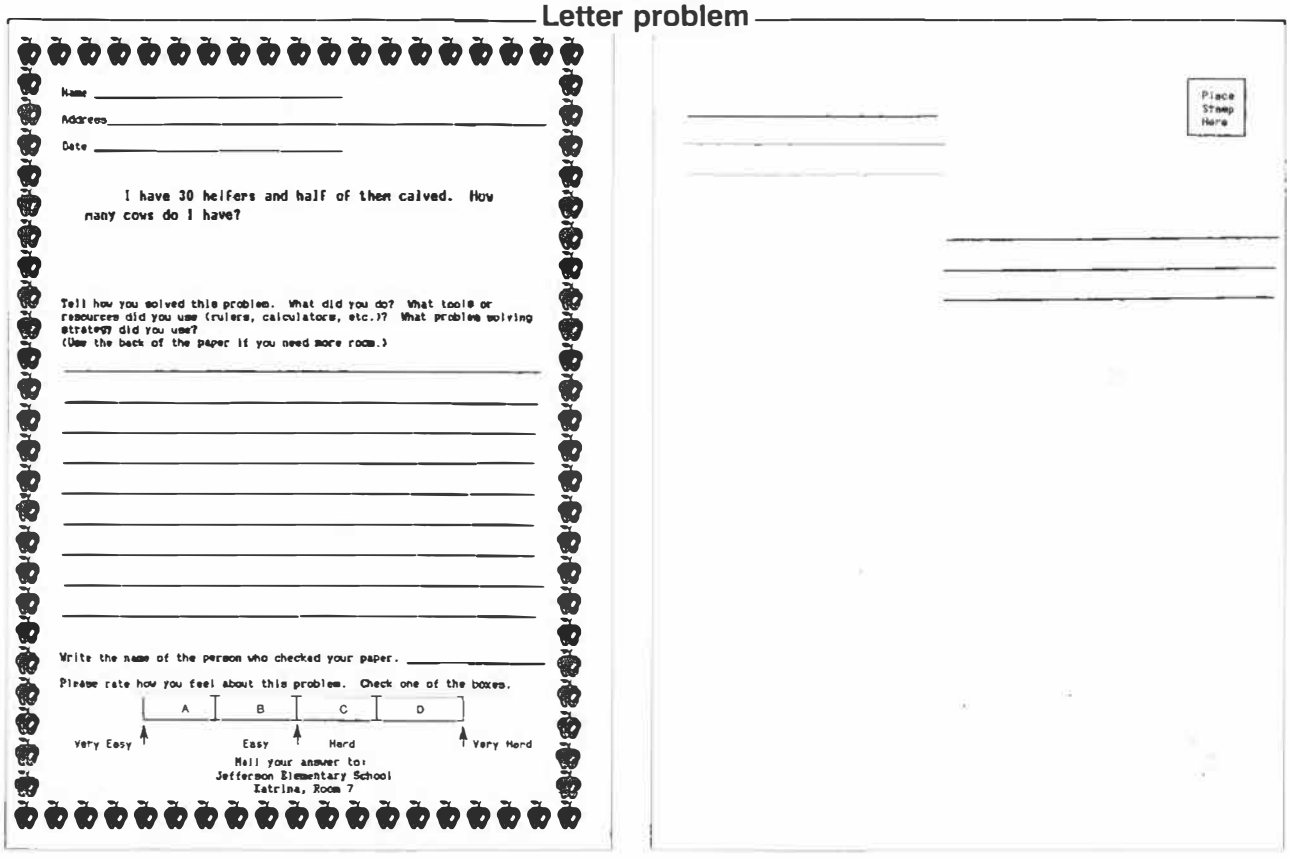
Structure 13

Present the students with a very open-ended problem and have them request the information needed to solve the problem.

- Directions to the student: (1) Request any information you need to answer the question. (2) When you think you have enough information, solve the problem. (3) If you find that you need more information, request help from your teacher.
- Sample problem: How much will it cost for the second-grade field trip? When introducing this type of problem to students, begin with a whole class activity using oral requests and responses

Figure 2

Letter problem



which are recorded and displayed on the overhead projector. As students become comfortable with making requests, ask them to work problems individually or in small groups and to make their requests in writing. By varying the type of information given to students, teachers can control the level of difficulty of the problem. For example, if a student asked, "How much does the bus for the field trip cost?" one of the following replies could be given: $150.00; each bus costs $75.00; each bus costs $55.00 for the driver and $20.00 for gasoline; or each bus costs $8.35 an hour for the driver and $0.05 a mile for the gasoline.

Structure 14

Ask the students to revise a fairy tale or folk tale to include numerical information. This new version of the tale can then be used as a source for generating story problems.

- Directions to the student: (1) Pick a favorite fairy tale or folk tale. Read the tale you have chosen to an adult. (2) Rewrite the tale by adding numerical information. (3) Write five story questions that someone else could answer using the added information.
- Sample tale: "The Five Bears" by Andrea Kachel, Grade 2, Jefferson Elementary School.

  Once upon a time there were five bears. The Papa bear was the oldest. He was 39 years old. Next came the Mama bear. She was 35 years old. Next came their son Andrew. He was 18 years old. Next came Andrea. She was 9 years old. Last came Jessica. She was 3 years old and a real brat. Every day Mama bear would leave for work at 7:00 in the morning and she would get home at 5:00 in the afternoon. The children left for school at 8:00 in the morning and came home on the hot, noisy, bumpy bus at 3:30. Papa bear took baby Jessica to the day care at 9:00 in the morning on his way to work in the honey factory.

Conclusion

Students need time to observe, to work together and to construct an understanding of the language of mathematics and to make it their own. Personal knowledge becomes useful and usable in social situations when combined with the knowledge of others. Thoughts, ideas and the meanings of words are focused and clarified when individuals engage in conversation.

As soon as students use words, they make their understanding of mathematics more precise and more general at the same time. Only by using words in many situations and contexts do students come to understand the full meaning of each word. When students write or talk about mathematics problems, they test, expand and extend their understanding of mathematics. When students write or speak, they do not use language just to express their thoughts; they use the process of communicating with others to engage in a conversation with their own mind.

References

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: Author, 1989.

Perkins, D. *Smart Schools: From Training Memories to Educating Minds*. New York: The Free Press, 1992.

Szetela, W., and C. Nicol. "Evaluating Problem Solving in Mathematics." *Educational Leadership* 49 (May 1992): 42–45.

Wiggins, G. "Creating Tests Worth Taking." *Educational Leadership* 49 (May 1992): 26–33.

Bibliography

Andrini, B., and S. Kagan. *Cooperative Learning and Mathematics*. San Juan Capistrano, Calif.: Resources for Teachers, 1989.

Cambourne, B. *The Whole Story: Natural Learning and the Acquisition of Literacy in the Classroom*. New York: Ashton Scholastic, 1988.

Goodman, K. *What's Whole in Whole Language?* Portsmouth, N.H.: Heinemann, 1986.

Johnson, T., and D. Louis. *Bringing It All Together: A Program for Literacy*. Portsmouth, N.H.: Heinemann, 1990.

Routman, R. *Invitations: Changing as Teachers and Learners K–12*. Portsmouth, N.H.: Heinemann, 1991.

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