Using Communication to Develop Students' Mathematical Literacy

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When students are given the opportunity to communicate about mathematics, they engage thinking skills and processes that are crucial in developing mathematical literacy. The importance of communication is evidenced through NCTM's recognition of this skill as one of the five process standards in mathematics, in both the 1989 and 2000 *Standards* documents (NCTM 1989, 2000). Students who are supported in their "speaking, writing, reading and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (NCTM 2000, 60). Communication, then, should be a fundamental component in implementing a balanced and effective mathematics program.

Consider a classroom episode in which students solve a version of a common problem, use a rubric (see Figure 1) to rate the written response of a peer and discuss their rationale for the score. Students were asked to describe their process for finding the dimensions of a pool having a perimeter of 18 and an area of 18 square meters. Several students drew diagrams showing how their use of square tiles led to the solution.

Figure 1

General rubric for open-ended responses

0 Answer is unresponsive, unrelated or inappropriate. Nothing is correct.

1 Answer addresses item but is only partially correct; something is correct related to the question.

2 Answer deals correctly with most aspects of the question, but something is missing. Answer may deal with all aspects but have minor errors.

3 All parts of the question are answered accurately and completely. All directions are followed.

Source: Adapted from the *1997–98 North Carolina Open-Ended Assessment, Grade 8*

The students' responses verified that they recognized the need to meet the two constraints of the problem. Hillary wrote, "You can figure this out by first solving for perimeter and saying to yourself—what times 2 plus what times 2 equals 18? After you have an idea of the dimensions, you can plug it into an area formula." Virginia organized her thinking differently and constructed a table listing such dimensions as 6 and 3, 2 and 7, 5 and 4, 8 and 1. For each dimension, she recorded the perimeter and area, concluding that "my study shows that the correct answer is 3 and 6." Lila used a set of 18 color tiles to explore the problem. She moved them around on her desk to form rectangles with a variety of dimensions. Later in her journal, Lila wrote that she "struggled with many different ways" before she decided to "fix the 18 area squares to get 18 around." These multiple methods demonstrate the richness of written communication as a means of helping students organize and consolidate their mathematical thinking.

In this episode, students discussed solutions and detailed decisions for rating their peers' papers. The teacher, Mrs. Weatherman, frequently directed students to give information about the strengths and weaknesses of the responses. One student challenged his partner, "You explained the perimeter very well, but nearly forgot the area. You need something in the problem about how to find area." Another student posed questions to get his partner to consider how her response could be more coherent and clear. "You covered all criteria but could use more detail. Why does it work? Why didn't you choose any other numbers? What factors don't work? Have you tried them all?" The oral dialogue that extended the written work gave students a forum for examining not only their mathematical skills but their ability to express their reasoning with details sufficient to convey the validity of their approaches.

An Overview of the Communication Standard

The previous discussion of a well-known problem shows how communication-rich environments, as described in NCTM's *Standards* documents, encourage students to reflect on their own thinking and gain insights as ideas become explicit through written and oral communication. Each of the five process standards in *Principles and Standards for School Mathematics* (NCTM 2000) presents the major areas of student competence that are important in sustaining and promoting students' mathematical growth. Whereas the earlier *Standards* document presented a differentiated list of skills for students at each grade-level division, the 2000 version, *Principles and Standards*, outlines coherent focus areas that highlight the skills that students from pre-Kindergarten through Grade 12 should have. The grade-band chapters then discuss the attributes of communication at the various levels and suggest how teachers can support communication. The Communication Standard for Grades pre-K–12 (NCTM 2000, 63) stresses that mathematics instructional programs should enable students to

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Expectations at the various grade bands differ primarily in the level of complexity and abstractness of students' communication. In addition, the norms for evaluating students' thinking become more stringent at ascending levels. Beyond presenting and explaining their problem-solving strategies, middle school students should be able to "analyze, compare and contrast the meaningfulness, efficiency, and elegance of a variety of strategies" (NCTM 2000, 268). Another distinguishing feature of communication at the middle school level involves classroom social norms. Adolescents are self-conscious and may be hesitant to expose their thinking to others; therefore, teachers should establish a classroom community that makes students feel free to share their thoughts without fear of ridicule.

Principles and Standards also includes a rich discussion of the teacher's role in promoting the development of the process standards. One fundamental role for the teacher in promoting communication is to create a classroom environment of mutual trust and respect in which students can critique mathematical thinking without personally criticizing their peers. This community atmosphere requires the teacher to be an active facilitator, guiding students as they explore mathematics together. The teacher monitors and facilitates discussions and directs students' conversations so that important learning objectives are met. Second, the teacher selects engaging tasks that require the students to think and reason about important mathematical ideas and concepts. The tasks might have more than one method of solution and use multiple representations. The tasks should further require that students justify, conjecture, interpret and correlate important mathematical ideas. The teacher also has an important role in guaranteeing that all students have opportunities to contribute at some level. Further, the teacher's feedback, about not only the mathematical content and ideas but also the quality of the communication, encourages classroom communities in which communication becomes a tool for thoughtful inquiry.

Illustrating the Four Focus Areas for Communication

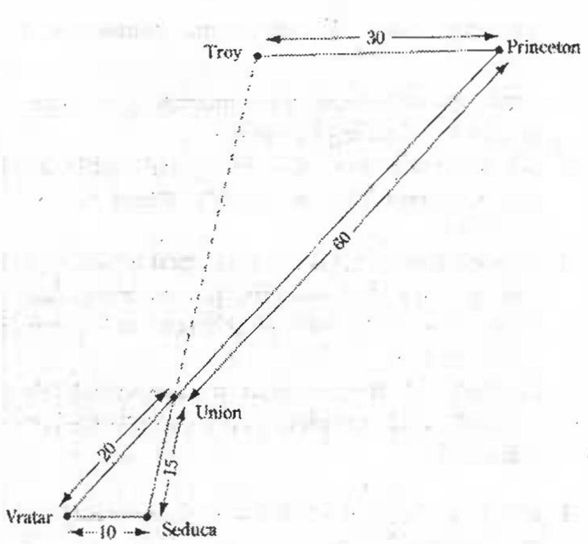
Let's revisit the four focus areas for the communication standard by continuing our illustration of how Mrs. Weatherman's classes worked to extend their mathematical communication. Since state and national tests focus on writing, one particular interest area involved helping students extend their written communication skills through both writing about their solutions and critiquing one another's work in class discussions. The problem in Figure 2 presented students with rich opportunities to develop their abilities to communicate mathematically.

This problem, like the pool problem described previously, required students to *organize and consolidate their mathematical thinking through communication*. Madison began her solution by stating that "this problem is solved by viewing the roads as two similar triangles," Hillary wrote, "I came up with this conclusion because the triangles are similar; therefore, each individual side is proportional." Both of these students then used proportions to solve for the missing side (that is, the distance from Troy to Union). Writing descriptions of mathematical processes encouraged students to reflect on their thinking.

Through sharing their mathematical thinking, students were asked to *communicate their mathematical thinking coherently and clearly to peers, teachers and others.* As part of the focus on communication, the teacher presented papers that had earned the highest rating, a 3, either from the teacher or in peer assessments, as models of good written communication. These papers were the impetus that allowed the class to engage in discussions about the approaches used in the problems. The comments written by the teacher on papers were intended to extend the students' abilities to communicate effectively. Sometimes the teacher simply wrote, "Where could you strengthen your description?" At other times, the papers contained more explicit comments, such as, "I see some computations in the middle of the paper. Could you elaborate on what you were doing and how that information was used in solving the problem?" and "You have included a diagram on your paper. Could you go back and incorporate that diagram into your discussion so that it is clear how the diagram supports your line of reasoning?" The clarity and cohesion of writing are evident in one student's work on the Road Problem: "I came to the conclusion that 45/15 is equal to 30/10. This problem could also be stated (Union–Troy)/(Union–Seduca) = (Troy–Princeton)/(Vratar–Seduca.)"

Figure 2
The Road Problem

The Department of Transportation for Montrose County is interested in building a four-lane highway to connect Troy to Union. The distance from Princeton to Union is 60 miles and from Vratar to Union is 20 miles. One other road between Union and Seduca is 15 miles long. The distance from Troy to Princeton is 30 miles and from Vratar to Seduca is 10 miles. Find the distance of the proposed highway from Troy to Union. Write a description of how you solved the problem.



Communication provides opportunities for students to *analyze and evaluate their mathematical thinking and strategies of others.* Although many students stated that the Road Problem presented similar triangles, students generally did not present any justification for their thinking. For example, Mallory "rotate[d] the big triangle using the Union axis 180° counter clockwise." She drew a diagram of the smaller triangle, now embedded in the larger one, with Union as the common vertex. Classroom discussion revolved around students' using properties of similar triangles to find their solutions but failing to state why the triangles were similar. At best, the students should have indicated that their assumptions of similarity provided a model for finding a "good" estimate instead of the exact distance from Union to Troy. Yet students were quick to assert that "these triangles are similar in shape but different in size." The comments on the students' papers, including the ones selected for a follow-up discussion, focused on this assumption. Through communication, students' assumptions were analyzed and the importance of justifying their mathematical reasoning was reaffirmed.

Students were able to *analyze and evaluate the mathematical thinking and strategies of others.* Using communication, students extended their use of the language of mathematics. Jonathan wrote, "The side from Troy to Union is unknown, so it is *x* (or any other variable). You must then match corresponding sides." Amanda justified her calculations by writing, "I know to solve proportions you take the cross products." Communication aided students in thinking about how to express mathematical ideas as those concepts became more visible. Amy summarized the importance of communication when she wrote, "Sometimes in math we just do, we don't even stop to ask 'why,' but on these papers you have to know why. So now, we know why we do things." In this example, reading the detailed responses of a peer and engaging in discussions of the ratings gave students occasions to consider other approaches to the problem. This process enabled students to consider, evaluate and build on the thinking of others. This complex interaction involved listening and talking and enabled students to develop facility with mathematical concepts by examining the methods and ideas that others used to determine relative strengths and limitations in those approaches.

Conclusion

Giving students opportunities to develop skills in communicating mathematically should be a natural outgrowth of a well-balanced mathematics program. As a result, students will become comfortable in expressing to others the results of their thinking in both written and oral form. Middle school students must also build skills in evaluating the thinking of others in a mathematical community. This foundation is essential at the secondary level, where students can further develop the ability "to structure logical chains of thought, express themselves coherently and clearly, listen to the ideas of others, and think about their audience when they write or speak" (NCTM 2000, 348–49). Providing such experiences is pivotal in developing communication processes that promote mathematical literacy for all students.

References

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: Author, 1989.

———. *Principles and Standards for School Mathematics.* Reston, Va.: Author, 2000.

North Carolina Department of Public Instruction. *1997–98 North Carolina Open-Ended Assessment, Grade 8.* Raleigh, N.C.: 1997–98.

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Two regular polygons are given. There are twice as many sides in the second polygon as there are in the first. The measure of the interior angles of the first polygon is 10° smaller than the measure of the interior angles of the second polygon. What is the number of sides and the measure of the interior angles of the two regular polygons?