NCTM Standards in Action
The Measurement Standard

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An underlying view of mathematics education expressed in the National Council of Teachers of Mathematics (NCTM 2000) *Principles and Standards for School Mathematics* is that students should be actively involved both mentally and physically in constructing their own mathematical knowledge.

Principles and Standards for School Mathematics identifies 10 standards that form an essential and comprehensive foundation for students from Kindergarten to Grade 12 as they learn mathematics. The standards are divided into five content standards (number and operations, algebra, geometry, measurement, and data analysis and probability) and five process standards (problem solving, reasoning and proof, communication, connections and representation). This article focuses on one content standard: measurement.

The Alberta program of studies for K–12 mathematics contains a measurement strand that emphasizes describing and comparing everyday phenomena using direct or indirect measurement. At the Kindergarten level, the general outcome focuses on demonstrating awareness of measurement, whereas the general outcomes at the elementary level include estimating, measuring and comparing in different units of measure, and solving problems using measurement concepts and various tools. At the secondary level, the measurement strand includes problem solving involving properties of circles and their connections to angles and time zones. At the start of the secondary level, students are introduced to indirect-measurement procedures and use of trigonometric ratios to solve problems involving right triangles. In the final years of the measurement strand, the general outcomes include demonstrating and understanding scale factors and their interrelationship with dimensions of similar shapes and objects. Furthermore, students are exposed to problems involving triangles (including three- and two-dimensional applications); the use of measuring devices to perform calculations in solving problems and analyzing objects and shapes; and processes to solve cost-design problems, coordinate-geometry problems, polygons and vectors, and conic-section problems.

The measurement standard is of central importance to the teaching and learning of mathematics in Alberta. It helps students to see that mathematics is useful in everyday life, and it is the basis for developing many mathematical concepts and skills. The measurement strand also leads students to naturally recognize the need for learning about fractions and decimals. Because measurement is a building block in the learning of mathematics, students must understand the attributes to be measured as well as what it means to measure. For such understanding to occur, students must first experience a variety of activities that focus on comparing objects directly, covering them with various units and counting the units. In short, students must have a variety of qualitative-measurement experiences. Teachers must guard against the premature introduction and use of instruments and formulas when teaching students the measurement strand. The qualitative-measurement experiences should include awareness of the size of units as well as situations that require estimates. Students should become aware that measurements, rather than being exact, are approximations, which later leads students to discuss and estimate the error of a measurement. Eventually, students encounter measurement ideas both in and out of school, in such areas as geometry, architecture, art, building and design, cooking, sports and map-reading. As students mature and become more conscious about the world in which they live, they develop a natural desire to explore the world and engage in measuring objects.

Geometry is exceptionally rich in opportunities for students to be involved not only in measurement

activities but also in activities related to other content and process standards. For example, tangram activities reinforce the idea of conservation of area. Not only young students find it difficult to accept that a square and a triangle can have the same area. A geoboard or geopaper problem can be used to explore this idea. Students may want to cut apart each figure created on geopaper and rearrange the pieces to form a rectangular area to verify their solutions. Pentominoes with areas of five squares can be used in enjoyable and worthwhile activities linking visualization and measurement. Students in higher grades can explore the Pythagorean relationship using triangles on geopaper. Drawing the triangles on geopaper allows students to investigate quickly and accurately the squares on the sides of several triangles. This investigation, in turn, allows them to draw conclusions and make generalizations based on several examples rather than just the two or three that traditionally appear in textbooks. Nonright triangles can also be explored to determine if the property holds. Students should be encouraged to ponder such questions as, Is there another way? and, What would happen if . . . ? For example, What would happen if we used isosceles triangles instead of right triangles? What if we drew semicircles on the sides instead of squares? What if we drew equilateral triangles (or other regular polygons) instead of squares?

Geometry and measurement are interconnected and support each other in many ways. The concept of similarity, for example, can be used in indirect measurement, and the perimeter and area of irregular figures can be determined using line segments and squares, respectively.

Measurement activities involve the handling of concrete objects and materials, which is helpful in gaining a deeper understanding of the concepts and skills related to measurement. As students advance through the grades, measurement concepts become more sophisticated and complex. Whatever the grade level, students should have many informal experiences in understanding measurable attributes before using tools or formulas to measure them. Measurable attributes, of course, increase in complexity as the grade level increases, as does the relationship between attributes. For example, cutting up a shape and rearranging its pieces may change the perimeter, but it will not affect the area.

The selection of different units to measure different attributes is sometimes difficult for students in the lower grades to understand. Hence, learning to choose an appropriate unit is an important part of understanding measurement. For example, the perimeter of a geometric shape is a linear measure, but the area of that shape is measured in square units and the volume is measured in three-dimensional or cubic units. Knowing how to select appropriate linear measures is also important. For example, measuring the distance from Edmonton to Calgary in centimetres would not be appropriate.

The application of appropriate techniques, tools and formulas to determine measurements is an important part of the measurement strand. Students must become proficient in the use of tapes, rulers, scales and clocks. Using formulas to measure an attribute is introduced as early as the elementary level and further developed in the later grades. Estimating as a legitimate measurement technique is also developed throughout the school years.

Measurement is one of the most widely used applications of mathematics. It helps to connect ideas within mathematics and ideas between mathematics and other disciplines. Measurement concepts and skills can be developed in the context of other strands and need not be taught as a separate unit of study. Other mathematics strands, science, art and technical courses provide natural opportunities to use and understand measurement in real-life and realistic contexts.

The four articles that follow deal with measurement in various ways and contexts and offer many practical suggestions for teachers. "Teaching Geometry and Measurement Through Literature" identifies texts that can support mathematical thinking. "How Big Is Your Foot?" is a hands-on discovery project that aims at improving students' understanding of measurement and showing that measurement is meaningful in the real world. "A Dynamic Way to Teach Angle and Angle Measure" addresses elementary students' lack of understanding of and attention to the concept of angle. "Let's Do It: Measurement for the Times" discusses measurement and multiplication using many practical applications.

Reference

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000. Available at http://standards.nctm.org/document/index.htm [accessed May 23, 2003].