Teaching Geometry and Measurement Through Literature

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As teacher educators, our work involves helping novice and experienced middle school teachers integrate literature in the mathematics classroom. We have noticed a recurring dilemma that can best be described through the voice of a composite middle school mathematics teacher:

> I'm really excited about the possibilities of using literature to teach math in my class. And yet, I'd be less than honest if I didn't say that the idea makes me feel a little uncomfortable. After all, I'm a math teacher. I haven't had any training or experience in this area, so I don't really know what literature is out there to teach math and where to find it.

Simply stated, mathematics teachers know mathematics but often have little knowledge of and experience with using literature to teach mathematics.

This article helps resolve this dilemma by making connections between literature and mathematics. We have three purposes: (1) to provide examples of learning experiences that use literature to teach aspects of geometry and measurement, (2) to discuss instructional implications of these experiences in the contexts of Van Hiele theory and differentiated instruction and (3) to identify and organize a variety of literature that has "text potential" (Burke 1995) for supporting mathematical thinking.

Looking Inside a Classroom

Teaching geometry in the middle grades traditionally includes many manipulative activities. Students use geoboards, pattern blocks, tangrams and other tools to explore such geometric concepts as properties of figures, area, perimeter and tessellations. In a literature-based geometry unit, stories can provide meaningful contexts for these experiences. For example, a common activity is to ask students to fit various shapes together in tiled patterns. The book *A Cloak for the Dreamer* (Friedman 1994) provides a context for this exploration.

This story involves a tailor and his three sons. The two older sons enjoyed working with their father; the youngest son dreamed of travelling the world. One day, the archduke ordered the tailor to make him three new cloaks for an important journey. The tailor in turn asked each son to sew a different cloak and to make sure that each garment would protect the archduke from the wind and rain. One son used the pattern of bricks on the floor to make a cloak by sewing rectangles together. Another son made a cloak by stitching triangles together. The youngest, however, cut circles to represent places he would love to visit and sewed his circles together into a cloak. When the sons showed the cloaks to their father, the youngest realized that his cloak was useless—it was full of open space. That night, the tailor and his two older sons snipped the circles into hexagons and sewed the hexagons together into a beautiful cloak for the youngest son, who used it to protect himself from the wind and rain as he set off to travel the world. This story illustrates that the successful completion of a cloak depends on the properties of the shape chosen to construct the cloak. The shape chosen for the cloak must fit together precisely; it must not contain open space, or the cloak will not protect its wearer. Students can use tools such as pattern blocks or attribute pieces to explore which shapes can be used to make a cloak and explain why some shapes tile and others do not.

The teacher could also extend this study of geometric figures and their properties by using the process of cutting circles into regular hexagons as a link to compass and straightedge constructions. After reading the story, ask students, "What mathematical skills do the father and his sons need to create accurate regular hexagons from the circles? What other shapes could they have made?"

Using Literature to Support Mathematical Thinking

As we read, we construct meaning from the text. The meaning that we create depends on many factors: our background knowledge, our purpose for

reading and the social context in which the reading occurs. As a family story, *A Cloak for the Dreamer* emphasizes supporting each child in his or her chosen path in life. When this book is read in the context of a mathematics class, it can become a launching point for investigating geometric figures and their properties. We encourage teachers to look for important mathematics in each text they explore, because the mathematics will not always be obvious.

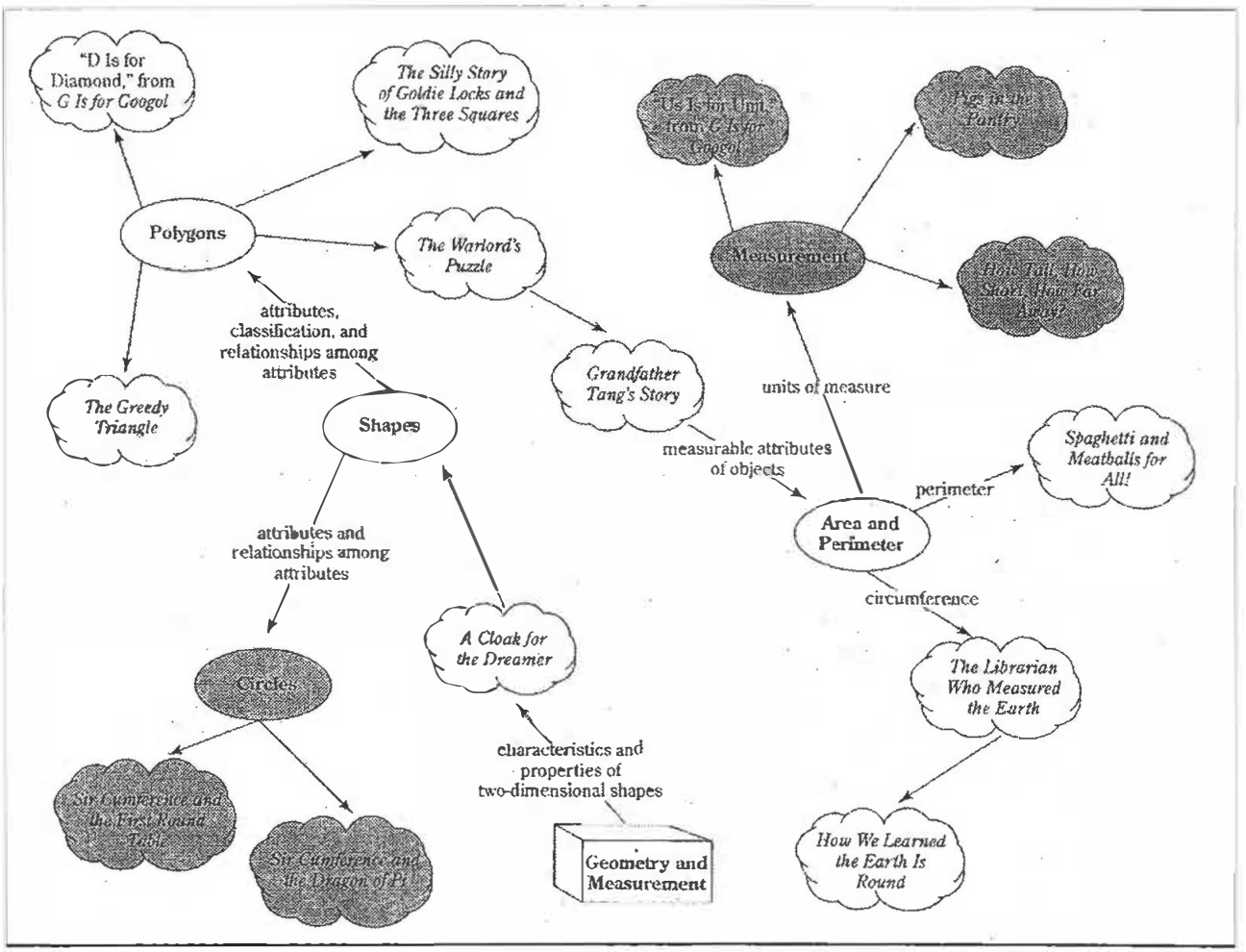
Figure 1 highlights clusters of literature that have potential for teaching concepts in geometry and measurement: circles, polygons, area and perimeter, and measurement. For simplicity's sake, we describe these clusters in a linear way; later, however, we discuss these clusters in terms of their possibilities for teachers to differentiate instruction.

The clusters in Figure 1 provide a framework for studying this facet of the National Council of Teachers of Mathematics (NCTM 2000, 233) geometry standard: "Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships." *A Cloak for the Dreamer* is a good starting point to introduce these concepts.

Circles and Polygons

In studying shapes, Figure 1 includes two clusters, one focusing on circles and the other on polygons. The cluster dealing with circles includes two interrelated texts. *Sir Cumference and the First Round Table: A Math Adventure* (Neuschwander and Geehan 1997) introduces basic concepts: parts of a circle and the terms used to label them. *Sir Cumference and the Dragon of Pi: A Math Adventure* (Neuschwander 1997) extends further the study of the properties of circles by using pi to show relationships between the radius, diameter and circumference of circles.

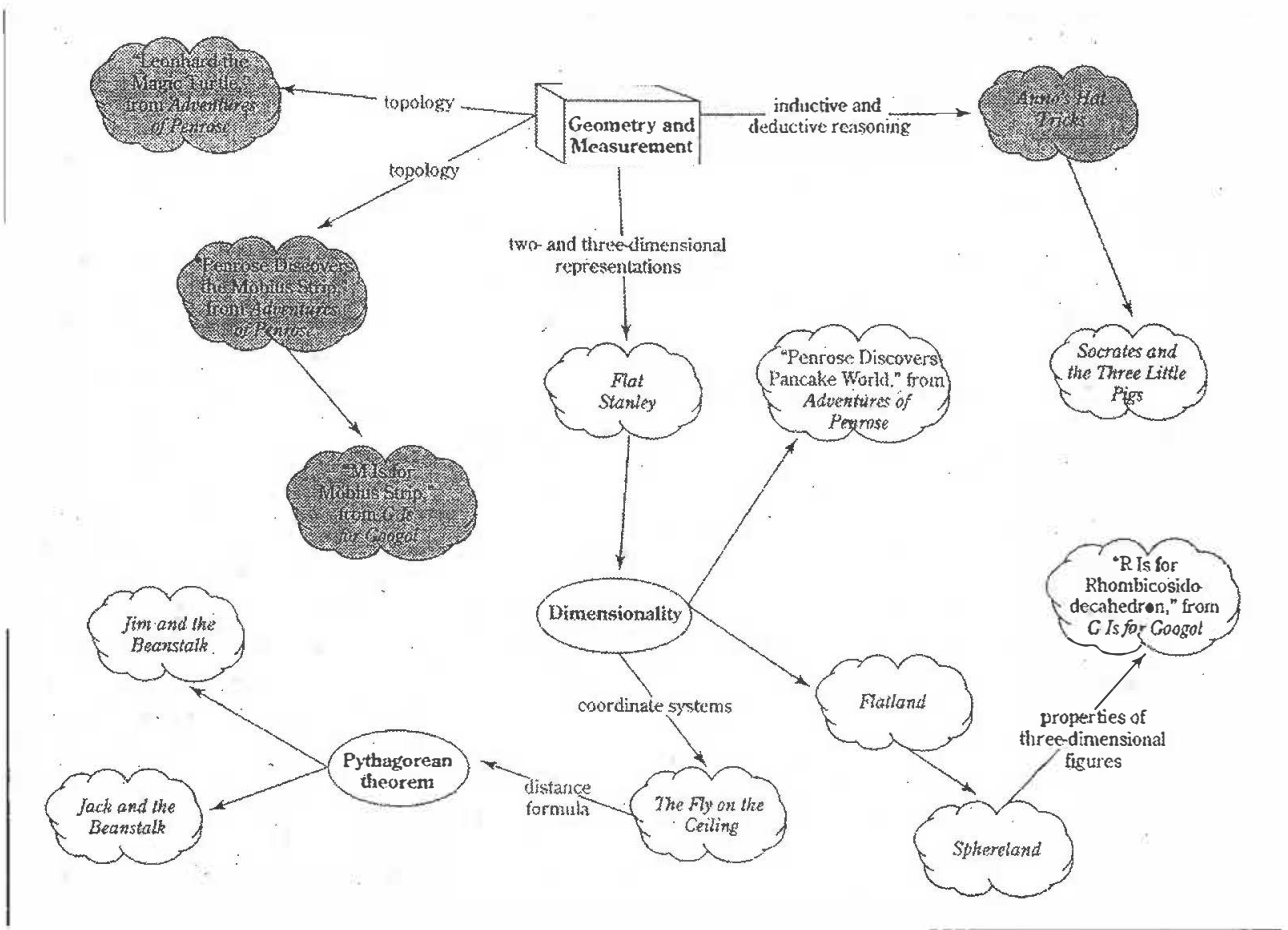
Figure 1
Literature Related to Shapes and Measurement



In addition, studying shapes can lead to the study of polygons. The cluster dealing with polygons relates to *The Greedy Triangle* (Burns 1994). This delightful book identifies and names a progression of polygons (triangle, quadrilateral, pentagon, hexagon and so on). If students have geoboards at their desks, they can follow along with the story. Defining properties of shapes can be discussed as students compare and describe the various triangles, quadrilaterals or other shapes that they make. "D Is for Diamond" (Schwartz 1998a), from the book *G Is for Googol,* identifies different properties of polygons (for example, one attribute of a square is that all four sides are congruent) and reinforces distinctions between colloquial names for figures (for example, *diamond*) and proper mathematical terms (for example, *rhombus*). *The Silly Story of Goldie Locks and the Three Squares* (MacCarone 1996) addresses relationships between properties (for example, the more sides to a regular polygon, the larger each interior angle).

Of course, some shapes tile a plane and some do not. *A Cloak for the Dreamer* describes shapes that do tessellate, or tile. *The Warlord's Puzzle* (Pilegard 2000) creates similar patterns (tiling with no gaps and no overlaps) but uses multiple shapes rather than a single, repeated shape. In this story, the challenge is to find out how to combine a variety of different pieces (specifically tangram pieces) to re-create a broken square tile. This book can be followed by *Grandfather Tang's Story* (Tompert 1997), in which the same pieces of a tangram set are used to create different figures. One relationship that we can explore is the relative areas of the pieces of the tangram set. If the small triangle represents one unit of area, students can describe the areas of the other shapes in the set and the figure that they create with some or all of the shapes in terms of this unit. This explanation provides a smooth transition to measurement.

Figure 2
Literature Related to Reasoning, Topology and Dimensionality



Area and Perimeter

In this cluster, we consider three concepts related to measurement: area, perimeter and circumference, and units of measure. Units of measure are particularly important because middle grades students do not always choose appropriate units for a particular measurement task. Readings such as "U Is for Unit" (Schwartz 1998d), from *G is for Googol, Pigs in the Pantry* (Axelrod 1997) and *How Tall, How Short, How Far Away?* (Adler 1999) help students explore units of measure—mass, capacity, length—appropriate for various contexts. These texts can also connect to science through measurement tools, such as a balance scale or a graduated cylinder, used in the science lab.

Spaghetti and Meatballs for All! (Burns 1997) extends students' mathematical thinking about the topics of area and perimeter by illustrating and describing how square units can be used to measure a shape's surface area. It also describes how to use units of length to determine the distance around tables of different shapes to seat varying numbers of dinner guests.

Finally, *The Librarian Who Measured the Earth* (Lasky 1994) and *How We Learned the Earth Is Round* (Lauber 1994) explore circumference—in particular, the challenge of finding the distance around the earth. These stories connect to science, as the earth's circumference is represented as the equator and prime meridian. They also connect to social studies, since the contributions of the ancient Greeks are described in part as we learn how Eratosthenes determined the circumference of the earth with astonishing accuracy.

Still More Possibilities in Geometry and Measurement

Figure 1 illustrates the literature that can tie the study of geometry and measurement to shapes and measurement. Figure 2 illustrates other texts that can help explore other facets of geometry and measurement: inductive and deductive reasoning, topology, two- and three-dimensional representations, and properties of figures.

Inductive and Deductive Reasoning

The NCTM's (2000, 232) geometry standard states that instructional programs should enable students to "develop mathematical arguments about geometric relationships." Specifically, middle grades students should "create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship" (p. 232). *Anno's Hat Tricks* (Anno and Nozaki 1985) can start a discussion on reasoning by introducing students to binary logic as they analyze their hat colour by looking at others' hats. *Socrates and the Three Little Pigs* (Anno 1986) extends this discussion of logical reasoning with more complex problem-solving situations. Students can critique the reasoning of the wolf, Socrates, as he attempts to find the three little pigs hiding in various houses. Although the fundamental mathematics involves combinations and permutations, teachers can frame questions so that students develop skills associated with "If . . . , then . . ." deductive reasoning and explore all possible cases of a given situation. These activities will be useful in developing formal proof skills in high school mathematics classes.

Topology

Although topology is not explicitly part of the middle school curricula, it is an engaging facet of mathematics for middle school students. A good place to start discussions about topology is "Leonhard the Magic Turtle" (Pappas 1993a). In this story, Leonhard attempts to traverse the seven bridges of Königsberg. Presented in story form, this classic problem is an effective introduction to network theory.

Similarly, the selection "Penrose Discovers the Möbius Strip" (Pappas 1993b) introduces the exploration of a single-surface Möbius strip. "M Is for Möbius Strip" (Schwartz 1998b) provides a more formal mathematical discussion and activity suggestions.

Two- and Three-Dimensional Representations

A number of texts explore properties of two- and three-dimensional figures and make connections to the coordinate plane. In *Flat Stanley* (Brown 1992), a little boy wakes up one morning to discover that his bulletin board fell on his bed during the night and that he is flat. The story describes the effect of Stanley's being two-dimensional on various events of his day. For example, Stanley poses unnoticed inside a picture frame to help catch an art thief. This story introduces students to the differences between two- and three-dimensional representations. For students who might need a simpler introduction, "Penrose Discovers Pancake World" (Pappas 1997) covers comparable ground. For students who are ready for a more sophisticated discussion, *Flatland* (Abbott 1992) and its "cousin" *Sphereland* (Abbott, Burger and Asimov 1994) not only provide a good

mathematical foundation but also can serve as an opportunity to connect to social studies through a discussion of Victorian society, particularly the social status of men and women. If a teacher wishes to extend *Sphereland* into a discussion of properties of three-dimensional figures, "R Is for Rhombicosidodecahedron" (Schwartz 1998c) introduces Euler's formula, which describes the relationship between faces, edges and vertices of a solid figure.

Finally, a two-dimensional world is explored further in *The Fly on the Ceiling* (Glass 1998). This entertaining piece of historical fiction hypothesizes about the circumstances under which Descartes developed his system of coordinates. If a teacher wishes to make connections to measurement, the distance formula serves as a link between the Pythagorean theorem and the coordinate plane. Both *Jack and the Beanstalk* (Kellogg 1991) and *Jim and the Beanstalk* (Briggs 1970) discuss situations in which these concepts can be applied. For example, after retelling *Jack and the Beanstalk,* students may determine the length of ladder needed to reach a golden egg resting at a particular height on the beanstalk. Although these explorations happen in mathematics class, the language arts teacher could talk about folklore, exaggeration in folktales and the balance necessary between reality and fantasy inherent in an effective story.

Using Literature to Differentiate Instruction

We do not expect teachers to use these texts exclusively or to work through these texts in the linear fashion in which we have described them. Rather, we offer this model as a way to organize a collection of texts about related mathematical ideas so that teachers can use literature to make mathematics more engaging and relevant to their students.

Van Hiele Theory

The work of Pierre van Hiele and Dina van Hiele-Geldof describes a sequential and hierarchical series of stages through which students progress as they learn about shapes, properties and relationships between properties. One aspect of the theory is that progression through the stages relates more closely to instruction than age or cognitive maturation (Van Hiele 1984, 1986; Van Hiele-Geldof 1984). The first cluster of books described in Figure 1, particularly the polygons group, is an excellent starter set to help students move from an early level where shapes are recognized holistically ("I know that's a rectangle because it is the same shape as the [rectangular] door") to recognition of properties ("A square has four right angles, four congruent sides, two pairs of parallel sides and so on"). These experiences can also support students as they learn to recognize relationships between properties ("If both pairs of opposite sides in a quadrilateral are parallel, then opposite sides are congruent"), the level that best supports success in high school geometry classes (Usiskin and Senk 1990).

Differentiated Instruction

A perennial challenge to middle school teachers is the fact that "one size fits all" instruction is not effective. Students come to mathematics classes perhaps grouped by skill in computation or algebra, but they range widely in their reading ability and knowledge of geometry and measurement. Differentiated instruction is the practice of creating related learning experiences in which students can be taught content matched to their instructional level (Tomlinson 1999). The texts about polygons mentioned earlier can create related learning experiences that are responsive to students at varying Van Hiele levels. For example, *The Greedy Triangle* can be discussed in terms of naming shapes, identifying properties of shapes or exploring relationships between properties by comparing shapes. The teacher could group students for these discussions depending on their levels of geometric understanding. The texts about two-dimensional worlds ("Penrose Discovers Pancake World," *Flat Stanley* and *Flatland*) can be used so that students of varying reading levels can all explore two-dimensional reality through literature. Each student would read the text that is most closely related to his or her reading level, then the students could work in groups across texts to complete tasks related to two- and three-dimensional representations. For example, teams could be given various two-dimensional drawings of a figure seen from different sides and use unit cubes to construct the three-dimensional figure that is consistent with the drawings.

Conclusion

In this article, our goal was to identify texts that can potentially support mathematical thinking. To this end, we illustrated and described two sets of literature that can be used to teach various aspects of geometry and measurement. We also discussed the connection between these texts and Van Hiele theory and shared examples of implementing these experiences in a differentiated classroom. Our hope is that this article will start new conversations and create new possibilities for using literature to teach mathematics.

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Twenty men did ¼ of a job in eight days. Then, because of an emergency, it became necessary to complete the job in the next five days. How many additional men were added to the crew of 20 to accomplish this?