How Big Is Your Foot?

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Concepts relating to fractions and measurement are difficult for students in the upper elementary and middle school grades to grasp (Bright and Heoffner 1993; Coburn and Shulte 1986; Levin 1998; Thompson and Van de Walle 1985; Thompson 1994; Witherspoon 1993). As a first-year teacher, I learned the value of relating difficult concepts, especially abstract concepts, to students' real-world experiences. The "How Big Is Your Foot?" project grew out of a question that I asked my Grade 8 students during my first year of teaching. We had just finished studying conversions in the metric system and had begun working with conversions in the customary system. As a warm-up question, I asked my students to describe the distance from my desk to the door of the classroom. I wrote their responses on the chalkboard as they called out estimates: 1 m, 60 m, 25 ft., 300 in., 300 cm. The students did not seem to have any grasp of the length of a metre or an inch. One of my more boisterous students got out of her desk and walked the distance heel to toe; the class counted with her and agreed that 9 "feet" was a fair estimate. I then asked the class whether the distance was greater or less than 9 ft. When no clear consensus was reached, I postponed my planned lesson and launched this activity involving comparisons of standard and nonstandard measurements.

Measure Your Foot

I asked each student to take out a pencil and two pieces of notebook paper. I then instructed the students to take off their shoes and determine which of their two feet was longer. Students traced the outlines of their longer feet, then cut their traces out carefully. My students quickly commented on how big or small their classmates' feet were. I asked students to form pairs. Each person was to determine distances using his or her own traced foot as the unit of measure. Partners were instructed to do the measuring. In other words, to determine the height of Person A, Person B would use Person A's traced foot to measure Person A's height. Then the roles would switch. I asked for the following measurements in the students' own "feet":

- Vertical height without shoes
- Arm span: tip of the third finger to tip of the opposite third finger when the arms are held out as far as possible and parallel to the floor
- Fingertip to shoulder: tip of the third finger to the shoulder when the arm is held out as far as possible and parallel to the floor
- Length of head: distance from chin to top of head
- Wrist to elbow
- Hand span: distance from pinky to thumb when the hand is stretched out as far as possible

The measurements that I requested were not random. The ratios of distances in the human body are well documented in anatomy books and work for most students as young as Kindergarten age. These anatomical relationships were examined and painstakingly sketched by Leonardo da Vinci in the late 1400s. I remembered these relationships from a design class that I had taken in college.

All but the hand-span measurement have the following relationships to the length of the foot: vertical height is 7 "feet"; arm span is 7 "feet"; fingertip to shoulder is 2.5–2.75 "feet," depending on the build of the body; length of head is 1 "foot"; and wrist to elbow on the inside of the arm is 1 "foot." The hand-span measurement has no predetermined relationship to the foot length because the span is affected by stretching of the fingers, such as when students play the piano. I included the hand-span measure because I wanted students to recognize that not every measurement on the human body is genetically determined. For middle school students, who are extremely concerned about their appearance, this knowledge of body proportions can lead to important discussions.

Measure Using Your Foot as the Unit

After giving the assignment, I watched as students groped with issues relating to the measurement tasks: "Should we round up or down? May we use fractions? Where do I start measuring the head? Do we measure the inside or outside of the arm?" One creative pair, who had made dots on the chalkboard to mark Person B's fingertips, complained, "It doesn't match!" The measurement on the chalkboard did not exactly match the measured distance when Person A measured Person B's arms and back. When everyone had finished measuring, we discussed some of these issues. I explained that, in measurement, mathematicians use certain conventions, such as starting at 0 rather than 1, and that the units we use help to eliminate some of the confusion. For this activity, we might label a measurement as a Beth-foot or a Robert-foot, and we would understand that 1 Beth-foot might not be equal in length to 1 Robert-foot.

With other issues for which no rule seemed to exist, the students explained and defended their choices. For example, one student argued that measuring the height of the head was not the same as measuring the distance from the top of the head to the chin because the first was an "up and down" measurement, whereas the other was a "perimeter" measurement. A quick consensus was reached to use the "up and down" measurement, and several students measured their partners' heads again. A more contentious problem, the question of how to measure the length between the wrist and the elbow, provoked heated discussion. Some students claimed that the task was "understood" to mean the inside of the arm; others countered that they did not understand this constraint at all. One student explained that, because we always want a large measurement, we should measure the outside of the arm. Another noticed that the distance changes when the wrist is bent. Still another student thought that averaging the length of the inside and outside of the arm would produce a fair result. Finally, a student who almost never participated suggested labelling the measurement "the inside of the arm from wrist to elbow." The class applauded, and we moved on.

As the discussion continued, I asked what the difference was between estimated and exact measurements. Some of the students said that a rounded measurement was an estimate. Others claimed that no measurement could be perfectly exact because humans are not exact. The class listed reasons for error when we cut out our "feet": (1) a person might have moved his or her foot during tracing, (2) the fingers that were used to hold the pencil and mark the paper had width, (3) the pencil point might not be sharp when drawing and a person might slant the pencil instead of holding it in perfect vertical alignment, and (4) cutting exactly on a line is difficult with scissors.

Unit Conversion

The next part of the activity asked students to compare their measurements to see if they could find any common answers or answers that were very nearly the same. Some students already knew that the arm span is nearly the same length as a person's height. Many did not know, however, that other relationships exist involving the foot as a unit of measure. As the students continued to discuss their results, they noticed that even though nonstandard measurements had been used, the ratios appeared to be about the same for everyone! With my help, they were able to express the relationships in algebraic terms. For example, $7 \times$ head = height and $2.5 \times$ wrist to elbow = fingertip to shoulder.

I then extended the lesson by asking students to predict the length of their own feet in both inches and centimetres. After students made their predictions, I handed out rulers with both centimetre and inch markings and instructed students to measure the length of their feet. At this point, several students, referring to the original question asked at the beginning of the lesson, called out, "It [the distance to the door] is less than 9 ft.!" When I asked students to tell me how they could determine, without measuring, the length in centimetres from fingertip to shoulder, nearly every hand went up. One student spoke for her classmates: "If 2.5 'feet' are needed and if my foot is 22 cm, I just multiply 2.5 by 22; 22 cm for each of my feet and 11 cm for half my foot." Her partner then used the ruler to determine that the first student's fingertip-to-shoulder measurement was 65 cm. When I asked whether the girl had overestimated or underestimated the length using her own foot, the class answered immediately that she had slightly underestimated.

I finally asked students to look at the lengths of their feet and describe objects in the classroom that were approximately 1 "foot" in length, 2 "feet" in length and so on. We then compared the longest "foot" and the shortest "foot" measurements with the measurements using a standard foot. I closed the lesson by asking students to estimate the length of the chalkboard in standard feet. Students wrote their estimates, then measured using rulers. This time, the answers ranged from 9 ft. to 15 ft. The actual length of the chalkboard was 12 ft.

That day, my students discovered that measurement is inexact. By paying close attention to the size of the unit, however, my students learned that making reasonable estimates is not difficult. Nonstandard units can help us estimate the lengths of objects in inches, centimetres, feet and so on, by giving a frame of reference. The students enjoyed this lesson and mentioned it frequently in subsequent weeks, especially when we revisited this idea by comparing 1 kg with the weight of two medium-sized apples, 1 g with the weight of a paper clip, a small bowl of cereal with the weight of about 30 g and the liquid in four large coffee mugs with the approximate capacity of 1 L.

Adapting the Project for Different Audiences

The first time that I presented this project, I had not carefully planned out what I would say or have the students do. As I wrote out the project for students in successive years, I changed the language that I used to reflect the difference between the student's foot length and the length of a standard foot. To emphasize the fact that every student's foot was different, I asked students to refer to the person's name followed by the words *shoe-unit.* Hence, Brian's body would measure about 7 Brian-shoe-units in length, and Tanisha's body would measure about 7 Tanisha-shoe-units in length.

Renaming the units was a cosmetic change to eliminate confusion. As I thought about this project, I wanted to make some conceptual additions that would connect with the mathematics that students had seen earlier that year and in previous classes. I wanted to connect work on ratios and proportions with measurements through the application of scale. I also wanted the students to do more of the activity on their own. Figure 1 shows the final form of the project.

Figure 1

Final Format of Project for Middle School Students

How Big Is Your Foot?

On a piece of paper, trace the outline of your largest foot *without your shoe on.* The heel should just touch the bottom edge of the paper. Now, cut out this outline of your foot. The length of this foot will now be used as a nonstandard measure. Name your nonstandard unit in the following way: <your name>-shoe-unit. For example, the length of Chad's foot will now be called a Chad-shoe-unit. Use your shoe-unit to measure yourself. For example, Chad would measure himself in Chad-shoe-units. Chad's partner would *not* use Chad-shoe-units. In pairs, complete the following information:

Name of Person A ______________	Name of Person B ______________
Person A will be measured in ______________-shoe-units.	Person B will be measured in ______________-shoe-units.
➢ I am ______________-shoe-units tall.	➢ I am ______________-shoe-units tall.
➢ I measure______________-shoe-units from the tip of my longest finger to my shoulder.	➢ I measure______________-shoe-units from the tip of my longest finger to my shoulder.
➢ I measure______________-shoe-units from the top to the bottom of my head.	➢ I measure______________-shoe-units from the top to the bottom of my head.
➢ I measure ______________-shoe-units from the tips of my fingers on my left hand to the tips of my fingers on my right hand when I hold my arms out parallel to the floor.	➢ I measure ______________-shoe-units from the tips of my fingers on my left hand to the tips of my fingers on my right hand when I hold my arms out parallel to the floor.
➢ I measure ______________-shoe-units from the tip of my thumb to the tip of my pinky when my hand is opened as far as possible.	➢ I measure ______________-shoe-units from the tip of my thumb to the tip of my pinky when my hand is opened as far as possible.
➢ I measure ______________-shoe-units from my wrist to my elbow.	➢ I measure ______________-shoe-units from my wrist to my elbow.

Complete the following:

- Write at least three observations that you made while measuring.
- Write any questions that you faced while doing this activity.
- If you came up with an answer to your question, write it and say how you came up with this answer.
- Compare answers with several other pairs. Do you notice any similarities or any differences for a specific measurement?

Estimate the length of your shoe-unit in inches and centimetres, then use a ruler to measure your foot. Describe how you can determine the lengths in the chart above in inches and centimetres without actually measuring.

Another change that I made was to incorporate the use of calculators to verify the estimates and measurement conversions. Although the computation is not difficult in this project, having calculators available allowed many students to complete the basic questions quickly and gave the class more time to examine the conceptual questions. Because of time constraints, I do not currently include spreadsheets with this project; however, one natural extension might be to have middle school students use spreadsheets to compare the average length of different measurements on the body in terms of "feet." These averages could then be displayed using bar graphs.

I have had the opportunity to do this project with more than 400 middle school students, more than 200 undergraduate elementary education students and more than 30 inservice teachers. Although the initial activity for preservice and inservice teachers is essentially the same as the student activity, additional questions are included to examine the prerequisite knowledge needed for successful completion of the project, and suggestions are given for adapting this activity for students who have special needs.

The preservice and inservice teachers recognized that all students need to go through a process of learning measurement. Teachers need to guide their students through this process by addressing the following prerequisite skills for this project, which involve

- a basic understanding of how to measure (that is, a single unit is placed end to end with no space and no overlap);
- gross- and fine-motor skills, including the abilities to trace and cut;
- an understanding of how to record data in a table;
- an understanding of how to find the arithmetic mean, or average, of a set of data, including knowing how to add and divide whole numbers;
- the ability to identify a ratio as a comparison of two related quantities or numbers;
- the ability to write and solve proportions, including in whole-number multiplication and division;
- an understanding of how to measure to the nearest inch and centimetre;
- an ability to make a reasonable estimate for linear measurement; and
- an awareness of how to interpret a scale measure.

If students have not attained these prerequisite skills, then this activity can easily be modified to include instruction in any of the mathematical topics listed above. Additionally, this activity can be used as a preassessment to gauge how well students understand these topics.

When asked how they would adapt this activity for students who have special needs, the preservice and inservice teachers made the following suggestions:

- For students who have vision impairments, use a rough piece of paper, such as sandpaper, to enable students to measure the length.
- For students who have vision impairments, let the partner say "hot" or "cold" to refer to how close to the end of the "foot" the student's finger is.
- Allow students who have hearing impairments to measure after their partners do, so that they can follow the example of the first partner. This same suggestion was made for students who do not speak English.
- Also for students who have hearing impairments, write the findings on the chalkboard as the discussion continues.
- For students who tend to need more time, make one or more of the measurements that are one "foot" long (such as the wrist to elbow or the height of the face) optional. Have students who tend to finish quickly estimate then measure other body parts, such as the distance from knee to ankle or from neck to waist.

Preservice and inservice teachers suggested tying this project to reading map scales in geography, because the notion of converting between one measurement system and another is related to this concept. Another adaptation suggested by the inservice teachers is to read *How Big Is a Foot?* (Myller 1962). This story illustrates why we need standard measurements. The teachers agreed that more emphasis should be placed on helping students write algebraic equivalents involving two lengths, such as $7 \times$ foot length $\cong$ arm span. Finally, the teachers suggested that students find out more about Leonardo da Vinci, who identified these and other anatomical relationships over 500 years ago. Students might be interested to learn that Da Vinci wrote his findings backward so that no one would steal them and that he drew more than 1,000 sketches relating to architecture, anatomy, maps, nature and art. In his sketches and artwork, he successfully used perspective, which requires the application of proportional reasoning and ratios.

What Have I Learned?

This project began in response to students' inability to reason about standard and nonstandard measurement and has grown to include the concepts of measurement conversion, ratio, proportion and scale. During my first year of teaching, I shied away from projects like this one precisely because I knew they would take extra time, which I felt I did not have, and would lead to difficult questions for which I did not always have answers. Such projects call for students to make and validate conjectures, neither of which is an easy task for middle school students. I worried about how students would feel if they could not do the assigned task quickly and accurately.

After doing this project year after year, I realize that students need to struggle with mathematical concepts. Memorizing conversion formulas, such as 1 yd. = 3 ft. and 1 m = 100 cm, is important, but if a student does not know how long a centimetre is, then the conversion is just another piece of mathematics trivia to be learned for the test and quickly forgotten. I now strive in my teaching to make the concepts relevant by connecting new mathematics to previously learned mathematics and, whenever possible, by connecting mathematics to other disciplines and the real world. Furthermore, I try to avoid reciting mathematics facts by incorporating these types of projects into my teaching as often as possible.

By watching my students work on "How Big Is Your Foot?" I discovered that they did not really mind the struggle when they understood what was being asked. For some of my students, the struggle required two or more periods. Although I had assumed that my students would rebel if they did not get the answer quickly, many of them begged me not to give away the answer!

My students have enjoyed the hands-on discoveries of this project, and I am convinced by their discussions in class, as well as their test scores, that their understanding of measurement and ratio is more complete. Additionally, this project forces students to acknowledge that mathematics topics can be meaningful in the real world, as well as in class.

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A salesman travelled at 110 km/h while making a 220 km trip to a client. He then returned home at 90 km/h. What was the average speed for the round trip?