Fun with Mathematics—Challenging the Reader

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Each issue of delta-K *will contain problem sets, which will also be posted on the MCATA website (www.mathteachers.ab.ca).*

The spring issue will contain a set of problems for January, March and May, and the fall issue will contain a set of problems for September and November. Teachers and students are invited to participate by submitting the full solution to each problem by the deadline stated on the website.

Note that the solutions to the problem sets will be published in delta-K *only, with the fall issue containing the solutions for the January, March and May problems and the spring issue containing the solutions for the September and November problems.*

Submit your full solutions to Andy Liu, Department of Mathematics, University of Alberta, Edmonton T6G 2G1; fax (780) 492-6826, e-mail aliumath@telus.net.

January 2003 Problems

1. The numbers 1, 2, . . ., 16 are placed in the cells of a 4×4 table as shown below:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

One may add 1 to all numbers of any row or subtract 1 from all numbers of any column. How can one obtain, using these operations, the table shown below?

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	16

2. There are four kinds of bills: \$1, \$10, \$100 and \$1,000. Can one have exactly half a million bills worth exactly \$1 million?
3. The king intends to build six fortresses in his realm and to connect each pair with a road. Draw a diagram of the fortresses and roads so that there are exactly three intersections and exactly two roads cross at each intersection.
4. If each boy purchases a pencil and each girl purchases a pen, they will spend a total of 1¢ more than if each boy purchases a pen and each girl purchases a pencil. There are more boys than girls. What is the difference between the number of boys and the number of girls?
5. A six-digit number from 000000 to 999999 is called lucky if the sum of its first three digits is equal to the sum of its last three digits. How many consecutive numbers must we have to be sure of including a lucky number if the first number is chosen at random?
6. Two players play the following game on a 9×9 chessboard. They write in succession one of two signs in any empty cell of the board: the player making the first move writes a plus sign (+) and the other player writes a minus sign (–). When all the squares of the board are filled, the scores of the players are tabulated. The number of rows and columns containing more plus signs than minus signs is the score of the first player, and the number of all other rows and columns is the score of the second player. What is the highest number of points the first player can gain in a perfectly played game?

March 2003 Problems

1. Initially, there is a 0 in each cell of a 3×3 table. One may choose any 2×2 subtable and add 1 to all numbers in it. Can one obtain, using this operation a number of times, the table shown below?

4	9	5
10	18	12
6	13	7

2. A teacher plays a game with 30 students. Each writes the numbers 1, 2, . . ., 30 in any order. Then the teacher compares the sequences. A student earns a point each time the same number appears in the same place in the sequences of that student and of the teacher. It turns out that each student earns a different number of points. Prove that at least one student's sequence is the same as the teacher's.
3. Is it possible to write the numbers 1, 2, . . ., 100 in a row so that the difference between any two adjacent numbers is not less than 50?
4. Do there exist two nonzero integers such that one is divisible by their sum and the other is divisible by their difference?
5. A game starts with a pile of 1,001 stones. In each move, choose any pile containing at least two stones and remove one of them, and then split any pile containing at least two stones into two nonempty piles, which need not be of equal size. Is it possible for all remaining piles to have exactly three stones after a sequence of moves?
6. A square castle is divided into 64 rooms in an 8×8 configuration. Each room has a door on each wall and a white floor. Each day, a painter walks through the castle repainting the floors of all the rooms he passes, so that white is changed to black and vice versa. Can he do this so that, after several days, the floors in the castle will be coloured like a chessboard?

May 2003 Problems

1. A jury makes up problems for an Olympiad, with a paper for each of Grades 7–12. The jury decides that each paper should consist of seven problems, with exactly four of them not appearing on any other paper of the Olympiad. What is the greatest number of distinct problems that could be included in the Olympiad?
2. A six-digit number (from 000000 to 999999) is called lucky if the sum of its first three digits is equal to the sum of its last three digits. Prove that the number of lucky numbers is equal to the number of six-digit numbers with a digit sum of 27.
3. Given 32 stones of distinct weights, prove that 35 weightings on a balance are sufficient to determine which are the heaviest and the second heaviest.
4. Find two six-digit numbers such that the number obtained by writing them one after another is divisible by their product.
5. Two players play a game of wild tic-tac-toe on a 10×10 board. They take turns putting either an X or an O in any empty cell on the board. Both players can use X or O, and not necessarily consistently. A player wins the game by making three identical symbols appear in consecutive cells horizontally, vertically or diagonally. Can either player have a winning strategy? If so, is it the player who moves first or the one who moves second?
6. Each section of tracks in a model railway is a quadrant of a circle directed either clockwise or counterclockwise, as shown below in the diagram on the left. One may assemble the track only in such a way that the directions of the sections are consistent along the whole track, as illustrated below in the diagram on the right. If such a closed track can be assembled using given sections, prove that this is no longer the case if one clockwise section is replaced by a counterclockwise section.