Probabilities: An Example of Linking Mathematical Ideas

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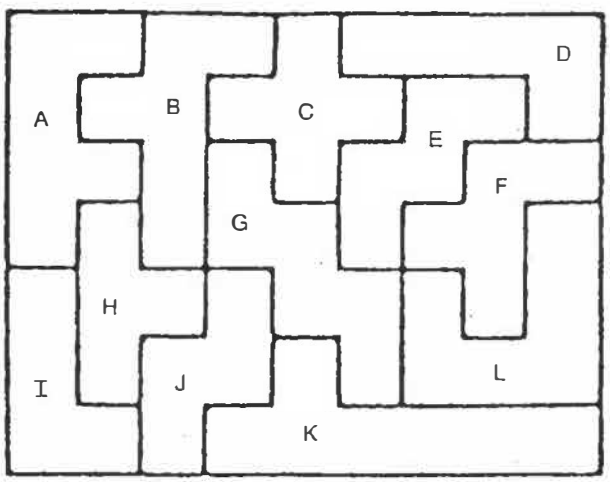
Teachers are always seeking situations in which mathematical ideas can be connected. In this article, we discuss ways in which a rectangular puzzle can be used to connect geometry and probability. These activities assume a background in geometry and a familiarity with the basic language of probability. They can be done by the whole class, in groups or for individual recreation at the teacher's discretion.

The puzzle to be discussed consists of 12 pieces that may be arranged to form a rectangle (see Figure 1).

Figure 1



Let us assign numerical values to the rectangle. Assume it is a 9 × 7 rectangle; then the perimeters and areas can be found (see Figure 2).

Figure 2

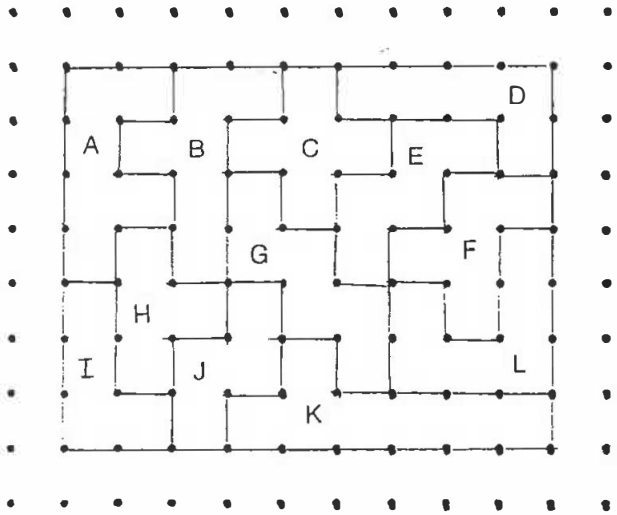


Table 1 shows the perimeter and area of each of the 12 pieces.

Table 1

Figure	Area (in square units)	Perimeter (in units)
A	6	14
B	6	14
C	5	12
D	5	12
E	5	12
F	5	12
G	6	14
H	4	10
I	4	10
J	4	10
K	7	16
L	6	14

We can now discuss links to probability. Suppose we cut the rectangle into its 12 pieces.

Problem 1

If we select two pieces at random, with replacement (that is, the first piece is returned before the second piece is selected), what is the probability that their areas are equal? What is the probability that their perimeters are equal? There are 144 possible pairings, which are listed in Table 2.

Area

Of the 144 equally likely pairs in Table 2, the following 42 pairs contain figures of equal area: AA, AB, AG, AL, BA, BB, BG, BL, CC, CD, CE, CF, DC, DD, DE, DF, EC, ED, EE, EF, FC, FD, FE, FF, GA, GB, GG, GL, HH, HI, HJ, IH, II, IJ, JH, JI, JJ, KK, LA, LB, LG and LL.

The probability of drawing two pieces with the same area is, thus, $\frac{42}{144}$, or 0.29.

Perimeter

Of the 144 equally likely pairs in Table 2, 42 contain figures of the same perimeter (the same 42 pairs as in the area problem). Thus, the probability of drawing two pieces with the same perimeter is $\frac{42}{144}$, or 0.29.

Problem 2

Let us redo the first problem, this time without replacement (that is, the first piece is not replaced before the second is drawn). Table 2 must be adjusted to eliminate all pairs in which the same piece is selected twice (see Table 3).

There are 132 pairs remaining.

Table 2

AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL
BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL
CA	CB	CC	CD	CE	CF	CG	CH	CI	CJ	CK	CL
DA	DB	DC	DD	DE	DF	DG	DH	DI	DJ	DK	DL
EA	EB	EC	ED	EE	EF	EG	EH	EI	EJ	EK	EL
FA	FB	FC	FD	FE	FF	FG	FH	FI	FJ	FK	FL
GA	GB	GC	GD	GE	GF	GG	GH	GI	GJ	GK	GL
HA	HB	HC	HD	HE	HF	HG	HH	HI	HJ	HK	HL
IA	IB	IC	ID	IE	IF	IG	IH	II	IJ	IK	IL
JA	JB	JC	JD	JE	JF	JG	JH	JI	JJ	JK	JL
KA	KB	KC	KD	KE	KF	KG	KH	KI	KJ	KK	KL
LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL

Table 3

	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL
BA		BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL
CA	CB		CD	CE	CF	CG	CH	CI	CJ	CK	CL
DA	DB	DC		DE	DF	DG	DH	DI	DJ	DK	DL
EA	EB	EC	ED		EF	EG	EH	EI	EJ	EK	EL
FA	FB	FC	FD	FE		FG	FH	FI	FJ	FK	FL
GA	GB	GC	GD	GE	GF		GH	GI	GJ	GK	GL
HA	HB	HC	HD	HE	HF	HG		HI	HJ	HK	HL
IA	IB	IC	ID	IE	IF	IG	IH		IJ	IK	IL
JA	JB	JC	JD	JE	JF	JG	JH	JI		JK	JL
KA	KB	KC	KD	KE	KF	KG	KH	KI	KJ		KL
LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	

Area

The 42 pairs of the same area in the first problem must now be reduced by eliminating the 12 pairs in which a letter is repeated, leaving 30 pairs. The probability of drawing two pieces with the same area is then $\frac{30}{132}$, or 0.23.

Perimeter

The same reasoning holds for finding the probability of drawing two pieces with the same perimeter. Again, the probability of matching perimeters is equal to that of matching areas: $\frac{30}{132}$, or 0.23.

Interesting Exercises for the Teacher and Students

1. If you throw a dart at a rectangular puzzle board, what is the probability of hitting any one of the seven pieces? For example, $P(G) = \frac{6}{63}$.
2. Use the fundamental principle of counting from probability to determine the number of figure pairs with the same area or the same perimeter.
3. Find and investigate other situations in which geometry and probability can be linked.

How much water should be added to 800 cm^3 of a 70 per cent solution of boric acid to make it a 40 per cent solution of boric acid?