The Probability of Winning a Lotto Jackpot Twice

Emeric T. Noone

Studying state lotto games allows students to apply basic probability concepts in a practical setting. Students enjoy the discussion and become enthusiastic about investigating the probabilities of winning the jackpot. The probability of winning the jackpot in one play of a state lotto game is quite easy to compute. A more interesting problem is computing the probability of winning it twice. Some state lotteries have produced two-time winners of large jackpots. For example, an Ohio man won \$4.3 million in August 1990, then won \$12 million in March 1991. This article addresses the probability of winning twice.

A typical state lotto game is played by selecting one six-number combination from among the integers 1, 2, 3, . . . , 44. If the six-number combination selected matches the winning six-number combination that is randomly chosen, the player wins the jackpot. The number of six-number combinations possible in a play of the game is computed by using the formula for a combination of N objects taken r at a time. For the typical game that has been described, the formula yielded the following results, which can easily be computed with a hand-held calculator:

$$C(N, r) = \frac{N!}{r!(N-r)!}$$

$$C(44, 6) = \frac{44!}{6!38!}$$

$$C(44, 6) = 7{,}059{,}052$$

A total of 7,059,052 different six-number combinations are possible in one play of the game. Consequently, the probability of winning in one play of the game is

$$\frac{1}{7{,}059{,}052}.$$

Often when students are asked to compute the probability of winning the lotto game twice, they simply compute the following product, since the probability of success in two independent trials is the product of the probabilities of success in each trial.

$$\left(\frac{1}{7{,}059{,}052}\right)\left(\frac{1}{7{,}059{,}052}\right) \approx 2.0068 \times 10^{-14}$$

The likelihood of such events as winning a lotto game twice is frequently expressed as odds against the event's occurring. The odds against the event of winning the lotto twice are the ratio of the probability of not winning, $1 - 2.0068 \times 10^{-14}$, to the probability of winning, 2.0068×10^{-14}. If students use that reasoning to compute the probability of winning twice, then the odds against winning the jackpot twice are approximately 49,830,576,040,000:1. What is wrong with that reasoning? Nothing is wrong if the game is played exactly two times. The answer 2.0068×10^{-14} would be a reasonable estimate of the probability of winning twice, and 49,830,576,040,000:1 would be a reasonable approximation of the odds against winning twice.

However, what would be the effect on the probability of winning twice if the game was played repeatedly over a longer period of time and multiple six-number combinations were played each time? Two-time winners of large jackpots had most likely played the game for an extended time and had played more than one six-number combination per play.

For illustrative purposes, suppose that a player plays the game twice a week for four weeks and plays five different six-number combinations each time. What is the probability that the player will win the jackpot twice? In that situation, eight independent plays of the game occur, with the probability of winning on any given play of

$$\frac{5}{7{,}059{,}052}.$$

The number of successes, x, in n trials is a random variable X, which follows the binomial distribution denoted by $X \sim Bi(n, p)$, where p is the probability of success on a given trial. The probability of x successes in n trials is given by $P(X = x) = C(n, x)p^x \cdot (1 - p)^{n-x}$. Hence, the probability of winning twice on eight independent plays of the lotto game when playing five six-number combinations each play is given by

$$P(X = 2) = C(8, 2)\left(\frac{5}{7{,}059{,}052}\right)^2\left(\frac{7{,}059{,}047}{7{,}059{,}052}\right)^6$$

$$\approx 1.40476 \times 10^{-11}.$$

Given the probability $P(X=2) \approx 1.40476 \times 10^{-11}$, the odds against winning twice under these conditions are approximately 71,186,537,200:1. When the number of plays of the game is increased and the number of combinations played on each play of the game is increased, the probability of winning twice is enhanced. Although still bordering on the miraculous, the odds against winning twice are not quite as astronomical.

It is interesting to examine the effects on the probability of winning twice when the number of plays is increased. Table 1 shows the binomial probabilities of winning the jackpot twice when the number of independent plays, n, is increased significantly. The probabilities were computed using the formula $P(X = x) = C(n, x)p^x(1 - p)^{n-x}$ and a hand-held calculator.

The random variable $X \sim Bi(n, p)$ where n is large and p is small tends to behave like a random variable Y, which follows the Poisson distribution denoted by $Y \sim Po(m)$, where $m = np$. The probability that the random variable Y assumes the values $x = 0, 1, 2, \ldots$ is given by $P(Y = x) = e^{-m} m^x/x!$ The Poisson distribution $Y \sim Po(m)$ makes the computations easier and less tedious. For example, if $p = 5/7{,}052{,}059$ and $n = 100$, then

$$m = 100 \left(\frac{5}{7{,}052{,}059}\right)$$
$$= .00007.$$

Therefore, the probability of winning twice in 100 plays is

$$P(Y=2) = e^{-.00007}\left(\frac{.00007^2}{2!}\right)$$

so $P(Y = 2) = 2.4498 \times 10^{-9}$.

Table 1 shows the Poisson probabilities for n independent plays of the game when playing five different six-number combinations each play. The probabilities are close to those of the binomial probabilities.

A total of 961.54 years would be needed to complete 100,000 plays of a lotto game when playing twice a week. If a player purchases five combinations on each play and each combination costs \$1, then the total cost would be \$500,000.

Using $Y \sim Po(m)$, one can investigate the probability of winning twice as the number of independent plays, n, approaches a much larger number and five different six-number combinations are played each time. If $n = 1{,}000{,}000$, then $P(Y = 2) = .12370$. The odds against winning would be 7.1:1. These odds are not too bad. However, if one played this version of the lotto game twice a week, 9,615.38 years would be needed to reach 1,000,000 plays. And at a \$1 per combination, the cost would be \$5,000,000.

Conclusion

State lotto games offer a wealth of problems and exercises for classroom activities. Students can create their own lotto games and study the probabilities associated with the game. For example, suppose that each student selects one three-number combination from among the integers 1, 2, 3, . . ., 21. The winning three-number combination is randomly chosen by drawing three slips of paper from among twenty-one slips numbered 1 to 21. Any student matching the winning combination wins the jackpot—perhaps a bag of M & M's. Before actually playing the game, the class can compute the probability of a given student's winning if the game is played repeatedly. Another interesting problem for students to explore is the likelihood that some student in the class wins on one play of the game. Also, they can compute the probability of winning twice if the game is played repeatedly. The students come away from this

Table 1
Comparison of Binomial and Poisson Probabilities

n	Binomial Probabilities of Winning Twice	Odds Against Winning Twice	Poisson Probabilities of Winning Twice
1,000	2.5043×10^{-7}	3,993,131:1	2.5124×10^{-7}
10,000	2.4906×10^{-5}	40,152:1	2.4956×10^{-5}
100,000	2.3369×10^{-3}	427:1	2.3349×10^{-3}

exercise with a much greater understanding of the remote chance that anyone has of winning a lotto game.

Yet another intriguing facet to study is the mathematical expectations of the random variables $X \sim Bi(n, p)$ and $Y \sim Po(m)$. The mathematical expectations $E(X) = np$ and $E(Y) = m$ are infinitesimally small and reveal further the futility of gambling.

Students can also simulate lotto games on the computer to determine empirically the probability of winning and compare the outcome with the theoretical probability of winning. As one can see, the possibilities for problems and exercises are numerous.

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Leonardo da Pisa (c.1175–c. 1250)

This Italian mathematician, who became famous under the name of Fibonacci, posed the following problem in his book *Liber Abaci*:

Determine five weights to be used to weigh objects, ranging from 1 kg to 30 kg, with the weight of the objects being whole numbers. A balance scale is being used. What must the weight of the five different weights be?