Problem Solving: Dealing with Data in the Elementary School

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Standard 11 of the K–4 recommendations of the National Council of Teachers of Mathematic's *Curriculum and Evaluation Standards for School Mathematics* and Standard 10 of the Grades 5–8 portion of this document suggest that students be given opportunities to

- collect, organize and describe data;
- formulate and solve problems that involve collecting and organizing data; and
- develop an appreciation for statistical methods as a powerful means for decision making (1989, 54, 105).

A basic assumption of the standards document is that students will learn better through problem-solving situations that involve them doing mathematics rather than having it done to them—so that they become producers of knowledge rather than merely consumers. The Elementary Mathematics Research Model furnishes a vehicle for problem solving through real data collection and analysis.

The Elementary Mathematics Research Model

To incorporate a research component into the curriculum, two aspects must be considered. First, students need a research model that is easy to understand and apply. Second, students must have an understanding of some basic statistical tools, such as mean, median, mode and range. At higher grade levels, measures of dispersion other than the range might also be included. Rather than being taught as isolated topics, the statistical tools are used in applying the research model to real situations. This concept is supported by Moore (1990), who emphasizes the need for statistics to be couched in realistic settings.

Getting Started

The Elementary Mathematics Research Model (Irby and Bohan 1991) has students move through seven steps to produce knowledge through mathematics. See Figure 1. In step 1 students must attempt to identify a problem. For the students to become involved and have ownership in the project, the first item of business is to let them think—of things that they would like to know, of some questions that they would like to answer or of some problems that they have observed in the school or community. During this brainstorming session, establish a rule that no one is to judge the thoughts of another. Let the ideas come freely. If someone repeats an idea already on the chalkboard, go ahead and write it. Never say, "We already said that," as this type of response stifles creative thinking. The job of the teacher is to see that a risk-free environment is maintained. After brainstorming, let the students take one of the generated ideas and work through the remaining steps in the design.

Step 2 is a natural outcome of step 1. One of the issues from the brainstorming session is chosen, a problem to be solved is developed and a research question is stated. The following is a problem formulated from a brainstorming session in a sixth-grade class:

> The students were concerned with the amount of garbage produced in the school cafeteria and its impact on the environment (the problem). The research questions was, What part of the garbage in our school cafeteria is recyclable?

In step 3 students hypothesize the expected outcome of the research. The teacher might ask, "What do you think will be the outcome of your research or investigation?" Students should be accustomed to hypothesizing from science classes. With regard to the first question, the students might answer, "We believe that half of the waste is recyclable."

Step 4 will find students developing a plan for how to test the hypothesis and answer the question. The following items will need to be considered in developing the plan: (a) permission—who will give us permission, the principal, the cafeteria supervisor, the maintenance director or others? (b) courtesy—when can we conveniently discuss this project with the cafeteria management? (c) time—how much time can we spend on this investigation, when should we do this project each day, how long do we think it will take to gather all the data? (d) money—will it cost anything, how can we get the money, do we need to write a grant proposal to request the money through the principal or the PTA? and (e) safety—what measures must we take to ensure safety, for instance, gloves and masks?

The students will need to develop an exact plan to address these concerns. In the process they may discover subquestions related to the original research question, such as, Which group is more environmentally aware—fifth or sixth graders? On which day do most students bring lunches? What buying trends should be observed by the cafeteria management on the basis of analysis of food in the garbage? Each question may call for different statistical treatments.

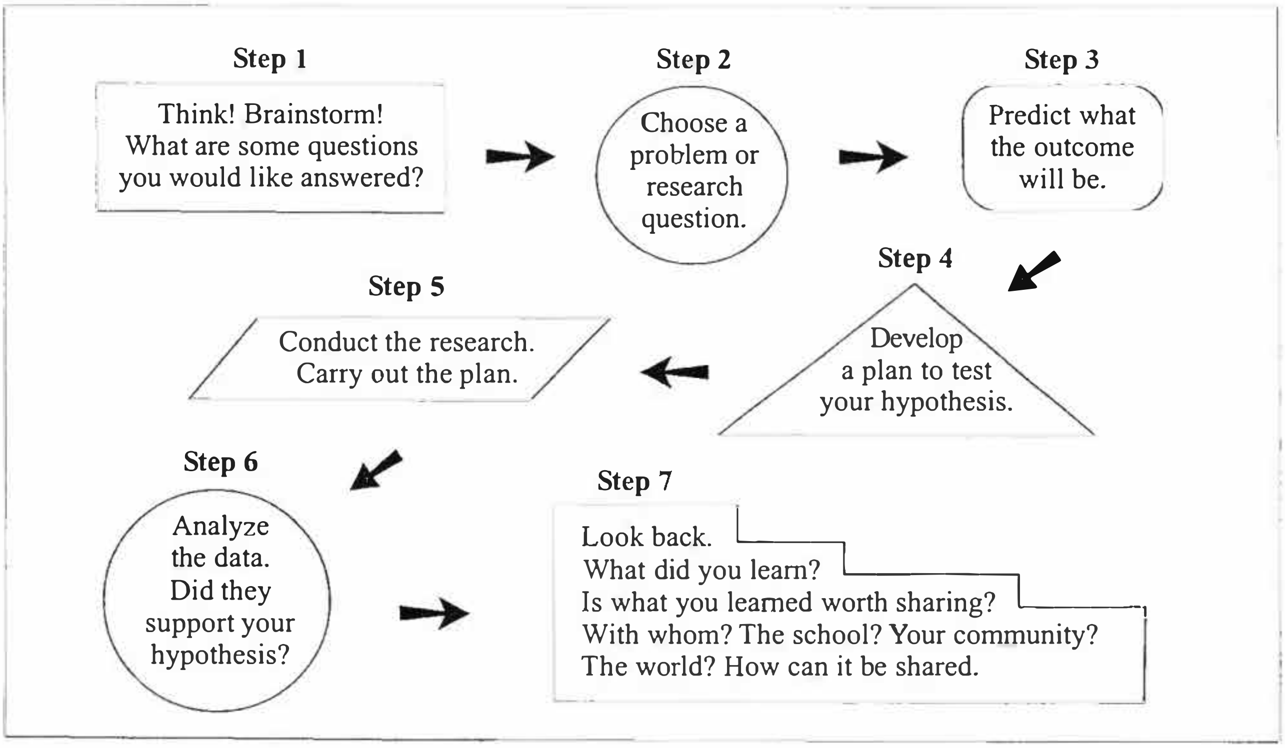
The teacher may have different groups in the class work on each related question, so that at the end of the research all questions will have been answered. Each subquestion will prompt development of a specific plan:

> We will have our study last for three weeks, giving us fifteen opportunities to collect data. We will check the garbage every day and request that it not be thrown out until we do so. We will request the help of our fellow students when throwing out their garbage in the cafeteria by requesting that they separate it into six different cans that are clearly marked—uneaten foods, partially eaten food, Styrofoam, paper, plastic and aluminum. We will weigh the amount of each can and keep the records each day. The number of aluminum cans will be counted.

As the students determine how they will gather the data, they need to determine what variables are involved in the research study. In this example, they might determine that the weight of each individual can be one variable, the length of the study might be another and so on.

Step 5 is "carry our the actual plan." During the time the data are being collected, discuss the ways in which the students might report the findings. Graphs should certainly be discussed as a possibility, as should types of graphs best used for various purposes (see, for example, Curcio [1981]). At this point, the need for statistical measures to describe the data becomes apparent. For example, since this study is to last 15 days, it is not probable that the same number of aluminum cans would be collected each day. How can the number of cans collected daily be described without having to list 15 numbers?

Figure 1
The Elementary Mathematics Research Model



Developing Measure of Central Tendency

The Mean

To teach the concept of mean, pose a situation for students in which 80 cans are collected one day and 60 the next. Have students use a metrestick and adding-machine tape to represent these numbers by cutting off pieces 80 and 60 centimetres in length. This tactic gives students a physical representation of their two-day collections. Have students attach the tapes end-to-end. Hold up the combined tapes and ask, "What does this paper represent?" (the total number of cans collected for two days). "Use this paper and the metrestick to decide the total number of cans you have collected."

"Suppose that on two other days you collected the total number of cans represented by the combined tapes. However, an equal number of cans was collected each day. Use the combined tape to decide what that number was." Since this paper represents two days of collecting, the combined tape can be folded into two equal parts and compared with the metrestick to find the number. Once the number 70 has been determined, define this number as the mean. Repeat this activity with different numbers of cans and days; this extension is necessary, as otherwise some students form the misconception that we always divide by 2 when finding the mean.

Present various situations in which students try to predict what would happen to the mean, if, on the next day, a greater or smaller number of cans was collected. Predictions can be investigated by using adding-machine tapes. The conceptual work done with the tape can readily be connected to the symbolic procedure for finding the mean. Connecting the tapes represents finding the sum of the numbers, and folding the combined tapes represents dividing the total into equal parts. The number of parts into which the combined tape is folded is determined by the original number of pieces of tape.

Figure 2
A Table of 15 Number Cards

				80				
	72		76	80	84	85		
61	72	74	76	80	84	85	87	91

Demonstrate the need for other measures of central tendency by pointing out the main weakness of the mean—the extent to which its value can be affected by extreme scores. This weakness can be demonstrated within the framework of activities discussed earlier by showing the effects that a day when no cans were collected would have on the mean of three days of collection averaging 80 cans per day. The median and mode can then be presented as different measures of central tendency that minimize the effect of extreme scores.

The Mode

To teach the meaning of the mode, have students write 15 numbers on index cards and place them in a box. For example, 76, 80, 84, 72, 85, 80, 74, 61, 72, 84, 76, 80, 91, 87 and 85. Have students pull a card at random from the box and place it on a chart. A second card is then extracted, and the question asked, "Is this number greater than, less than or equal to the first number?" This second card is placed on the chart to the right, to the left or above the first card depending on whether the number on it is, respectively, greater than, less than or equal to the number on the first card. (See Figure 2.)

The students continue to pull cards, asking the same question over and over until all cards are arranged on the chart in order, left to right, from smallest to largest. Looking at all the cards on the chart, ask, "What number appeared the greatest number of times?" After identifying the tallest column, define the number of cards in that column as the *mode.*

The Median

To teach the concept of median, have students use the 15 numbers they have placed on the chart. Ask, "Where have you heard the word *median* used before?" (The median of the highway is the part that divides the highway into two equal parts.) "In mathematics the word *median* is used to tell us something about a set of numbers. What do you think it tells us? (It is the point that separates a set of numbers into two equivalent subsets.) Have students work with a partner to find the median of the set of numbers represented by the cards on the chart. One way is to begin removing cards from either end of the set simultaneously, one with each hand. This process is continued until only a single card remains. Have groups share their method with the class. Identify the number on the middle card as the median.

"If we find the median by eliminating cards from each end, will we always get to a point where a single card remains?" After getting a consensus that we would not, discuss the conditions under which this outcome would or would not occur, capitalizing on the opportunity to review the concept of even and odd numbers. Next place an even number of cards in the box, place them on the chart in order as indicated previously and eliminate cards simultaneously from either end until only two cards remain. Give students an opportunity to discuss how the median might be identified. Try to get the class to agree that the best solution would be to call the point halfway between the two remaining cards the median. Introduce situations in which

- the median is not a whole number, as when the two remaining cards contain such consecutive numbers as 87 and 88 (median 87.5); and
- the median is a whole number but not a number on one of the cards; for example, the two remaining cards have such whole numbers as 84 and 88 (median 86).

In either case, the median is the mean of the two remaining numbers.

Dealing with the Data

In step 6, at the end of the three weeks, analyze the data. The question to be answered is, "Did the test support our hypothesis?" The data will be analyzed on the basis of the statistical tools previously developed.

As they look back in step 7, students should ask such questions as the following:

- What did we learn?
- Will our findings contribute to our school, our community or our world?
- How can we share our findings with others?
- If we repeated this experiment during a different three weeks, would we expect the same results? Why or why not?
- Who might be interested in our results?"

The teacher should assist the students in presenting the findings to a particular audience. In the example presented here, the students presented the information to the fifth-grade students, the cafeteria workers and the teachers. A formal presentation with charts and graphs is important in showing students that research is valuable when it can be related to the real world and put into practice. Additionally, it emphasizes the need to communicate mathematically.

A Real-Life Research Winner

Using the Elementary Mathematics Research Model, Dolly Vogel, a Grade 6 mathematics teacher at Houser Intermediate School, Conroe, Texas, presented her students with the opportunity to conduct research. She encouraged her students to focus their studies on science and mathematics. All studies had to be submitted with a written report in research format and had to include statistical data that were graphically displayed. The three studies submitted for review were as follows:

Group 1: Which pollutants are most harmful (Survey research)
Group 2: Does life exist on other planets? (Survey research)
Group 3: How much trash can the students at Houser Intermediate School eliminate by recycling aluminum and Styrofoam? (Observational research)

Significant findings were reported by each group, with the findings from group 3's research of particular interest. The students found that, by recycling only the aluminum and Styrofoam, the school's garbage could be cut in half. As it turned out, the research was award-winning, with the school receiving a set of statistical software from the American Statistical Association, which sponsored the competition. (For information on this national contest, write to the American Statistical Association, 1429 Duke Street, Alexandria, VA 22314-3402.) Mrs. Vogel and her students are to be commended for their award-winning efforts.

Conclusion

The Elementary Mathematics Research Model allows the students to begin early to collect, organize and describe data. The model is founded on problem-finding and problem-solving behaviours and is designed to support the development of higher-level thinking as students' thoughts diverge and converge throughout the research process. Additionally, the appreciation for statistical methods used in problem solving can be emphasized. Zawojewski (1988) reported that when students applied memorized algorithms for finding measures of central tendency in a role manner, they tended to make predictable errors that they did not tend to make when these measures were presented in the context of real-world situations.

The research studies may be completed as group or individual projects. As studies are developed they may be concentrated in the community or in the school. Although in some instances the teacher's assistance may be required, research topics should preferably be chosen by students. The teacher's responsibility should be to assist with the design of the study so that the students will be able to use statistical treatments, tools and terms in the analysis of the data collected in the study. The format for a report and record keeping throughout the project is open.

The final thought to leave with students is that they can be researchers and products of new information and that new knowledge can be produced and communicated through mathematics. Their findings may contribute to the knowledge base of the class, the school, the community or society as a whole. Their findings may affect their school or their world in a very positive way.

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Johannes Buteo (c. 1525)

This French mathematician wrote the following in his book *Logistica*:

If the price of 9 apples, reduced by the price of one pear, is 13 Dinar, and the price of the 15 pears, reduced by the price of one apple, is 6 Dinar, what is the price of an apple and a pear?