Activities for the Middle School Math Classroom: Games and Problem Solving

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There is an unmistakable similarity and strong link between problem solving and mathematical games. First let's explore problem solving. Consider this definition of problem solving: Problem solving is a process that occurs when a set of conditions and a goal are given, but the solver must provide or develop the means to achieve the goal. In other words, problem solving is an active task that has both a starting and (at least one) end point.

Process

The whole notion of problem solving as process is extremely important because we must teach our students that most problems do not have an obvious or necessarily quick solution. Indeed most problems in real life are not resolved quickly at all. Wouldn't it be nice if they were? Many of our students become handicapped by the two-minute rule: they say or think, "If I can't solve it in the first two minutes, I will never solve it," and they concede defeat too quickly. Unfortunately we often encourage the two-minute rule through examples we present (always solved correctly within the time limits) and our evaluation practices (20-minute quizzes comprising 10 questions). Real problem solving takes and deserves adequate time. In fact, we may have to accept that real problem-solving abilities can not realistically be measured in a typical testing format; it simply contradicts the process dimension of the activity.

Conditions and Goal

Problems always come with a set of conditions. Sometimes the conditions are obvious and straightforward, and sometimes we have to collect our own data, sort through the information given to weed out extraneous statements and so on. The goal in a problem becomes the resolution of these conditions. When we have somehow reconciled all of the information given, collected or found (and thus met the task presented), we consider the problem to be solved. Typically the conditions in a problem are given as a series of statements, and the goal is given as a question that follows those statements. We can dramatically alter the complexity and interest of problems by altering the number, nature and format of both the conditions and goal.

Developed Means

If it is intuitively obvious to the solver how the conditions and the goal(s) may be reconciled; that is, how the problem is solved, then it is probably fair to say the given activity is merely a task (in classroom terms, a drill-and-practice exercise) and not a problem at all. In other words, in real problem solving we expect to have to wrestle a little. We expect the activity to be challenging and fraught with frustration, some disappointments and failures and, of course, some successes. The nature of problem solving is that the solver brings forward his or her own insights, skills and strategies into the process while developing new methods, thus experiencing a number of small failures and victories before the ultimate success (a solution) is finally attained.

The above elements constitute a basic description of problem solving, but let's not forget that problem solving is ultimately a complex task. For example, remember that the solver must also be motivated to solve the problem. The problem must have some relevance or inherent interest about it before the solver will be willing to engage it. Let's also not forget that another higher level of mental processing is associated with problem solving: the ability to monitor progress, the ability to evaluate the effectiveness of strategies, the flexibility to modify the chosen strategies that have proven to be ineffective and so on. While it is not the purpose of these games specifically to address these higher mental abilities, the good news is that these metacognitive talents can be encouraged and can be (should be) actively taught in our classrooms.

But what does all this have to do with games in the math classroom? Simply put, a game is in fact a type of problem. Like a problem, a significant process is involved in playing the game, and like a problem a game usually requires a strategy and a means to achieve the goal. It is fair to think of the rules of the game as the conditions of a problem, and the goal of the game is usually how the game is won or finished. Sometimes the goal is to beat your opponent and sometimes it is just to work well with your team or to make it further in the game than the last time. A game is a complex problem that often contains many smaller problems embedded within it. The advantage a game has over more typical problem-solving activities falls in the motivation element inherent within it. My students used to groan when I announced we would try some problems, yet they would cheer when I announced I had a game for them instead.

Problems come in many different forms, and games represent one motivating way to engage our students in some significant and challenging problem-solving activities.

Here are some suggestions and considerations for integrating games in your math classroom:

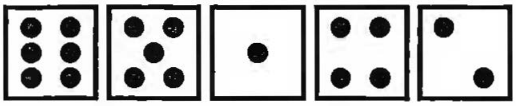
- Where possible emphasize the mathematics vocabulary used in any game. To work with others, we need to communicate clearly, and students will need to use appropriate mathematical terms to communicate effectively during all classroom activities.
- Try to collect games that can be easily adjusted to make them more or less difficult. Highly flexible games can also let you apply a familiar game structure to an alternative math concept. Games that are flexible in this way help reduce the amount of time spent introducing a new game to the class.
- Recognize that games are appropriate for all grade levels; young children and adults alike enjoy these types of activities. Students may have some suggestions for how certain games can be modified or adapted, consider getting your students to modify, develop and create their own games, too.

Five Dice

Objective: Solve problems involving multiple steps and multiple operations (Alberta mathematics program of studies, Number [Number Operations], Grade 5, Outcome 13)
Materials: Five regular six-sided dice for each team or player, pencil, paper and calculator.
Players: Two or more

Rules

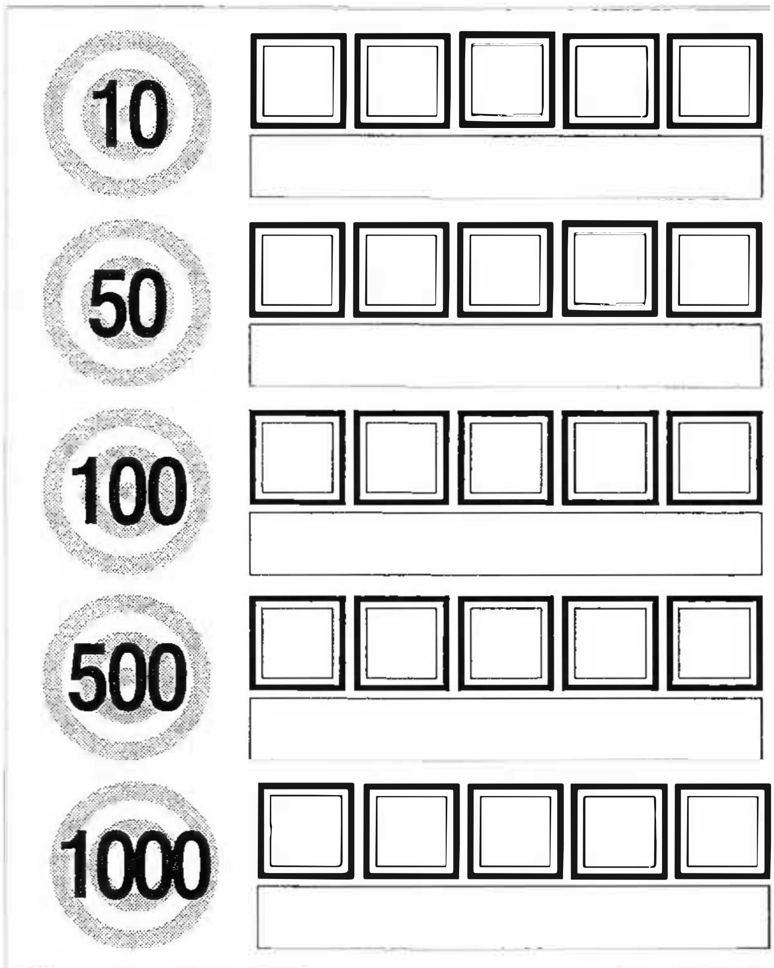
1. Each player or team rolls five dice and records the values rolled.
2. These values are now used to construct an equation with the result as close to the target as possible using the rules stated below. Assume we rolled the values shown:



   a. Numbers may be used to specify place values. For example, using the digits above, we could make the values 56, 16 or even 156 or 651 and so on.
   b. Brackets may be used wherever desirable.
   c. Exponents are not permitted.
   d. The values may be used to create whole numbers only.
   e. A number may be used only once unless doubles are rolled.
   f. All numbers must be used.
3. Players roll the five dice for each new target number.
4. After completing each target, teams or players compare results to see which equation has a value closest to the target. The closest player scores one point for each target.
5. The team with the greatest point total at the end of the game wins.

Adaptations

1. Substitute 10-sided dice for the 6-sided dice.
2. Have players use the values to build equations with results between 0 and 100. The player who can build the greatest range wins.
3. Permit players to use exponents.



Factors

Objective: Distinguish among and find multiples, factors, composites and primes, using numbers 1 to 100 (Alberta mathematics program of studies, Number [Number Concepts], Grade 6, Outcome 3).
Materials: Factors game board, two crayons, calculator
Players: Two

Rules

1. The objective of this game is to shade the greatest number of spaces on the Factors game board as possible, which, when summed, have the highest total.
2. On a turn a player will select any available number (a number not already shaded) and will proceed to shade it as well as all remaining numbers that are a factor of his or her number. For example, if a player chooses the number 75, he or she could shade all of the following (assuming they have not already been shaded on an earlier turn or by another player):

75	1	3	5	15	25

3. Players alternate until each player has had three turns. At the end of the third turn, players take the calculator and add up the numbers they shaded.
4. If a player fails to shade a factor that he or she could have shaded, the opponent may point out the error and claim that number. If a player shades a number incorrectly, the opponent claims that number.
5. The player with the highest sum after three turns wins the game.

Adaptations

1. Instead of playing for the highest sum of shaded numbers, play to see who can shade the greatest number of spaces.
2. Allow players to use factors more than once. If a player selects 100, the number 100 is out of the game, but the factors (1, 2, 4, 5, 20, 25 and 50) can be used again.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Pattern Detective

Objective: Represent, visually, a pattern to clarify relationships and to verify predictions (Alberta mathematics program of studies, Patterns and Relations [Patterns], Grade 6, Outcome 1).
Materials: Pattern Detective game board (one per player)
Players: Two

Rules

1. To begin the game, each player places a series of letters arranged in a pattern in the boxes down the left side of his or her game board. Players may only use the letters A, B, C and D to construct their patterns, and letters must be arranged in a pattern. Examples:
   - A B C A B C A B C A B C A B C A B C A B
   - C A A C A A C A A C A A C A A C A A C A
   - D B D D B D D B D D B D D B D D B D D B
   - D C A B D C A B D C A B D C A B D C A B
2. Players now take turns guessing what letters are written in their opponent's boxes. Example: "Do you have a C in box 3?"
3. If a player guesses correctly, he or she gets to keep guessing until a mistake is made. After a mistake, play passes to the opponent.
4. The first player to correctly reveal his or her partner's entire pattern wins.

Adaptations

1. Include more players. Players drop out of the game as their patterns are uncovered by other players. Last player in the game is the winner.
2. Allow a greater range of letters or symbols for more complex patterns.
3. Use numbers instead of placing letters in the boxes. Players must provide to their opponent the first two numbers in their pattern at the start of the game. This allows for much more complex patterns, such as growth patterns:
   - 0, 1, 1, 2, 3, 5, 8, 13, 21, ...
   - 1, 2, 4, 7, 11, 16, 22, 29, 36, ...
4. Play cooperatively as teams rather than as individual players.

My Pattern	Correct	Incorrect Guesses
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	
9	9	
10	10	
11	11	
12	12	
13	13	
14	14	
15	15	
16	16	
17	17	
18	18	
19	19	
20	20	

Triplet

Objective: Make a connection between the number of faces, for various dice, and the probability of a single event (Alberta mathematics program of studies, Statistics and Probability [Chance and Uncertainty], Grade 6, Outcome 11

Materials: A variety of dice with different numbers of faces (for example, 4, 6, 8, 10, 12, 20-sided dice), Triplet game board, two crayons.

Players: Two

Rules

1. On a turn, a player selects one die and rolls it. This player now colours a space in any one of the three game boards corresponding to the value rolled.
   Example: If the player selects the eight-sided die and rolls a seven, the player could cross off any one of the following:
   - The space labelled >6 (greater than 6) in the top board
   - Either the 7 or odd space in the middle board
   - Either the prime or <20 (less than 20) space in the bottom board
2. Play passes to the opponent.
3. A player may cross off only one space on a given turn. If there is no remaining space the player can claim, the player passes the turn.
4. The first player to cross off three spaces in a line (horizontal, vertical or diagonal) in any of the three game boards is the winner.

6	<4	11
16	10	>6
even	13	3

<3	20	7
12	odd	>10
9	6	4

18	1	prime
11	5	>2
<20	2	8

Adaptations

1. At the start of the game, have players select one game board as their own. Players now take turns rolling and racing to cross off all of the spaces in their respective game boards.
2. Use plastic bingo markers of two colours. Instead of shading spaces, use the bingo markers to show chosen spaces. Your opponent can claim that space back from you on a later roll by removing your marker and adding his or her own.
3. Imagine that the game boards are stacked one on top of the other and allow three in a row in three dimensions. For example, the even, odd and prime spaces would be three in a row.
4. Allow players to select two dice and add or subtract the values rolled.

Four in a Row

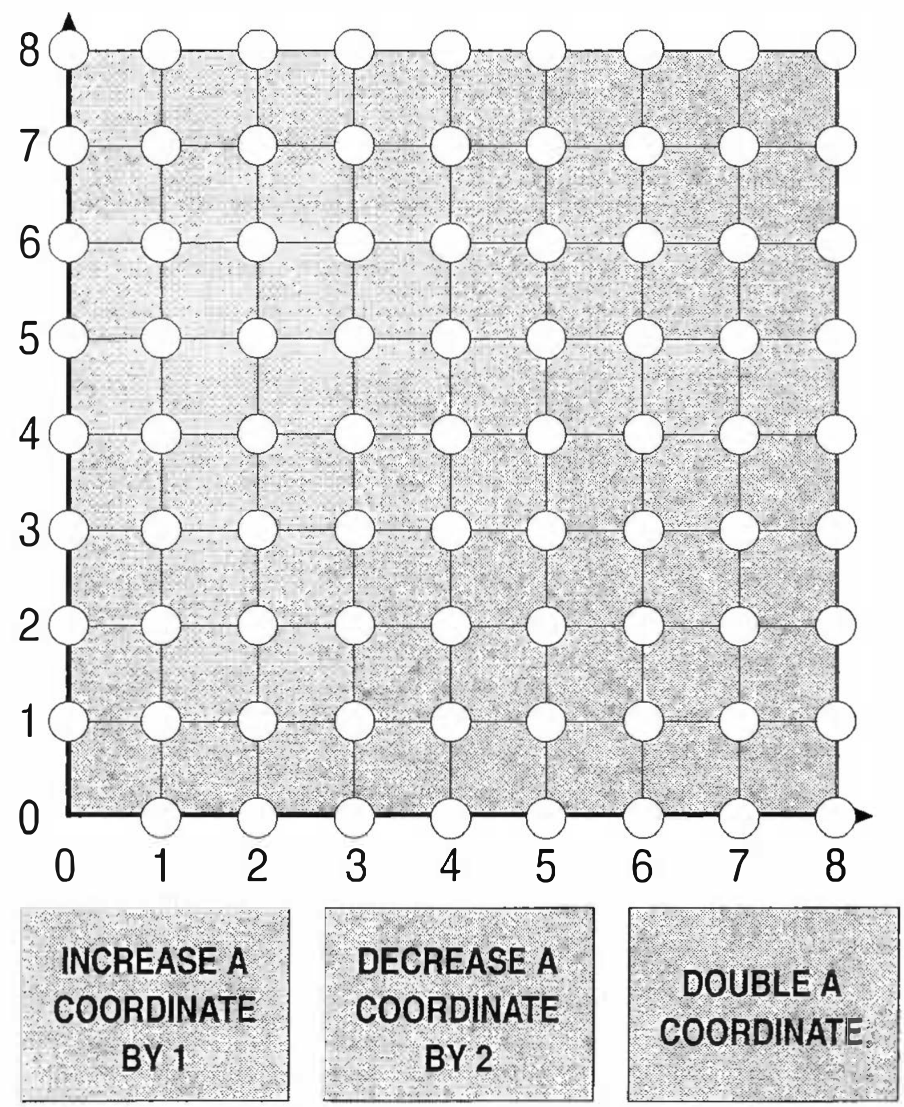
Objective: Graph relations, analyze the result and draw a conclusion from a pattern (Alberta mathematics program of studies, Patterns and Relations [Patterns], Grade 7, Outcome 3)
Materials: Four in a Row game board, two eight-sided dice (blue and red), two crayons
Players: Two

Rules

1. The objective of this game is to plot four points that fall in a straight line on a Cartesian coordinate system.
2. On a turn a player rolls both dice. The blue die represents the x-coordinate of a point, the red die the y-coordinate. The player may opt to plot the point as rolled, or may opt to alter either the x- or y-coordinate, but not both.
3. Either coordinate may be altered using the rules shown.
   Example: If the player rolls a three on a die, it could be used as a one (by selecting the first box: "decrease the coordinate by two"). Likewise, it could be used as a four or as a six.
4. The player now plots his or her point in his or her colour crayon. Players continue taking turns plotting points according to these rules until either player has positioned four points in a row horizontally, vertically or diagonally. This player is the winner.

Adaptations

1. Substitute or add other rules that may be used to alter coordinates.
2. Establish two sets of formulas for altering coordinates. One set applies to the x-coordinates only, the other to the y-coordinates. A player may select from either list, neither list or both lists on a turn.
3. Include two 4-, two 6- and two 8-sided dice. A player may select any combination of two dice to roll on a turn.
4. Players work cooperatively to try to colour all spaces in the fewest number of rolls.
5. Allow players to apply more than one rule on a turn.



Percentage Snap

Objective: Estimate and calculate percentages (Alberta mathematics program of studies, Number [Number Operations], Grade 7, Outcome 18)
Materials: Deck of cards (jack, queen, king removed, A=1), calculator
Players: Two

Rules

1. To begin the game, shuffle the cards and split them into two equal-sized piles. Each player takes one-half the deck and holds the cards face down in his or her hand.
2. At the same time each player turns over one card and places it on the top of the table.
3. Each player now estimates the percentage equivalent to the fraction formed by the two cards (to the nearest 10 per cent). For example, if the cards were a two and a six, the players would estimate a value as close as possible to 33.33 per cent (that is, 30 per cent).
4. The first player to call a value wins the two cards if there are no closer estimates. The second player must provide a closer estimate or concede the cards.
5. The closest estimate to the actual per cent (use a calculator to calculate actual percentages if necessary) wins those two cards.
6. Play continues until both players have exhausted their decks. The player with the most cards at the end of the game wins.

Adaptations

1. Play the same way, but let the fractions created all be improper fractions, that is, the larger number over the smaller.
2. Play the same way, but instead of calling out percentages, call *snap* every time the fraction built has a percentage equivalent to any one of the following: 25 per cent, 33 per cent, 50 per cent, 67 per cent, 75 per cent or 100 per cent
3. Estimate to the nearest 5 per cent.

Target Game

Objective: Estimate, compute and verify the sum, difference, product and quotient of rational numbers (Alberta mathematics program of studies, Number [Number Operations], Grade 8, Outcome 10)
Materials: Calculator
Players: Two or more

Rules

1. To begin the game, have one player type into the calculator a random four-digit number, no two digits alike. For example, start with 2753 in your calculator.
2. The players will now take turns multiplying the value in the calculator by any decimal they wish, trying to obtain a product that falls in the range specified below:
3. $499 \leq x \leq 501$
4. The calculator is not cleared between players, but instead, each player starts with the value left behind by the previous player.
5. The first player to hit the target is the winner.

Adaptations

1. Play again, but this time you may only divide (no multiplying) to hit the same target.
2. Start with a larger (5- or 6-digit number) or with a value between 0 and 1.
3. On a turn, before you may multiply or divide, you must hit the square root key.
4. Play as a solitaire game: what is the fewest number of turns necessary to hit the target?
5. Play against a friend. Play the game as above except each player has his or her own calculator. Each player may take four turns. Whoever is the closest after four turns is the winner.

The Right Stuff

Objective: Use the Pythagorean relationship to calculate the measure of the third side of a right triangle, given the other two sides in two-dimensional applications (Alberta mathematics program of studies, Shape and Space [Measurement], Grade 8, Outcome 2)
Materials: The Right Stuff game board, ruler, pencil, calculator, one six-sided die, one ten-sided die
Players: Two

Rules

1. On a turn, a player rolls both dice (that is, both the six- and ten-sided dice). These values represent the base and height of a right-angled triangle.
2. The player now draws this triangle on the grid provided and, using the Pythagorean relationship, calculates the length of the third side. The accuracy of the calculation can be checked by his or her opponent by measuring the length of the third side (in centimetres).
3. Using the table provided, the player now determines his or her score for this roll, based on the length of the third side.
4. Players both take three turns, rolling the dice, drawing the triangles, calculating the length of the hypotenuse and scoring points. The player with the highest score after three turns wins.

Example

Assume a player rolls a five (on the 6-sided die) and a seven (on the 10-sided die). The player would draw a triangle with a base of 7 cm and a height of 5 cm.

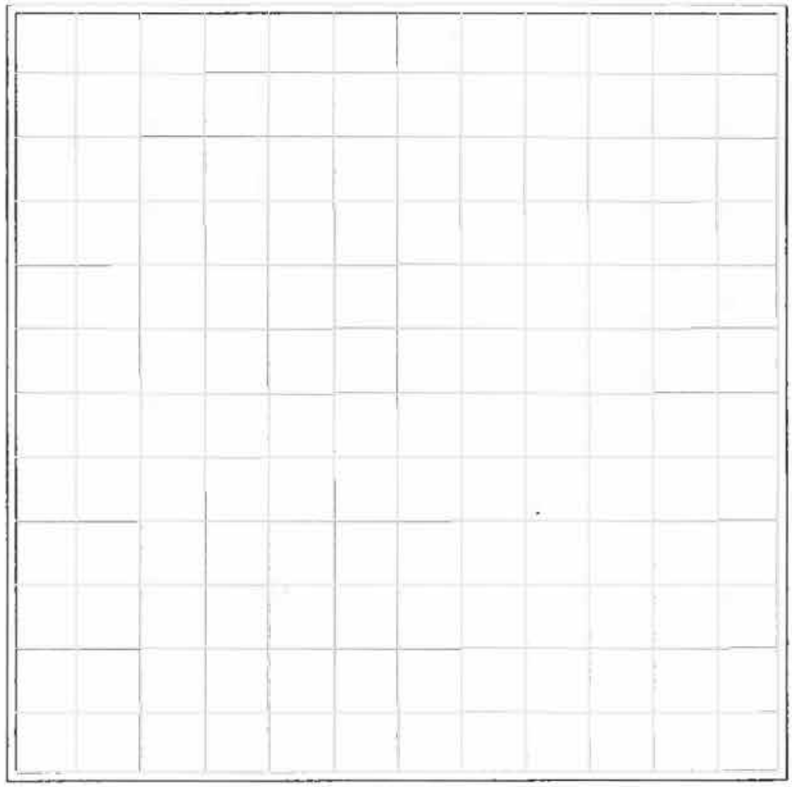
The third side would have a measure of 8.60 cm, and the player would score two points.

Adaptations

Change the rules so that the player scores the same number of points as the length of the third side.

For a solitaire game, allow a player to take as many turns as she or he can up until she or he can not fit the triangle on the grid. Triangles may not overlap, cross, be contained within another triangle and so on.

Score double points if the third side of your triangle has a whole number value.



1.00 – 3.00 cm	4 points
3.00 – 5.00 cm	2 points
5.00 – 8.00 cm	1 point
8.00 – 10.00 cm	2 points
10.00 – 12.00 cm	3 points

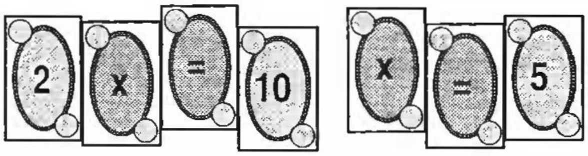
Your score:

Equation Rummy

Objective: Solve and verify one- and two-step first-degree equations (Alberta mathematics program of studies, Patterns and Relations [Variables and Equations], Grade 8, Outcome 5)
Materials: Equation Rummy cards
Players: Two or more

Rules

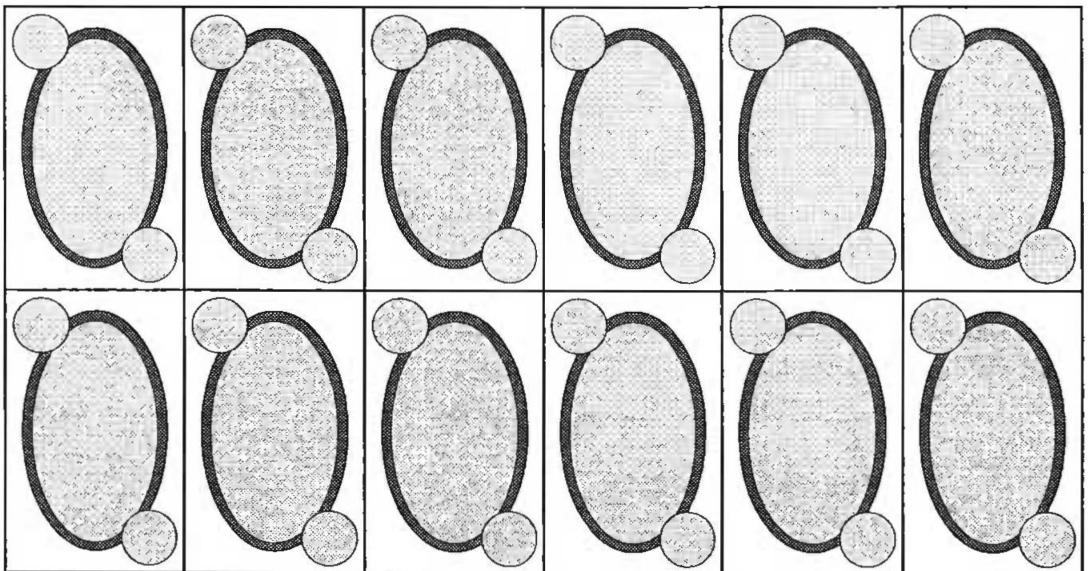
1. Shuffle the cards well and deal each player seven cards, which will serve as that player's hand. Place the remaining cards face down in a pile. Turn over the first card to start the discard pile.
2. On a turn, a player may draw either the top card from the deck or the top card from the discard pile. This card may either be discarded or exchanged for one card in the hand. Discarded cards are placed face up on the discard pile.
3. Players continue exchanging cards until they construct a collection of cards that build two equations of the form $ax=b$ and $x = b/a$. An example:

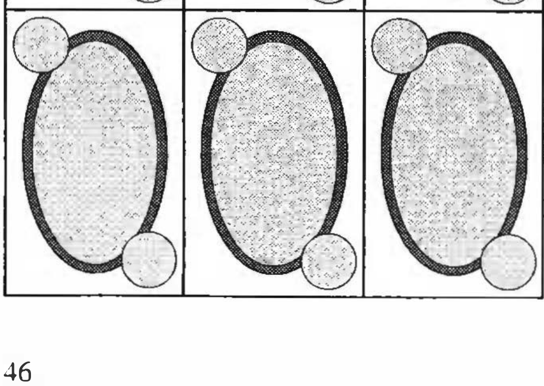


4. The first player to construct the pair of equations is the winner.

Adaptations

1. Add some "+" and "–" cards to the deck and start each player with eight cards. Build equations of the form $x + a = b$ and $x = (b - a)$.
2. Add the "+" and "–" cards and build equations of the form $ax + b = c$ and the root $x = c - b/a$.
3. Add two wild cards.





To make Equation Rummy cards:

2: make two	**3:** two	**4:** three	**5:** two
6: three	**7:** one	**8:** two	**9:** two
10: two	**12:** three	**14:** one	**15:** one
16: two	**18:** two	**20:** two	**21:** one
24: two	**27:** one	**28:** one	**30:** one
32: one	**36:** one	**40:** one	

10 variable (x) cards 10 equals (=) cards