A Focus on Fostering Number Sense Makes a Lot of Sense

Werner Liedtke

Number sense is the most important pre- and corequisite for numeracy. Number sense contributes to flexible thinking in numerical situations and the ability to solve problems. Without a conscientious focus on fostering number sense through key aspects of the curriculum (the teacher, resources, materials and so on), it is unlikely that most students will develop this sense. Consideration of number sense must go beyond unit topics and should, whenever possible, be part of the ongoing program. When students are not able to make sense of the numbers being manipulated, mathematics learning becomes rote or, as research shows, overwhelming and anxiety inducing.

In *A Handbook on Rich Learning Tasks,* Flewelling and Higginson (2001) label the term *rote learning* an oxymoron (p. 24), identify rote learning as a major source of anxiety (p. 28) and suggest that it is an impediment to problem solving (p. 26). The authors state, "Rote-learning-plus-practice techniques train problem solvers as well as paint-by-numbers techniques train artists" (p. 27).

In this article, I identify the important components of number sense and illustrate them through examples using two-digit numbers. Most of these ideas can easily be adapted to other whole numbers and to fractions, decimals and integers.

Visualization

When students hear or see a two-digit number, they should be able to visualize the number. In responses to interview questions about numbers, students uttered phrases such as "I see it in my brain" and "I see it in my mind." For example, when they hear the words *thirty-four* or see the numeral *34,* students should be able to "see" the smallest number of base-10 blocks, dimes and pennies, or $10 and $1 bills needed to represent that number.

To develop this sense, students should learn to make as many groups of 10 as possible from the number, recording the result in a box titled "Tens." What is left goes in a box titled "Leftover Ones." To enhance the association of digits with the appropriate place values, the students record the digits again below and beside these boxes.

Some resources suggest that the main reason for grouping by tens and ones is that it is faster and easier. Nothing could be further from the truth. For children in the early grades, it is much faster to count by ones and easier, or less work, not to group the objects. In fact, the main reason for adopting this procedure is that it allows us to use only 10 digits (an accident of nature?) to record an infinite number of number names.

Flexible Thinking

Students learn that, using only tens and ones, they can show two-digit numbers in at least two ways. For example, students can be given the following problem:

> What are the different ways to show 42 using tens and ones? How do you know that you have found them all?

Students can solve and even create riddles such as the following:

> I have only dimes and pennies. I have six coins. How much money do I have? How do you know that you have recorded all the possible answers?

Connecting

Students should be shown that two-digit numbers connect to many aspects of their experience—money, games, books on shelves, book orders, children in classrooms and so on.

Relating

When we talk about or compare two-digit numbers, we use terms such as *greater than, less than, close to, between, far apart, ones place, tens place, odd, even, sum of the digits* and so on. Teachers can solve and create riddles for missing, hidden or secret numbers on a 99 chart (see Figure 1).

Figure 1
99 Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Over the years, I have collected many excellent secret-number riddles created by Grades 1–7 students. Samples provided by the same group of students over a period of time indicate that sophistication and accuracy increase as students are given more opportunities to create riddles.

The following is an example of a secret-number hunt:

The number . . .
. . . is less than 54.
. . . is not between 36 and 54.
. . . does not have a 3 in the tens place.
. . . is greater than one dozen.
. . . has an odd number as the sum of its digits.

As hints of this type are presented, one at a time, students look at the 99 chart and are invited to respond to the questions, "Which number(s) do you think it could not be?" and "Which number do you think it could be?"

Estimating

A key strategy for learning how to estimate involves using a referent—in this case, a group of 10 or 10 fingers. For example, when students are asked to look at a group of objects and estimate how many objects there are, they use their fingers as a referent and pretend to put the objects in groups of 10. They then report their estimates to the nearest ten (that is, "About __ tens"). For a variation, rather than recording an estimate, students could be given three choices, asked to select the best estimate and be ready to justify their choice. Should any estimates be deemed unreasonable or illogical? Teachers should take great care in assessing estimates. Number sense develops slowly, and estimates may differ greatly from student to student. Labelling an estimate, if it is an estimate, as illogical seems illogical to me.

Resources for teachers include ideas for estimation tasks that pose a question such as "How many marbles are in the jar?" The greatest value of these types of tasks lies not in the numerical responses provided by the students but, rather, in the strategies students use to arrive at their responses and in the follow-up discussion comparing the strategies.

Subitizing

The term *subitizing* refers to the ability to recognize the numerousness of small sets and to attach the appropriate name to the sets without having to count each member. I would like to think that a type of subitizing is possible for two-digit numbers. After students have had many opportunities to represent two-digit numbers in different ways, the teacher could try the following task:

> Ask three children to come to the front of the room. Ask two of them to hold up 10 fingers each and one of them to hold up four fingers (whispering in their ears so the rest of the students can't hear). Then, ask the remaining students, "How many fingers do you see?"

Depending on their previous experiences, many students will be able to identify the number without having to count each finger. As they look at the numbers displayed, students will also reach a stage at which they can state numbers that are one more, one less, 10 more, 10 less, or even double or half of the numbers shown.

Mental Calculation

The task described under the category of subitizing involved visualization or aspects of mental calculation.

It is discouraging to encounter students who, when asked to describe how they might proceed to find an answer without using pencil and paper, explain something that is the same as a recorded algorithm. That is not the intent or the goal of mental calculation. Opportunities for flexibility exist, and different levels of number sense can be accommodated. For example, the teacher can give students the following task:

> Let's pretend that we have 24 books and we order 13 more. How many books will we have?

After two groups of students have been asked to represent these two numbers at the front of the room, different ways to find the answer without using pencil and paper can be illustrated. An interesting discussion can revolve around the question, "Which starting point did you like best? Why?"

Practice

As I pointed out earlier, rote practice is of little value. In *Future Basics: Developing Numerical Power*, Charles and Lobato (1998, 17) state that "appropriate practice can promote the development of numerical power." Number sense is an important aspect of this numerical power. The authors define *appropriate,* in this case, as something involving reasoning, communicating, connecting and problem solving. One of their examples asks students to explain through words and pictures two different ways to find the solution to 43 – 18 without using a calculator.

The calculator can be used in fostering the development of number sense—if it is used in ways other than to check answers (which leads students to ask, "Why did we not use the calculator in the first place?"). Students have to think about the numbers when, for example, a question asks them to think of four different ways to enter the numbers to get the answer and to record how these numbers were entered. Pointing out incorrect answers and determining what went wrong and why can make students think about the numbers (for example, "Someone got an answer of 4,318. What do you think happened?").

Conclusion

With a little imagination, teachers can generate high-order questions that will provoke students to think about the numbers they are working with and, thus, further develop a sense of number. The time and effort required to select and modify activities to make them more appropriate or effective in developing number sense will pay great dividends, because such tasks also foster self-confidence and a positive attitude.

References

Charles, R., and J. Lobato. *Future Basics: Developing Numerical Power.* Golden, Colo.: National Council of Supervisors of Mathematics, 1998.

Flewelling, G., and W. Higginson. *A Handbook on Rich Learning Tasks: Realizing a Vision of Tomorrow's Mathematics Classroom.* Kingston, Ont.: MSTE Group, Faculty of Education, Queen's University, 2001.

Werner Liedtke is a professor emeritus at the University of Victoria in Victoria, British Columbia.

This article is based on key ideas from the author's session "Number Sense: The Key to Numeracy," presented in October 2003 at the 2003 Northwest K–12 Mathematics Conference: "Mathematics: A Peak Experience" hosted by the British Columbia Association of Mathematics Teachers (BCAMT) in Whistler, B.C., and in November 2003 at the MCATA/NCTM Regional Conference: "Taking Mathematics to the N*th Degree" in Edmonton, Alberta.*