

Activities for the Middle School Math Classroom: Games Using Manipulatives

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We have all taught students who can do mathematics yet who do not really understand that mathematics. Such students succeed with one problem or task, but if given a similar but slightly altered or camouflaged task, they are lost. They have probably focused on the routines or procedures (algorithms) associated with concepts but have not really learned the concepts themselves. It is entirely too easy for students to see mathematics as a disconnected collection of algorithms rather than a meaningful body of knowledge overflowing with connections and integrally related to life outside the classroom. Often our students are happy to learn only the algorithms (until their memories are taxed), and we lapse into the contentment of teaching to this inclination. The students' focus on algorithms is probably related to a desire to quickly finish or avoid homework.

But if there is so much more to mathematics than algorithms, what more is there to know? Or, in other words, what does it mean to understand mathematics? The answer to this question is neither short nor simple. However, it is fair to say that incorporating manipulatives in our mathematics instruction usually represents a genuine intent to teach for meaning and to go beyond the first level of instruction, the algorithm.

When we teach with manipulatives, we allow students to learn through more senses and to literally see mathematics in action. Manipulatives lend themselves to exploration, conversation and investigation. In general, we use manipulatives to provide a representation of a concept that enables learning at a

greater depth. The use of manipulatives enables conceptual learning as opposed to procedural or algorithmic learning.

However, like any other useful instructional method, manipulatives are not meant to stand alone. Let's remember that we want our students to know the algorithms in addition to the concepts underpinning those algorithms. A well-balanced program includes opportunities to explore ideas; draw conclusions; and formulate, test and practise algorithms. Manipulative-based games provide motivation and a wonderful learning context in which students can work together to explore ideas, generate new ideas and practise what they have learned.

Here are some considerations for integrating games in your math classroom:

- Where possible and appropriate, include manipulatives in your games. Students must learn how to work with representations and tools, and games provide a motivating and nonthreatening environment in which to do so.
- Consider collecting student game sheets, cards and notes as a way to catch student errors. Be sure to watch the students play the games, and note where misconceptions become evident.
- Be prepared for a more active, noisier classroom. Motivating activities such as games are often noisier than traditional activities.

Note: The objectives in the following games are based on outcomes from Alberta Learning's (1996, 1997) mathematics program of studies.

Fraction Relay

Objective: Represent and describe proper fractions (Number [Number Concepts], Grade 5, Outcome 7)

Materials: Base-10 blocks (small set for each team), hi-lo cards, 6-10 fraction cards

Players: Two or more teams

Rules

1. Before introducing the game, the teacher must make a hi-lo card for each team and 6-10 fraction cards. Hi-lo cards are index cards labelled "Too high" on one side and "Too low" on the other. Fraction cards are index cards with a fraction on one side. The

fractions should be of the form $\frac{x}{1,000}$
—for example, $\frac{156}{1,000}$ or $\frac{24}{1,000}$.

2. Each team selects one member to start. This person comes to the front of the classroom with the starters from the other teams. The teacher shows the starters the first fraction card.
3. The starters return to their teams to help their teammates build the fraction using the base-10 blocks. The starter may not talk but may give his or her teammates clues by showing them the appropriate side of the hi-lo card.
4. Once the team has built the fraction, a team member other than the starter comes to the front of the room and tells the teacher the fraction. If the fraction is correct, the teacher shows the player the next fraction to be built. This player now uses the hi-lo card to provide clues to his or her teammates as they try to build the fraction (as in Step 3).
5. The team that works through the whole set of fraction cards first wins.


Adaptations


1. Instead of limiting the fractions to the form $\frac{x}{1,000}$, use fractions of the form $\frac{x}{10^n}$, such as $\frac{7}{10}$, $\frac{56}{100}$ or $\frac{854}{1,000}$.
2. Play several rounds with every team working on the same fraction at the same time. The team that wins the most rounds wins the game.


FRACTION RELAY

fraction Cards

$\frac{455}{1000}$	$\frac{12}{1000}$
$\frac{886}{1000}$	$\frac{3}{1000}$
$\frac{500}{1000}$	$\frac{352}{1000}$

 = $\frac{1}{1000}$

 = $\frac{10}{1000}$

 = $\frac{100}{1000}$

Risky Patterns

Objective: Construct and expand patterns in two and three dimensions, concretely and pictorially (Patterns and Relations [Patterns], Grade 5, Outcome 3)

Materials: Tiles, a six-sided die

Players: Two or more

Rules

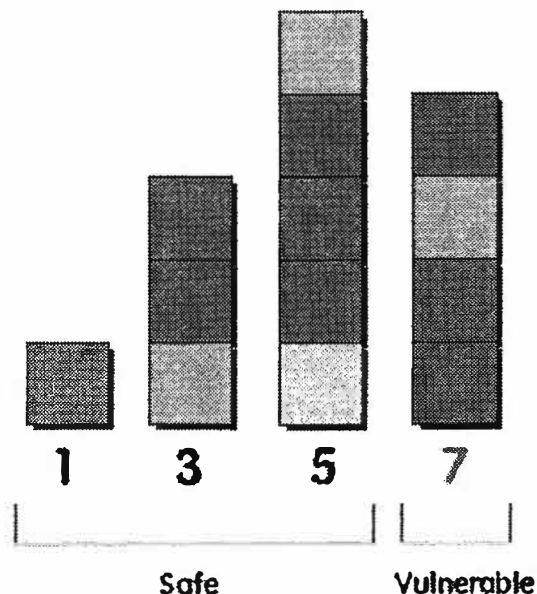
1. Players will attempt to construct the first six elements in the sequence 1, 3, 5, 7,
2. On a turn, a player rolls the die and then adds the specified number of tiles to columns representing the elements in the sequence (see the figure at right). He or she may complete a column and start a new column on the same turn. Alternatively, the player may remove the specified number of tiles from an incomplete column belonging to an opponent. The player may not both add tiles to his or her own columns and remove tiles from an opponent's column on the same turn.
3. The first player to construct all six columns wins.

Adaptations

1. Have students construct other sequences, such as 2, 4, 6, 8, . . . (six columns); 1, 1, 2, 3, 5, . . . (seven columns); 1, 2, 4, 7, 11, . . . (six columns); 5, 2, 6, 3, 7, 4, . . . (nine columns).
2. Change the rules so that a player can add to only one column on a turn and must complete one column before starting another (constructing the columns in sequence).

RISKY PATTERNS

Example



Shaping Up

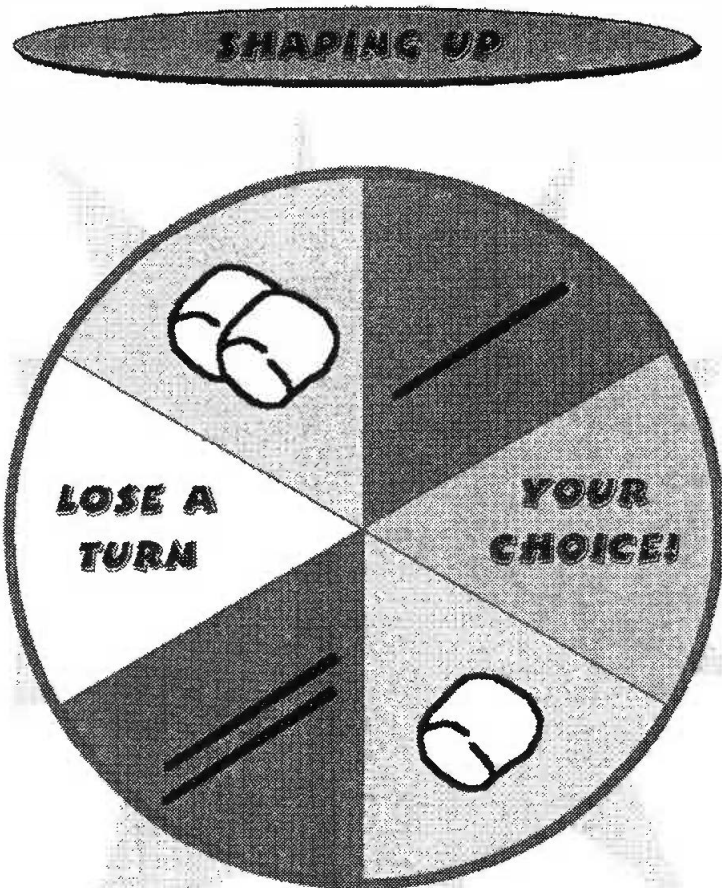
Objective: Build, represent and describe geometric objects and shapes (Shape and Space [3-D Objects and 2-D Shapes], Grade 5, Outcome 15)

Materials: Toothpicks, mini-marshmallows, the Shaping Up spinner mat, an overhead spinner

Players: Two or more

Rules

1. In the first round of this game, players will race to build a cube out of eight marshmallows and 12 toothpicks.
2. On a turn, a player spins the spinner and adds toothpicks or marshmallows according to the result of the spin. If the spinner lands on Lose a Turn, the player adds nothing to the shape. If it lands on Your Choice!, the player may choose to add either one or two marshmallows or one or two toothpicks.
3. If the spinner lands on something the player does not need, play passes to the left.
4. After the player has built the cube, he or she starts building the triangular prism out of six marshmallows and nine toothpicks.
5. After the player has built the triangular prism, he or she starts building the square pyramid.
6. The first player to build all three geometric shapes wins.



Adaptations

1. Change the rules so that, if the player does not need the part spun, his or her opponent (or the player to the left) may use it instead.
2. Build a spinner that changes the odds of landing on certain elements.
3. Have the students build compound shapes (for example, a clock tower comprising a cube with a square pyramid on top).

Spin to Win

Objective: Describe events using the vocabulary of probability: *always, more likely, equally likely, less likely, never* and so on (Statistics and Probability [Chance and Uncertainty], Grade 5, Outcome 10)

Materials: Spin to Win spinner mat, an overhead spinner, pattern blocks

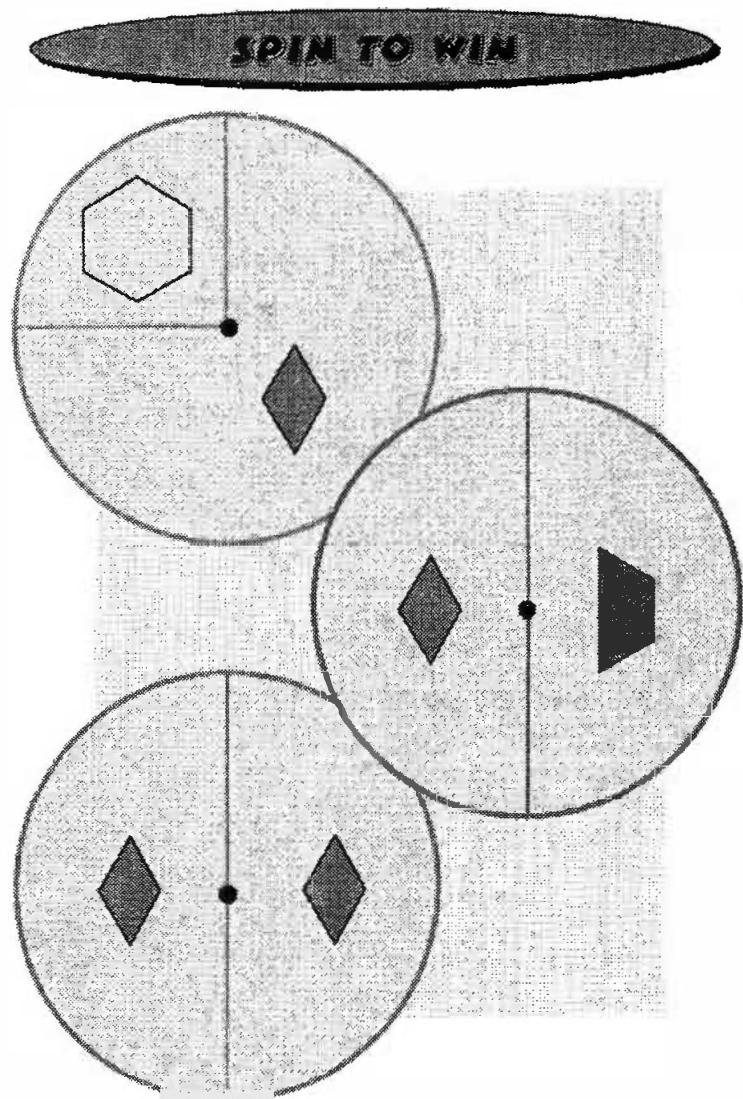
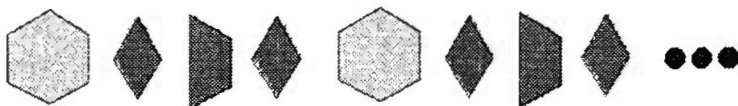
Players: Two or more

Rules

1. On a turn, a player selects one of the three spinner mats (shown at right) and spins the spinner. The player then adds the block specified by the spinner to his or her set. (Here, the diamond represents the blue block, the quadrilateral represents the red block and the hexagon represents the yellow block.) If the player rejects the block, any other opponent can claim it.
2. When a player has three blue blocks or two red blocks, he or she may trade them for one yellow block.
3. The first player to collect four yellow blocks wins. (The player must collect exactly four yellow blocks—that is, he or she may not accept any block that would build a collection greater than four yellow blocks).

Adaptations

1. Change the rules so that a player must trade up to a yellow block before starting to gather blocks for another yellow block. In other words, if a player starts collecting blue blocks, he or she must complete the collection (three blue blocks) before accepting a red block.
2. Change the spinners to include green blocks.
3. Change the game such that a player tries to build (in sequence) the first 15 elements in the following pattern:



So Very Close!

Objective: Determine the volume of an object by measuring the displacement of a liquid by that object (in cubic centimetres or in millilitres) (Shape and Space [Measurement], Grade 6, Outcome 8)

Materials: So Very Close! game board, a graduated cylinder or beaker, water, five objects of various sizes

Players: Any number

Rules

1. To begin the game, the players will need to agree on five objects that can be immersed in water and that fit inside the graduated cylinder or beaker.
2. Each player estimates the amount of water (in cubic centimetres or in millilitres) that will be displaced by the object.
3. After players have recorded their estimates, each object is immersed in water and the displacement is determined.
4. Each player records the displacement and calculates the positive difference between his or her estimate and the actual displacement.
5. After the displacements for all five objects have been measured, each player sums the differences from his or her five estimates. The player with the lowest sum wins.

Adaptations

1. Change the rules so that a player scores one point if his or her estimate is within 10 mL of the actual measurement. After five rounds, the player with the most points wins.
2. To increase the challenge of estimating, use some regular objects (such as cubes), some irregular objects, some objects that float and some objects that don't float.
3. Change the scoring so that a player scores a point when his or her estimate is closer than the previous estimate.

SO VERY CLOSE!			
	Estimate	Actual	Difference
1	<input type="text"/>	<input type="text"/>	<input type="text"/>
2	<input type="text"/>	<input type="text"/>	<input type="text"/>
3	<input type="text"/>	<input type="text"/>	<input type="text"/>
4	<input type="text"/>	<input type="text"/>	<input type="text"/>
5	<input type="text"/>	<input type="text"/>	<input type="text"/>
		SUM:	<input type="text"/>

SO VERY CLOSE!			
	Estimate	Actual	Difference
1	<input type="text"/>	<input type="text"/>	<input type="text"/>
2	<input type="text"/>	<input type="text"/>	<input type="text"/>
3	<input type="text"/>	<input type="text"/>	<input type="text"/>
4	<input type="text"/>	<input type="text"/>	<input type="text"/>
5	<input type="text"/>	<input type="text"/>	<input type="text"/>
		SUM:	<input type="text"/>

Tangled Angles

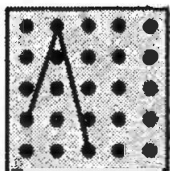
Objective: Classify given angles as acute, right, obtuse, straight and reflex angles (Shape and Space [Measurement], Grade 6, Outcome 12)

Materials: A geoboard, elastic bands, an overhead spinner, the Tangled Angles spinner mat

Players: Two or more

Rules

1. To play this game, players take turns spinning the spinner on the spinner mat (shown at right).
2. On a turn, a player takes an elastic band and, using any three pegs on the geoboard, builds the type of angle specified by the spinner. For example, if the spinner lands on Acute, the player might build an angle like the following:

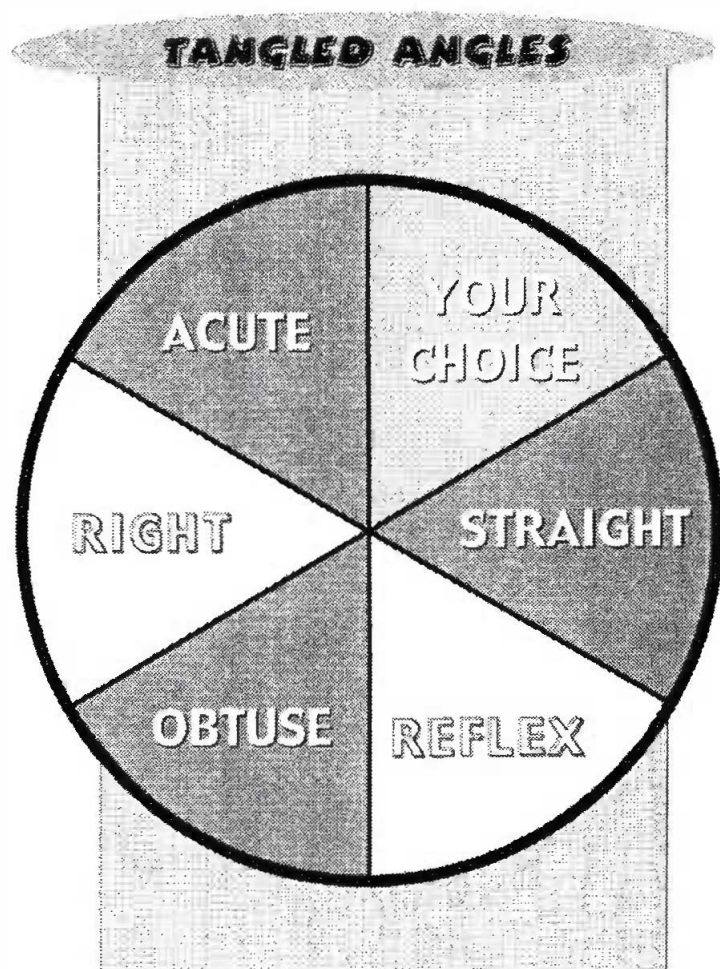


Players must follow these rules when building angles:

- The arms of any two angles must not cross each other.
 - A peg must not be used to build more than one angle (that is, once a peg is used, it is no longer available for use in another angle).
 - The arm of any angle may span more than two pegs, but all such pegs are considered used and, therefore, no longer available.
3. Play continues until a player does not have pegs in the appropriate orientation to build the specified angle. This player must then drop out of the game. The last remaining player in the game wins.

Adaptations

1. Replace the angle types on the spinner with specific angles for players to build and measure.
2. Have students play cooperatively to see how many angles they can build before the requisite collection of pegs is no longer available.



Tangram Checkers

Note: The idea for the original version of this game belongs to James Reynolds.

Objective: Create, analyze and describe designs using translations (slides), rotations (turns) and reflections (flips) (Shape and Space [Transformations], Grade 7, Outcome 11)

Materials: The Tangram Checkers checkerboard, tangrams (two sets, different colours)

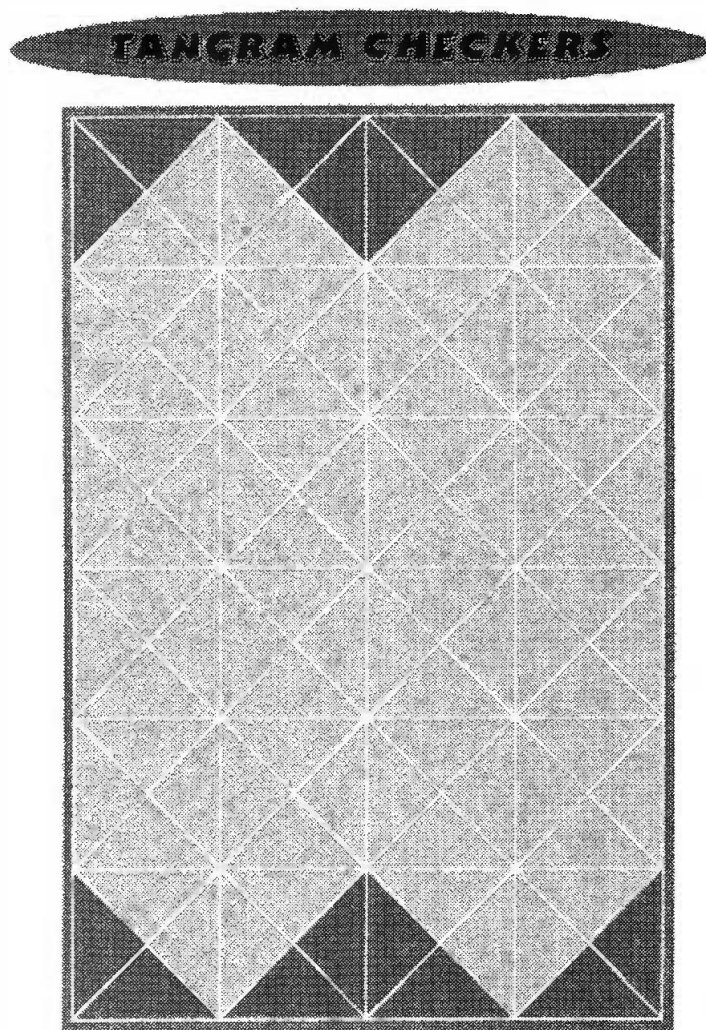
Players: Two

Rules

1. To start the game, each player places the appropriate triangular tangram pieces (one medium-sized triangle and two small-sized triangles) on the dark spaces on his or her side of the checkerboard.
2. On a turn, a player may make one of the following moves:
 - Slide one piece one space horizontally, vertically or diagonally.
 - Flip one piece over an identified line of reflection (which must be an edge of the piece to be moved).
 - Rotate one piece a quarter-turn clockwise or counter-clockwise around any corner of that tangram piece.
3. Before a player may move a piece, the player must identify the move he or she will be making, including identifying the corner used in the rotation or the edge used in the reflection.
4. Two tangram pieces must never overlap, and no part of a tangram piece may rest off the checkerboard.
5. The first player to reposition his or her triangles in the home positions originally occupied by the opponent wins.

Adaptations

1. Substitute or include shapes other than triangles.
2. Place a tangram silhouette in the centre of the checkerboard and ask players to build it together (following the same rules).
3. Modify the board by adding home positions on the right and left sides of the checkerboard. Play the same game with four players.
4. Darken some spaces on the checkerboard to represent obstacles around which players must navigate.



Tic-Frac-Toe

Objective: Convert, mentally, among fractions, decimals and percentages to facilitate the solution of problems (Number [Number Operations], Grade 7, Outcome 21)

Materials: Tic-Frac-Toe game board, two-colour chips, bingo chips, a four-sided die, a six-sided die, a calculator

Players: Two

Rules

1. In this game, players will build models of fractions using the two-colour chips. For example, the fraction shown below represents $\frac{3}{8}$:

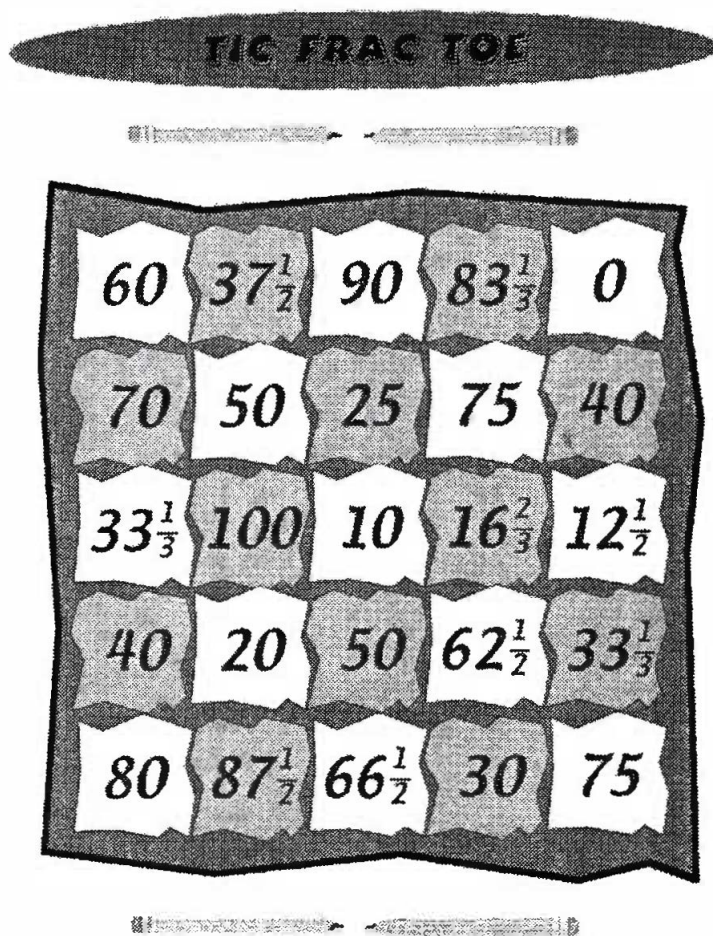


The players will then convert the fractions to percentages (mentally or with the aid of a calculator).

2. Players start with no chips. On a turn, a player rolls either the four-sided die or the six-sided die. The player then adds to or removes from his or her set the same number of chips as the value rolled. For example, if the player rolls a 2, he or she may add two chips with the red side up or two chips with the white side up. Alternatively, the player may remove two red chips or two white chips. If the value rolled is not favourable, the player may pass the turn. A player may not add or remove a combination of red and white chips.
3. When the player has built a new fraction, he or she converts the fraction to its equivalent percentage and places a bingo chip on the corresponding value on the game board (shown at right). Play passes to the left.
4. If the appropriate percentage is already occupied, the player passes his or her turn.
5. If a player is shown (with the calculator) to have converted incorrectly, his or her chip is removed from the game board and the player passes the turn.
6. The first player to get three chips in a row—vertically, horizontally or diagonally—wins.

Adaptations

1. Replace the percentages on the game board with their decimal equivalents.
2. Change the rules such that, if a player creates a fraction for a percentage that is already occupied, the player can steal the space, replacing the chip with his or her own. Four chips in a row wins.



Transformation Puzzle

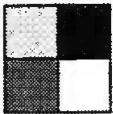
Objective: Create, analyze and describe designs using translations (slides), rotations (turns) and reflections (flips) (Shape and Space [Transformations], Grade 7, Outcome 11)

Materials: 16 gram cubes (four each of four colours)

Players: One

Rules

- To begin the game, the player takes the 16 cubes and builds four squares, each comprising four blocks (one block of each colour) as shown below:



- The player now arranges the four sets in a larger square like the starting arrangement shown in the figure at right.

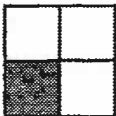
- Each of the following counts as a single move:

- Turning a set of four blocks a quarter-turn clockwise or counter-clockwise
- Flipping a set of four blocks vertically or horizontally
- Flipping two adjoining sets of four blocks vertically or horizontally
- Turning the entire puzzle (all four sets of four blocks) a quarter-turn clockwise or counter-clockwise
- Flipping the entire puzzle (all four sets of four blocks) vertically or horizontally
- Switching two adjoining sets of four blocks without changing their orientation

- Using the prescribed number of moves (and returning the blocks to the starting arrangement before beginning each puzzle), the player tries to re-create each of the patterns shown at right.

Adaptations

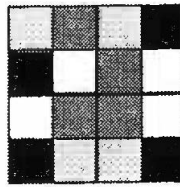
- Substitute different combinations of blocks, as below:



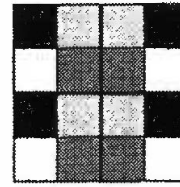
- Increase the number of moves required.

- Have students make cards showing patterns requiring up to three moves. The students then exchange decks of cards with other players and race to see who can solve each other's pattern puzzles first, working through the decks in sequence.

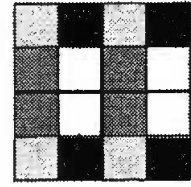
TRANSFORMATION PUZZLE



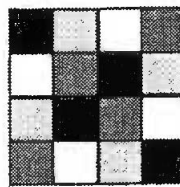
1 move



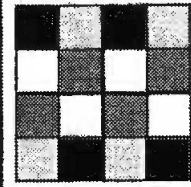
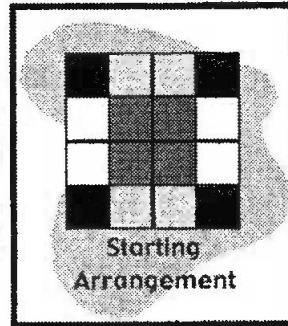
1 move



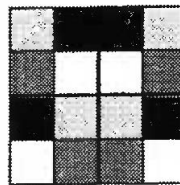
1 move



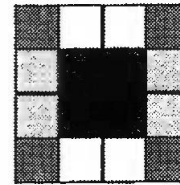
2 moves



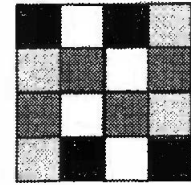
2 moves



3 moves



3 moves



3 moves

Geoboard Algebra

Objective: Solve and verify one- and two-step, first-degree equations (Patterns and Relations [Variables and Equations], Grade 8, Outcome 5)

Materials: Geoboard Algebra game board, geoboards, elastic bands, a four-sided die, a six-sided die, an eight-sided die, a pencil

Players: Two or more

Rules

1. On a turn, a player selects a die and rolls it. The player may now enter the value rolled in any of the blanks in the Equation column of the game board (shown at right). A value can only be entered into an equation if it constructs a whole number root (for example, $3x + 1 = 10$ is acceptable whereas $3x + 2 = 10$ is not). Players will have to think carefully before entering any value.
2. Play continues until an equation has both missing values entered.
3. When both values for an equation are determined, the player models his or her equation using the geoboard, thus demonstrating the value of the variable in the completed equation. The player scores points equal to the value of the variable. The example shows $2a + 3 = 15$, and the player scores six points for determining that $a = 6$.
4. Players continue completing and solving equations until one player has solved all seven equations. Players now total their points. The player with the most points wins.
5. If a player cannot (or chooses not to) enter a rolled value, the player passes that turn.



GEOBOARD ALGEBRA

EQUATION	POINTS
<input type="text"/> $x +$ <input type="text"/> $= 10$	$x =$ <input type="text"/>
<input type="text"/> $a +$ <input type="text"/> $= 15$	$a =$ <input type="text"/>
<input type="text"/> $c +$ <input type="text"/> $= 16$	$c =$ <input type="text"/>
<input type="text"/> $f +$ <input type="text"/> $= 12$	$f =$ <input type="text"/>
<input type="text"/> $h +$ <input type="text"/> $= 21$	$h =$ <input type="text"/>
<input type="text"/> $n +$ <input type="text"/> $= 11$	$n =$ <input type="text"/>
<input type="text"/> $p +$ <input type="text"/> $= 14$	$p =$ <input type="text"/>

SCORE:

Adaptations

1. Change the rules such that a player must enter a value on every turn. If an equation does not have a whole-number root, then the player scores no points.
2. Omit the geoboard model and allow roots that are not whole numbers.

References

- Alberta Learning. *Mathematics Grades 7–8–9*. Edmonton, Alta.: Author, 1996. Available at www.learning.gov.ab.ca/k_12/curriculum/bySubject/math/jhmath.pdf (accessed April 19, 2004).
- . *Mathematics Kindergarten to Grade 6*. Edmonton, Alta.: Author, 1997. Available at www.learning.gov.ab.ca/k_12/curriculum/bySubject/math/elemmath.pdf (accessed April 19, 2004).

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