Embodied Mathematics and Education

Elizabeth M. Mowat

Does understanding mathematics involve nothing more than learning symbols, axioms and theorems? For George Lakoff and Rafael Núñez (2000), authors of *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*, understanding mathematics means comprehending how mechanisms of the brain and mind enable people to reason mathematically. They use results of research in cognitive science to explore how mathematical ideas are possible and why they make sense.

Lakoff and Núñez suggest that the teaching of mathematics may be enhanced by an understanding of this cognitive perspective of embodied mathematics. In this article, I will attempt to show how ideas put forth in *Where Mathematics Comes From* may be helpful in mathematics education. The article is divided into three parts. The first section describes some aspects of embodied mathematics, based on ordinary human cognitive and bodily mechanisms, which are presented in *Where Mathematics Comes From*. The second section reviews ways in which the theory of embodied mathematics explains sources of student difficulties and the third section discusses how teachers can use these ideas in designing effective instruction for their classrooms.

Embodied Mathematics

Where Mathematics Comes From can be considered a study of the nature of mathematical intuition. The authors claim that automatic, unconscious understanding is developed and refined through activities and experiences in the real world. Lakoff and Núñez provide empirical evidence that this intuitive understanding is neither vague nor ill-defined, but is precise and rigorous enough to form a foundation for mathematical thought.

They assert that mathematics exists by virtue of the embodied mind. Cognitive structures used in mathematical thinking are based on physical sensations and activities. The brain receives input exclusively from the rest of the body. Therefore, what the body is like and how it functions in the world determine the form and content of thought. The mind emerges from distinctive characteristics of the human brain and body; it is embodied. "The detailed natures of our bodies, our brains and our everyday functioning in the world structures human concepts and human reason. This includes mathematical concepts and mathematical reasoning" (Lakoff and Núñez 2000, 5).

Lakoff and Núñez base their assertions of this thesis on the empirical findings of scientists from a wide variety of disciplines: developmental psychology, cognitive neuroscience, neuropsychology, cognitive linguistics and cognitive psychology. Convergent evidence from these fields is used to support and structure the theory presented in *Where Mathematics Comes From*. This book shows how mathematics is embodied through innate arithmetic abilities, the mind's cognitive mechanisms and its basis in bodily experience through grounding metaphors.

Innate Arithmetic Abilities

Lakoff and Núñez argue that humans are born with certain arithmetic capacities. The very notion of *number* is engraved on our brains. Highly specialized sets of neural circuits enable us to subitize; that is, instantly and accurately recognize very small numbers of objects. At an early age, people possess an understanding of limited addition and subtraction, capacities needed for simple counting and numerosity, which is the ability to make rough consistent estimates for larger numbers. Areas of the brain involved in these activities are thought to be located in the inferior parietal cortex which links vision, hearing and touch.

Cognitive Mechanisms

Knowing which parts of the brain are activated when people use these very limited innate capacities does not explain where normal arithmetic and more sophisticated mathematics come from. Lakoff and Núñez explain that mathematical thinking engages the same conceptual structures used by humans in other kinds of sense making. These cognitive mechanisms, used automatically and unconsciously in reasoning, are referential systems that assist people in understanding and employing concepts.

Abstract reasoning using cognitive mechanisms is grounded in basic bodily experiences. For example, balance is part of everyday life for all humans. We first encounter balance as babies wobbling across the floor. Over the years, balance becomes such an intrinsic part of our lives that we are hardly aware of it, but it is extremely important for our coherent perception of the world (Johnson 1987). This type of universal body-based experience becomes a cognitive mechanism that can be used to reason about many things like cheque books, relationships or solving equations. Lakoff and Núñez discuss three cognitive mechanisms that are particularly important: the image schema, the conceptual metaphor and the conceptual blend.

The Image Schema

Image schemas, for qualities like balance, straightness or verticality, represent the spatial logic inherent in physical situations. Image schemas are not just mental pictures, but are general and flexible patterns developed through sensori-motor experiences that make our perceptions of the world meaningful.

The container image schema is of particular importance in mathematics. Because our experiences with physical containers involve sight, touch, language and reasoning, the container image schema utilizes the corresponding regions of the brain. Lakoff and Núñez use the image of a set as a "cognitive" container to represent the container image schema (see Figure 1).

The logic of the physical container is projected onto the cognitive container, which can be used to reason about nonspatial situations (Johnson 1987, 34). Normal language use illustrates how common this is. Statements often refer to components of the container: its boundary (he's on the *brink* of disaster), its exterior (she's *out* of her league) and its interior (he's always getting *into* trouble). Modes of reasoning developed through experience with ordinary containers are an essential part of the image schema. Figure 2 shows how the container image schema can link physical experience to mathematics.

The power of the image schema is that it can introduce new ideas or extensions that do not arise from the original physical experience. We can imagine two sets overlapping (Figure 3) even though two physical containers cannot intersect in this way.

Figure 1
Container Image Schema

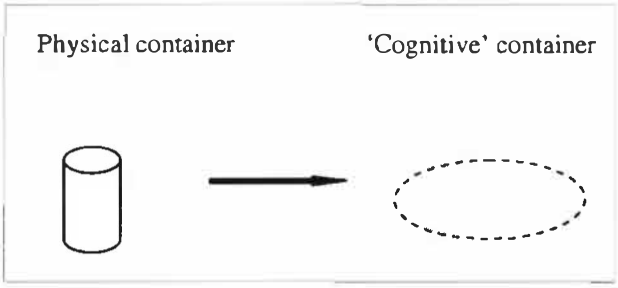


Figure 3
Concept of intersecting sets introduced by the abstract container image schema

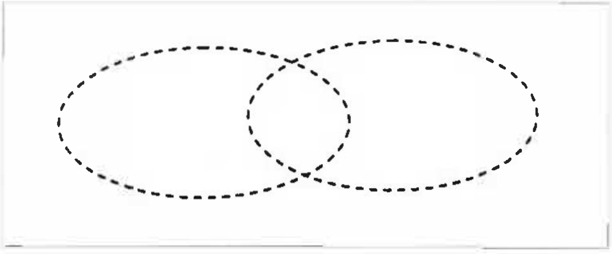
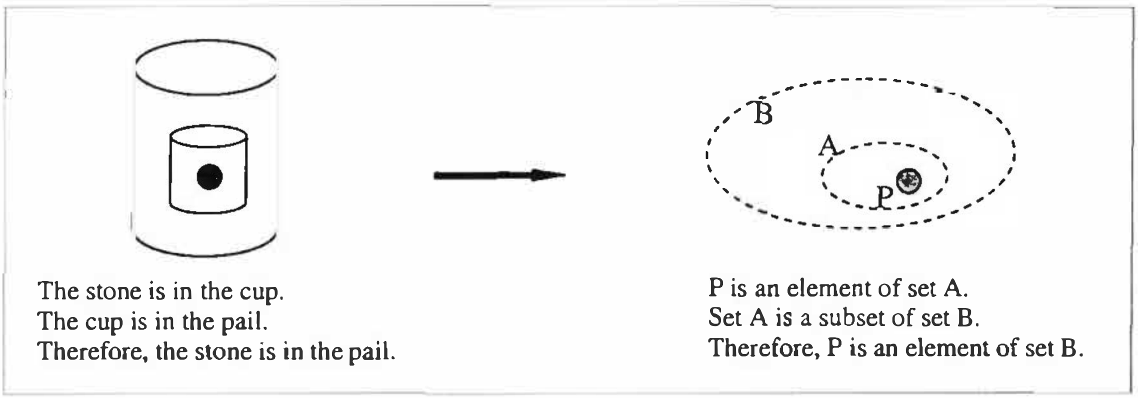


Figure 2
Reasoning from physical experience transferred to abstract mathematics through the container image schema



The Conceptual Metaphor

Image schemas are linked together by another important cognitive mechanism, the conceptual metaphor. Conceptual metaphors are the basic means by which conceptual thought is made possible. An essential part of all types of human understanding, conceptual metaphors enable people to think about an unfamiliar, abstract concept as if it were familiar and concrete. Many conceptual metaphors arise initially from the everyday experiences of children. A child, held in his mother's arms, feels both love and warmth. Associating affection with cuddling leads to the metaphor of affection as warmth. Evidence of the existence of the metaphor is seen in everyday language. We say "they warmed up to each other" or "she gave him an icy stare." Experiences in the source domain of warmth are mapped onto relationships in the target domain of affection.

Conceptual metaphors are not just linguistic devices, but empirically observable mechanisms of the mind. The simultaneous activation of two different areas of the brain establishes new neural connections between them and generates a single complex experience. Because the inferential structure inherent in these experiences is preserved, the abstract concept of affection can be understood in terms of the concrete experience of warmth.

Conceptual metaphors can also introduce new elements or extensions in the target domain. The statement, "I had to work hard to get that question" is evidence of the metaphor of learning as a job. Subtle aspects of this metaphor, like those set out in Figure 4, are absorbed and unconsciously influence thinking.

Figure 4

Implications of the metaphor of learning as a job

Learning is work.
Learning is routine.
Learning is difficult.
I deserve some compensation for learning.

Learning is not play.
Learning is not fun.

The Conceptual Blend

Two conceptual metaphors can be combined through a conceptual blend. Lakoff and Núñez offer this example: the unit circle is a conceptual blend of a circle in the Euclidean plane and a Cartesian plane with coordinate axes (see Figure 5)[1]. In the Euclidean plane, a circle consists of all points in the plane a fixed distance, called the radius, from a fixed point, called the centre. The two-dimensional Cartesian plane is defined by two axes set at right angles to each other. The horizontal or x-axis and the vertical or y-axis intersect at a point called the origin, O. By using a unit length on each axis and forming a grid, the position of any point on the Cartesian plane can be described using (x, y) coordinates. The unit circle conceptual blend combines characteristics of both of these metaphors.

In the unit circle conceptual blend, new connections are formed between the neural structures related to the two original types of geometric planes. Thus the blend possesses characteristics of both of the original domains. A circle is still composed of points a set distance from the centre. But now this centre is at the origin, the radius has a length of one unit and coordinates are used to describe points on the circle. Moreover, new concepts or extensions arise. The unit circle blend has properties related to trigonometry that are not part of either of the original metaphors (see Figure 5).

Grounding Metaphors

Grounding metaphors are conceptual metaphors that establish correlations between physical activities of the body and innate arithmetic. In mathematics, the grounding metaphor is the primary tool that enables the extension of innate numerical abilities to arithmetic within the set of natural numbers and ultimately to more sophisticated concepts. Lakoff and Núñez pay special attention to four grounding metaphors:

- Arithmetic is object collection
- Arithmetic is object construction
- The measuring stick metaphor
- Arithmetic is motion along a path.

Human understanding is grounded in already acquired understanding of ordinary actions. A child who puts blocks into piles is establishing neural connections between areas of the brain responsible for the physical action and innate arithmetic. This initiates the metaphor of arithmetic as object collection, whereby numbers are identified with collections of objects. Adding involves putting two collections together, while subtracting involves taking a small collection from a larger one. The natural number system, which includes numbers too large to be subitized (instantly recognized), is formed. Properties of number-collection entities are consistent with those of innate mathematics, but are extended to include new properties. Since the sum of any two collections is another collection, the sum of any two numbers must be another number. Thus, the natural numbers possess the property of closure, which is not part of innate arithmetic.

A similar grounding metaphor is arithmetic as object construction. Children start to form this metaphor when playing by putting things together to construct a new object. Numbers are identified with wholes made up of parts. Imagine a child building a tower out of blocks; a tower five blocks high represents the number five. Addition means adding more parts to the object. Subtracting means taking some of the parts away. This metaphor ties innate arithmetic to the natural numbers, but can be extended farther. A whole object can be broken up into smaller equal parts giving an embodied meaning to the concept of fractions.

Lakoff and Núñez's third grounding metaphor is the measuring stick metaphor in which objects are measured using physical segments. Blocks might be used to measure the size of a new toy or hands to measure the size of a pony. Number-physical segment entities are created. Addition is putting two segments together end to end, and subtraction is taking a smaller segment away from a larger one. This metaphor is similar to arithmetic-as-object-collection and arithmetic-as-object-construction metaphors, but has different extensions. In this metaphor, any physical segment or anything that can be measured can be considered a number. Consequently, some irrational numbers are grounded.

Figure 6

Grounding √2 and π using the measuring stick metaphor

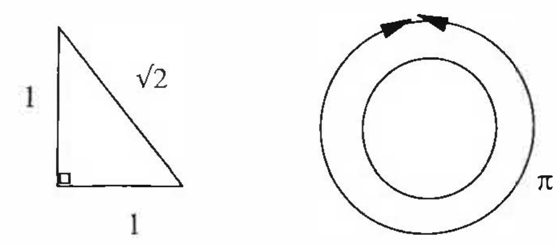
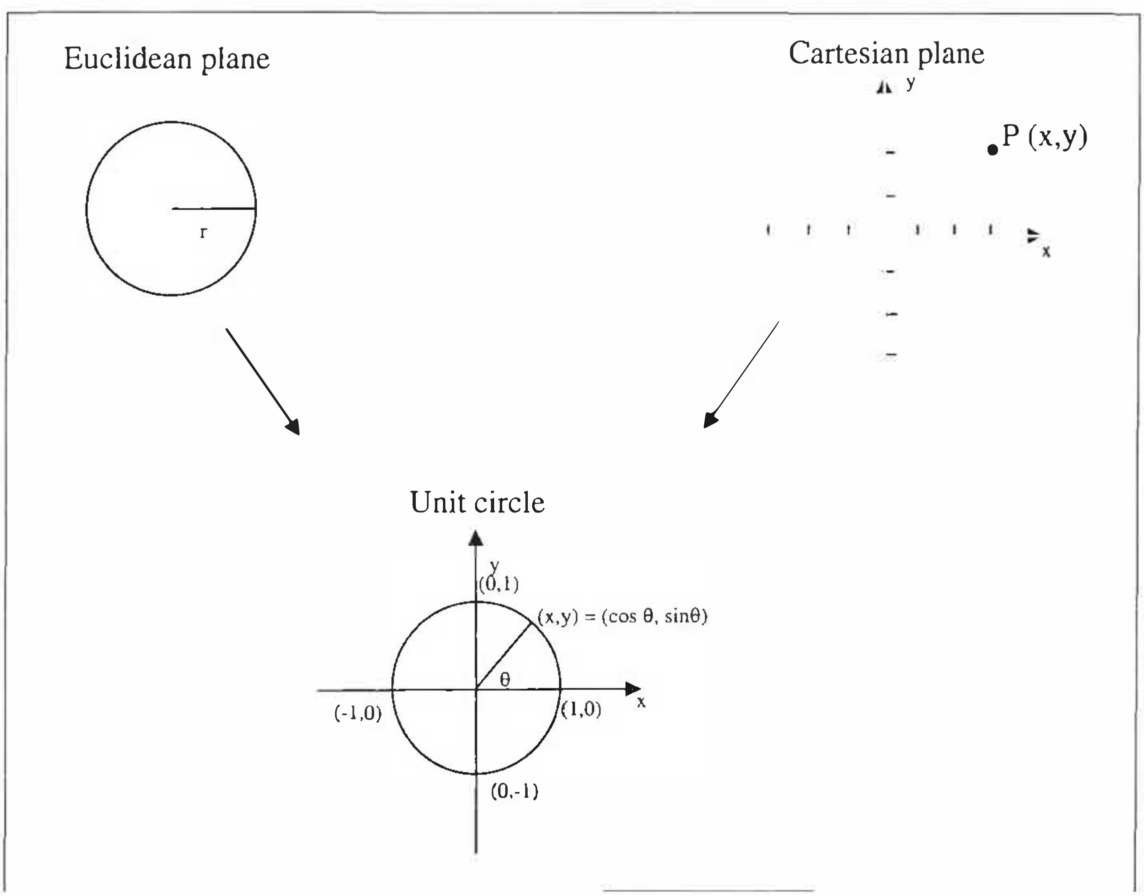


Figure 5

Features of Euclidean and Cartesian geometry combined into the unit circle



The fourth grounding metaphor is arithmetic as motion along a path. Numbers are point locations on a line. Addition involves moving from a position on the line away from the origin, whereas subtraction involves moving toward the origin. This metaphor, which grounds natural numbers, fractions and irrational numbers, has two unique extensions. Allowing motion on either side of the origin provides a physical basis for integers. Moreover, because a path-number can have any length, the metaphor provides a grounding for the real numbers.

Arithmetic as motion along a path differs significantly from the other grounding metaphors. Lawler and Breck (1998) point out that this metaphor, built on early experiences of crawling and walking, is based on ambulation, involving legs and feet, while the first three grounding metaphors are based on manual manipulation. Moreover, it implies continuous motion rather than discrete objects or segments. The arithmetic-as-motion-along-a-path metaphor is the only grounding metaphor that cannot be combined with subitizing (Chiu 2000). Another unique characteristic is its inherent concept of zero, which is located at the origin of the path.

These four grounding metaphors are not imaginary. Evidence of their existence is found in language and in mathematical constructs of the past. The metaphor of arithmetic as object collection appears in such expressions as "*add* some lettuce *to* the salad" and "*take* a log *from* the woodpile." Arithmetic as object construction is seen in Roman numerals like IX and VII where parts are being added to or subtracted from a whole. The measuring-stick metaphor is shown in units of measurement like cubits, feet and paces. Arithmetic as motion along a path appears in expressions like "6 is *close* to 8" and "*starting* at 20, count to 50."

These four grounding metaphors are not arbitrarily chosen. Of the many grounding metaphors that exist, Lakoff and Núñez found that only these four have physical sources with properties and logic sufficient to form a connection with inborn numerical capacities. "Each of them forms just the right kind of [correlation] with innate arithmetic to give rise to just the right kind of metaphorical mappings so that the inferences of the source domains will map correctly onto arithmetic . . ." (Lakoff and Núñez 2000, 102) and ultimately onto more complex mathematics.

Where Does Mathematics Come From?

From a rather limited set of inborn skills, mathematics is extended through an ever-growing collection of metaphors. These cognitive mechanisms, which are neurally embodied structures of the mind, abstract patterns of inference from physical experience. Grounding metaphors form correlations between innate arithmetic and physical action to make elementary arithmetic possible. Other conceptual metaphors link arithmetic to more abstract mathematical concepts. Each layer of metaphors carries inferential structure systematically from one domain to another. Complex networks grow as domains that are connected to each other by conceptual blends, and new metaphors involving these blends are formed. Even the most abstract mathematical concept bears traces of its origin in physical perception and motor activity and is, thus, embodied. "The only mathematics that human beings know or can know is a mind-based mathematics, limited and structured by human brains and minds" (Lakoff and Núñez 2000, 4). Hence, the study of embodied mathematics sheds light on difficulties experienced by students in the mathematics classroom.

Table 1
Characteristics of the Four Grounding Metaphors

Grounding metaphor	Object collection	Object construction	Measuring stick	Motion along a path
Numbers are …	Collections	Wholes with parts	Physical segments	Points on a line
Addition is …	Adding items	Adding parts	Putting segments together	Moving away from the origin
Subtraction is …	Taking items away	Removing parts	Taking a segment away	Moving toward the origin
Number systems	Natural numbers	Fractions	Irrational numbers	Integers, real numbers
Physical experiences	Manipulation	Manipulation	Manipulation	Ambulation
Properties	Discrete	Discrete	Discrete	Continuous Zero is the origin

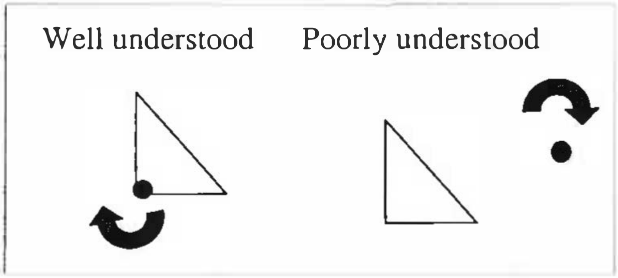
Understanding Student Difficulties

Because metaphors are the basis of embodied mathematics, the study of their use in the classroom may reveal problems experienced by students in learning mathematics. Metaphors are not usually learned through formal instruction, but arise through informal everyday experiences and develop gradually over time. While conceptual metaphors make mathematics possible and very rich, they can also cause confusion and even apparent paradox if they are not made clear or are taken literally (Núñez 2000). Students may not understand everything that is implicit in a metaphor, what it hides and what it introduces. Research examining how students use and misuse common metaphors has identified some common difficulties experienced by students in their use of metaphors.

Using an Inappropriate Metaphor

Use of an inappropriate metaphor can cause difficulties for students who are trying to comprehend a mathematical idea. Edwards (2003) found that children and adults studying transformation geometry had difficulty fully understanding the concept of rotation. Rotations of an object about a point that was inside the object were well understood. But all learners, regardless of age, had trouble with situations where the centre of rotation was outside the object (see Figure 7).

Figure 7
Types of rotations about a point



Edwards realized that learners were using their embodied understandings of turning to make sense of rotations. When students considered babies rolling over or skaters spinning on ice, they thought of themselves as the centre of rotation. Stating that human perception tends to place the body at the centre of the universe, Johnson (1997) clarifies why the metaphor of rotation as turning is used for reasoning about transformations.

For Edwards, this explained why problems in which the centre of rotation was inside the object being rotated were easily understood. Even situations in which a physical link existed between the object being rotated and the centre of rotation were grounded in experiences like playing on a swing and, consequently, were fairly straightforward. But when the centre of rotation and the object being turned are not in physical contact, the questions were harder to deal with. The metaphor rotation as turning was not useful in understanding these types of rotations in transformation geometry.

Misunderstanding the Source Domain of the Metaphor

The source domains of metaphors provide the foundation for mathematical reasoning. If students do not clearly understand these fundamental patterns of thought, they are unlikely to be able to understand related concepts. "Inadequate understanding of the source domain of a metaphor limits a person's reasoning through that metaphor" (Chiu 2000, 7).

Students may have trouble using a metaphor whose source domain has subtle extensions. For example, difficulties are often experienced in the study of probability, particularly in questions containing the word *or*. These questions often make use of the categories-are-containers metaphor. Consider the following problem: If you draw one card from a deck of 52, what is the probability that it is red or a queen? In this question, learners are dealing with two categories of cards, those that are queens and those that are red. Students see these two categories as mutually exclusive (see Figure 8) when in reality they intersect (see Figure 9).

Figure 8
Students' View of Categories as Physical Containers

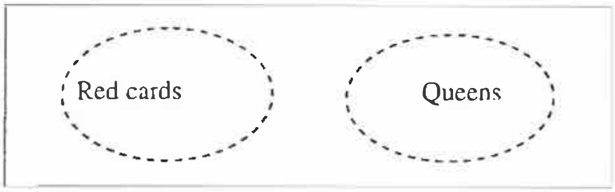
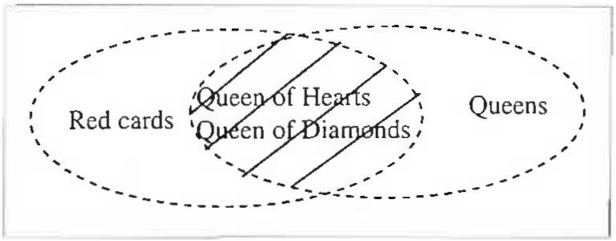


Figure 9
Categories as Containers Using Container Image Schema



Students' experiences with physical containers limit their thinking about cognitive containers, which are identified in this example with categories. They do not understand the container image schema, which is the source domain of the categories-are-containers metaphor.

This misunderstanding reflects a problem in the thinking processes of students, not in their mastery of the mathematical techniques. In discussion with a colleague, Mr. Michaels, a social studies teacher, found that understanding the container-image schema helped him to understand why his Grade 9 class had difficulty responding to a question about the Russian Revolution. When comparing how Russian people lived under the Czarist and the Communist regimes, students were able to list differences in lifestyles, but could not identify any similarities. Many existed, but their misunderstanding of the source domain of the categories-are-containers metaphor held students back. As in probability, they thought of physical containers (see Figure 10) rather than the cognitive containers of the container image schema (see Figure 11).

Not Recognizing Limitations of Metaphors

Because metaphors are used unconsciously, learners may fail to recognize their inherent limitations. Tall (2003) found that automatic use of previously mastered metaphors may cause confusion. Young children tended to feel that adding two numbers should always yield a larger sum and that multiplying should lead to a very much larger product. These properties are true for *arithmetic as object collection*. But addition of integers can lead to a smaller sum (2 and –7 makes –5) and multiplication by fractions can lead to a much smaller product ($6 \times 1/12 = 1/2$). Confusion arose in students' minds because they could not realize the limitations of the metaphor they are using. They were held back in their development of arithmetic skills by their reliance on what Tall calls "met-befores."

Figure 10
Students' View of Categories as Physical Containers

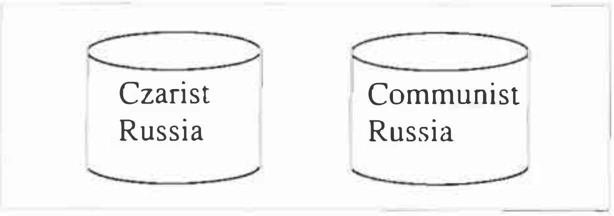
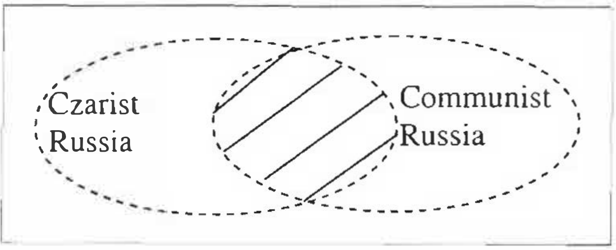


Figure 11
Categories Are Containers Using Container Image Schema



Relying Exclusively on a Single Metaphor

In studies of students doing arithmetic with signed numbers, Moses and Cobb (2001) found that children failed to progress because of their reliance on the arithmetic as object collection metaphor. He felt that the arithmetic as motion along a path would be more useful in this situation and developed activities using experiences familiar to students, like riding on the subway, to strengthen this metaphor. With such techniques, he was successful in improving children's understanding of integer arithmetic.

Using Two Metaphors That Conflict

Núñez, Edwards and Matos (1999) are particularly interested in conflicting metaphors used in the study of continuity of functions. High school students are introduced to "natural" continuity, which is defined as

$$\lim_{x \to a} f(x) = f(a)$$

This notion of continuity, used by Newton and Leibniz, is often described using Euler's idea of "a curve freely leading the hand" (Núñez 1997). Such a perspective is based on motion and uses the metaphor *a line is the motion of a traveller tracing that line*. The line does not move, but to the learner's understanding it does. Expressions commonly used in mathematics reflect this: a function *reaches* its maximum at (1,1); the line *crosses* the x-axis; two curves *meet* at a point; the line *goes through* (2,3); and the limit exists as x *approaches* 2.

At the university level, students are introduced to a new interpretation of continuity. The Cauchy-Weierstrass portrayal of continuity is very different.

$$\lim_{x \to a} f(x) = L$$

if and only if for every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

This definition is based on the metaphor *a line is a set of points*. The idea of continuity here is in terms of preserving closeness: for every x close to a, $f(x)$ is close to L.

While both definitions arise from metaphors grounded in experience, they are not compatible. Natural continuity is dynamic, based on properties of motion. The Cauchy-Weierstrass definition is static, based on closeness in containers. Although both definitions are useful, they have very different inferential structures, and this causes difficulties for learners.

Students of calculus are never told that the Cauchy-Weierstrass definition of continuity has a completely different embodied foundation than natural continuity (Núñez 1997; Núñez, Edwards and Matos 1999; Lakoff and Núñez 2001). Indeed, they are often told that it captures the essence of natural continuity. To compound the problem, both techniques talk of a limit as *x approaches a*, even though this terminology is inconsistent with the metaphor that the Cauchy-Weierstrass definition is based on. Because the two metaphors are not integrated into a coherent whole, it is understandable that students have trouble adapting to the Cauchy-Weierstrass method.

Not Integrating Multiple Metaphors

Metaphors have their own inferential structures and can "lead to different conscious and unconscious beliefs that can cause obstacles to drawing various aspects into a central core concept" (Watson, Spyrou and Tall 2003). Students commonly learn two methods for adding vectors: the parallelogram method and the triangle method, as illustrated in Figure 12. Both methods are based on embodied metaphors and, although the underlying metaphors are different, both techniques are useful in understanding operations with vectors.

The parallelogram method is based on the *vector as a force* metaphor. Situations like two people pulling a sled or having two friends grab your arms and drag you along are within the experience of students. Both result in the sled or the person moving forward as if one force pulls it. It is natural therefore to think of the combination of two forces as a single force acting between the two forces. On the other hand, the triangle method is based on the *vector as a journey* metaphor. The sum of two vectors consists of two successive moves. We first move from A to B and then from B to C. The result is a journey starting at A and ending up at C: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

Students who are introduced first to addition of vectors using the triangle method may have difficulty understanding general properties of vectors like the commutative law. In a journey where the order of the two components does matter, $\overrightarrow{BC} + \overrightarrow{AB}$ does not make sense. Consequently, the vector as a journey metaphor is not helpful in making sense of commutativity. In contrast, from the perspective of the vector as a force metaphor lying behind the parallelogram method, the commutative law is obvious. Watson and Tall (2002) found that emphasizing the vector as a force metaphor in this context was of benefit to students.

In turn, the parallelogram method does not easily explain subtraction of vectors. As shown in Figure 13, the difference $\vec{a} - \vec{b}$ lies on the diagonal of the parallelogram. It joins the endpoints of $\vec{a}$ and $\vec{b}$ and ends where the minuend $\vec{a}$ ends (see Figure 13). Nothing in everyday experience corresponds to this force. The vector as a journey metaphor explains subtraction much better. Students can think of $\vec{a} - \vec{b}$ as $\vec{a} + -\vec{b}$ by reversing the direction of the second component of the journey as shown in Figure 14.

Figure 12
Two Approaches to Adding Vectors

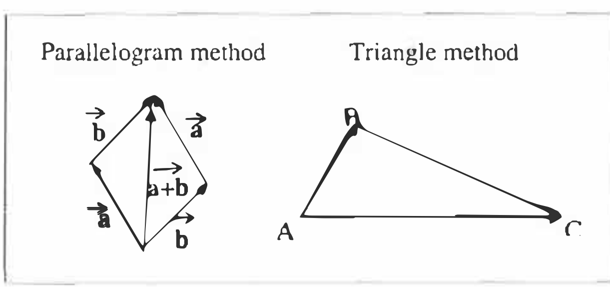


Figure 13
Subtraction of Vectors Using the Parallelogram Method

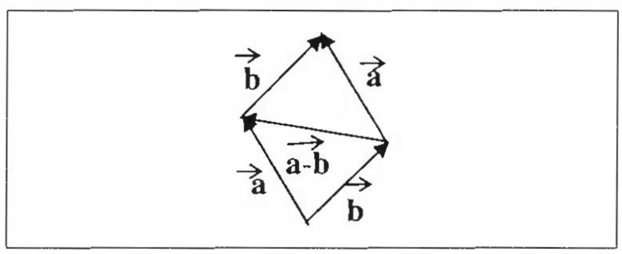
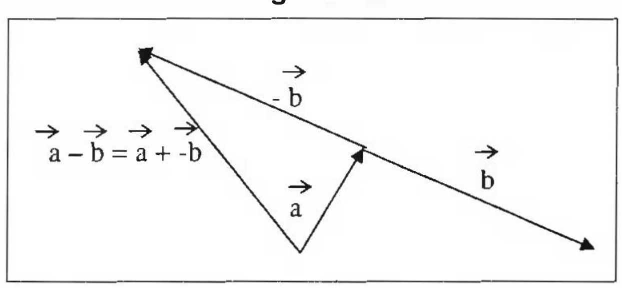


Figure 14
Subtraction of Vectors Using the Triangle Method



Knowledge of both methods with their underlying metaphors is necessary for a thorough understanding of operations on vectors. Hsu and Oehrtman (2000) found that students became confused when they were not able to integrate multiple metaphors that could be used to structure a mathematical concept.

Implications for Teaching

"Mathematical concepts on the surface may seem to be neat and well organized, but underneath, in the workings of the brain, all sorts of conflicts and confusions occur" (Tall 2003). Many theorists feel that teachers would find knowledge of cognitive structures inherent in mathematical concepts useful (Núñez 2000; Núñez, Edwards and Matos 1999). With this understanding, they could assist students to better understand mathematical concepts through appropriate use of metaphors.

Activities can be designed to provide initial grounding for conceptual metaphors (Núñez, Edwards and Matos 1999; Tall 2003). For example, working with scales can provide experience with balance thus developing a basis for metaphoric thinking when solving equations. Grounding metaphors that rely on everyday experiences of students, like playing or even taking part in cultural activities, have been found to have a powerful effect on student understanding (Chiu 2000).

Teachers can strongly encourage the use of metaphors in classroom communication. Using metaphors in classroom discussions encourages students to accept metaphoric thought as a normal method in mathematics. Madden (2001) mentions the importance of social interaction in determining the efficacy and usefulness of patterns of metaphoric thought. Communities of learners, like communities of mathematicians, can share and explain the metaphors they use and adopt or correct them as needed. When metaphors are legitimated and spread among students, metaphoric thought is strengthened (Bazzini 2001).

The importance of metaphoric thinking in the history of mathematics can be highlighted. Making students aware of different metaphors used at various times in the development of concepts like calculus will help them understand why conflicting metaphors sometimes appear in mathematics.

Mathematics is traditionally taught as a collection of techniques, skills and attitudes that students must acquire. Pure logic holds a dominant position. "The body has been ignored because reason has been thought to be abstract and transcendent, that is, not tied to any of the bodily aspects of human understanding ... [Our] bodily movements and interactions in various physical domains of experience are experiential in structure ... and that structure can be projected by metaphor on to abstract domains" (Johnson 1987, XV). A better understanding of the hidden, very ordinary origins of complex concepts in mathematics can only result in more effective learning and teaching.

Notes

1. Figure 5 was my own attempt to illustrate the unit circle conceptual blend. Later, I discovered that it has a remarkable similarity to figures on pages 390–392 in *Where Mathematics Comes From*. Independent development of the diagram illustrates how particular metaphors compel certain interpretations. It is likely that any graphic representation of the unit circle conceptual blend would closely resemble Lakoff and Núñez's images.

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Elizabeth Mowat has been a teacher with Edmonton Public Schools since 1974. During this period, she has taught both the junior and senior high levels and has served as department head of mathematics at Strathcona Composite High School. Since going on leave in 1997, she has maintained an interest in mathematics education and is currently studying mathematical cognition and complexity science at the University of Alberta in Edmonton.