Explorations with Simulated Dice: Probability and the TI-83+

A. Craig Loewen

The probability strand of the mathematics curriculum is most likely the one strand above others that begs for a great deal of exploration and creativity. Many different, fun activities and games can be implemented across the grade levels. The TI-83+ is a useful tool for conducting those explorations while engaging in a wide variety of creative problem-solving activities.

The TI-83+ calculator has several nice features. First, many different applications are available and can be downloaded free from the Texas Instrument Internet site. One is called probability simulation, and I encourage the reader to explore this piece of software, as it is quite powerful and versatile. This application simulates rolling dice, flipping coins, drawing cards, twirling a spinner and drawing coloured marbles from a defined set with or without replacement. A creative teacher could devise many interesting and fun explorations from this application alone. However, one disadvantage is that it restricts the number and type of dice—the user may select up to three dice only, and all dice must have the same number of sides.

A second positive feature of the TI-83+ is that it is programmable, and this dramatically expands our possibilities—we can use the programmability of the calculator to overcome the limitations of the probability simulation application. It is quite easy to devise simple programs to explore situations with many dice with different numbers of sides. The calculator can store the results of the exploration in tables and lists for later analysis, if desired.

Whether the explorations are quite simple or more complex, the TI-83+ calculator can be a powerful and flexible tool.

Writing a Simple Program on the TI-83+

The user will need to know a few key commands to program most probability explorations. First, the user will need to know how to select a random number. The TI-83+ makes this easy—there are two possible commands: **rand** and **randInt(v1.v2)**. Both commands can be accessed from the fourth menu (PRB) under the Math key.

Try this: Select the **rand** command and press the enter key repeatedly. The calculator will display 10-digit random values between 0 and 1. Now, return to the PRB menu and this time select **randInt(**. Complete the command by adding two values separated by a comma and enclosed by parentheses. Again, press enter several times to see the result. The calculator will provide a series of random integers in the range specified, including the chosen values.

- How could you use this command to simulate the rolling of a 12-sided die?
- What happens when one (or more) of the values you provide is less than zero?

Now, in order to conduct an experiment we will need a command to have the calculator roll the die several times. The **For** command is very useful here. The following mini-program causes the calculator to roll a six-sided die five times and display the results on the screen.

```
PROGRAM:PROB1
:ClrHome
:For(A,1,5)
:Disp randInt(1,
6)
:End
:■
```

When the program is run, the following might appear on the screen.

```
                5
                2
                3
                6
                1
             Done
■
```

Assume we want the calculator to keep track of the running total for 100 rolls of a six-sided die and display the average. One possible program follows.

```
PROGRAM:PROB2
:ClrHome
:0→T
:For(A,1,100)
:T+randInt(1,6)→
T
:End
:Disp T/100
:
```

- What value should that average approximate? Why?
- What would the average approximate for 100 rolls of an eight-sided die? Why?
- Run the experiment several times. Find the average of these averages. What does this value represent?

We now have enough simple commands to solve a variety of fun and challenging problems.

Lucky 11

On the table you have a selection of 4, 6, 8, 10, 12 and 20-sided dice. There are several of each kind of dice. You may select any two of the dice and roll them. Your opponent will do likewise, taking turns. The first person to roll lucky 11 wins. Which two dice should you select?

- Try playing the game with your classmates. Have each person select a different combination of dice. Which combination(s) of dice seem to be the best?

A very basic program can be constructed that rolls two dice 500 times and keeps track of how often the value 11 turns up. Presumably the program that generates the result 11 most frequently is also the combination most likely to generate it first. In the following program, simulating the rolling of two 6-sided dice, T is a counter that is increased each time an 11 appears.

```
PROGRAM:PROB3
:ClrHome
:0→T
:For(A,1,500)
:randInt(1,6)+ra
ndInt(1,6)→R
:If R=11:T+1→T
:End
:Disp T
:
```

- When rolling two 6-sided dice, about how many 11s would you expect to appear in 500 rolls?
- How could this program be modified to simulate rolling two 8-sided dice?
- How many different combinations of dice are possible in this problem? Can any combinations be eliminated at the outset of the problem? Why?
- Are you better off to roll two 6-sided dice or a 4-and 8-sided die together?
- Which combination of dice is optimal for rolling an 11? Can you explain your result theoretically?
- Rewrite the program above such that it will calculate the experimental probability (simulated) of rolling an 11 with any combination of the same two dice.

The Game of Pig

This game appears in the book *About Teaching Mathematics: A K–8 Resource* by Marilyn Burns (1992, 71). In this game a player rolls two 6-sided dice and sums the results. The player must now make a choice: accept the score or roll again. If the player accepts the score, the points are added to the total collected from previous turns and play passes to the left. If the player rolls again, those points are added to the total from the first roll. The player may continue to roll as many times as he or she wants, but if at any time the player rolls a one on either dice, the player loses all the points collected on that turn and play passes to the left. The points collected thus far on each turn are risked each time a player rolls. If at any point in the game a player rolls double ones, the score is reset to zero and the dice pass to the left. The first player to collect 100 or more points wins. This game is appropriately titled because if you get too greedy, eventually you will get caught. What is the probability of rolling at least one 1 on any given roll?

Again, the rolling of dice within this game can be simulated on your calculator. In this program, we simply have the computer check the results of both dice and if either dice equals one then our counter increases by one.

```
PROGRAM:PROB4
:ClrHome
:0→T
:For(A,1,500)
:randInt(1,6)→B
:randInt(1,6)→C
:If B=1 or C=1:T
+1→T
:End
:Disp T
:
```

- What is the probability of rolling at least one 1 on a turn in this game? Can you prove this result theoretically as well?

- How would it change the game (and your strategy in playing the game) if you used two 8-sided dice instead of two 6-sided dice? Two 4-sided dice?
- How would it change the game (and your strategy) if you used three 6-sided dice instead of two 6-sided dice?
- Challenge: What is the likelihood of completing the whole game on one turn (using 6-sided dice); that is, making it all the way to 100 without rolling a single 1?

Even the Odds

Even the Odds (Loewen and Firth 1994) is a very simple game played with four regular 6-sided dice and a game board that includes a list of the values 1 through 20. On a turn, a player rolls all four dice and may take the value of any one die, sum any two dice, sum any three dice or sum all four dice. The player crosses the chosen value off of his or her game board. The player must cross off all of the even values before crossing off any of the odd values. The first player to cross off all 20 values wins.

When playing this game, it quickly becomes apparent that the most difficult values to cross off are 19 and 20. In fact, typically, crossing off 20 takes several turns. What are the odds of rolling a sum of 20 in a single roll with four regular 6-sided dice?

This problem is very easy to model with the TI-83+. Here the calculator simulates rolling a die four times and sums the values; if the sum equals 20 the counter is increased by one. We need to ask the calculator to do this several times in order to get a reasonable estimate of the probability.

```
PROGRAM:PROB5
:0→C
:For(A,1,1000)
:randInt(1,6)+ra
ndInt(1,6)+randI
nt(1,6)+randInt(
1,6)→B
:If B=20:C+1→C
:End
:Disp C
:
```

The first time the program was run, I obtained a result of 1, implying the probability of rolling a sum of 20 with a single roll is about 1 in 100. The second time the program was run, the result was 4, implying 4 in 100.

- How many rolls of the four dice are necessary to get a reasonable estimate of this probability?
- How could you modify the program to simulate rolling the dice 1,000 times?

- What is the theoretical probability of rolling a sum of 20 with four dice?

How can we calculate a theoretical probability for this problem? To do this we need to know the number of possible combinations of dice that sum to 20 and the number of possible outcomes.

$$\text{Probability}(20) = \frac{\text{Possible Combinations of 20}}{\text{Total Possible Outcomes}}$$

The number of possible outcomes is easy to calculate as it is simply $6 \times 6 \times 6 \times 6$ or 64. There are 1,296 possible outcomes.

There are a variety of ways to determine the possible combinations of 20, but I turned to my calculator again and used a simple routine.

```
PROGRAM:AAA
:0→T
:For(A,1,6)
:For(B,1,6)
:For(C,1,6)
:For(D,1,6)
:If A+B+C+D=20:T
+1→T
:End
:End
:End
:End
:Disp T
:
```

By running the program it was shown that there are 35 combinations that total 20. Using our formula, we can show that the probability of rolling a 20 is

$$\text{Probability}(20) = \frac{35}{1296}$$

The theoretical probability of rolling a sum of 20 is about three times in 100 tries.

- Calculate the probability of crossing off the value 19 with a roll of four dice.
- How can you calculate the probability of crossing off the value 18 on a turn? Remember, this can be done with combinations of three or four dice!
- Challenge: Knowing that you can select any one die, the sum of any two dice, the sum of any three dice or the sum of all four dice, calculate the probability of being able to cross off any given number on a turn.

The Unusual Die

Two players are trying a simple dice game in which they select one die at the start of the game and roll it five times, summing the values rolled. The player with the highest sum after five rolls wins. There are two different dice from which the player may select.

One is a regular six-sided die; the other die has six sides, but unusual values on its sides. The net for die is shown below. Which die should you select to maximize your chances of winning?

The easiest way to simulate this game is to have the calculator roll both dice five times and compare the sums. If the calculator played the game several times, we could get an indication of the better die to choose.

Note that rolling the unusual die is rather like flipping a coin where one side of the coin has a value of six, the other has a value of one. As an alternative, it may be interesting to try acting out this problem with a die and a coin.

In the program below, the values for the regular die are summed and stored in the variable C, while the values for the unusual die are summed in D. The variable M counts the number of times the regular die wins, while the variable N counts the number of times the unusual die wins. The variable Q records ties. What is the variable T used for in this program?

```
PROGRAM:PROB6
:0→M:0→N:0→Q
:For(A,1,500)
:0→C
:0→■
:For(B,1,5)
:randInt(1,6)+C→
C
:randInt(1,2)→T
:If T=2:6→T
:T+■→D
:End
:If C>D:1+M→M
:If D>C:1+N→N
:If C=D:1+Q→Q
:End
:Disp M,N,Q
:■
```

- Try running the program several times to calculate an average for M, N and Q, or try modifying the program to play the game a larger number of times. Can you draw any conclusions from these simulated experimental probabilities?
- For each die, what is the theoretical average sum after five rolls? What is the theoretical probability of the regular die winning? Of the unusual die winning? Of obtaining a tie?
- Which die is the better die for this game?

Horse Race

The Horse Race game is played with two or more players. Players each roll a single regular six-sided die simultaneously and repeatedly. A player must roll a one before he or she may roll for the two and so on. The first player to roll all of the values on the die in order wins. On average, how many rolls are required to roll each of the values one through six in order?

Before we construct a program, it is worth considering the given task a little more closely. At first the task feels a bit overwhelming because it is hard to comprehend the number of rolls necessary to finish the race. But, ask yourself this: is there any difference in the likely number of rolls necessary to obtain a two as compared with a one? No! The die does not care what we choose as our target value. In other words, the task is really just the same as racing to be the first to roll a one—six times over! We can simplify our problem, then, by asking this question: what is the average number of rolls necessary to roll a one?

The following program instructs the calculator to keep picking random digits between one and six inclusive until a one is selected, keeping track of the number of selections in the variable T. This program simulates rolling a die until a one appears.

```
PROGRAM:PROB7
:0→T
:0→X
:While X≠1
:randInt(1,6)→X
:1+T→T
:End
:Disp T
:■
```

Of course, when we run the program as it is shown above, we only find the number of rolls until the first one appears. We may be lucky and get it on the first roll, or it may take several rolls. I ran the program several times, and some experiments required in excess of 25 rolls! Sometimes I got it on the first try. We can have the TI-83+ calculate an average for us by modifying the program slightly, placing our **While** loop inside of a **For** loop so as to repeat the experiment several times. The variable Q is used to calculate an average of the number of rolls required in each experiment.

```
PROGRAM:PROB7
:0→Q
:For(A,1,100)
:0→T
:0→X
:While X≠1
:randInt(1,6)→X
:1+T→T
:End
:Q+T→Q
:End
:Disp Q/100
:■
```

- Modify the program above to simulate rolling until you have obtained 500 ones.
- Modify the program above to simulate rolling for a two instead of a one. How does this affect the required number of rolls?
- Calculate the theoretical average number of rolls required to roll each value from one to six in sequence.
- What is the probability of rolling your way straight through to the finish line in six rolls?
- How are the probabilities affected if you must roll all of the even values in sequence before you may roll the odd values, also in sequence?
- What is the average number of rolls needed to complete the horse race with an eight-sided die? with a ten-sided die?

Conclusion

Unquestionably, there are a few limitations in programming on the TI-83+, primarily among them the calculator's processing speed and data entry functions. However, the TI-83+ is a surprisingly powerful tool in conducting simple probability explorations. With a surprisingly small number of commands, the calculator can easily simulate rolling dice and thus enable the exploration of a wide variety of fun and challenging problem-solving activities.

References

Burns, M. 1992. *About Teaching Mathematics: A K–8 Resource.* Sausalito, Calif.: Marilyn Burns Education Associates.

Loewen, A. C., and B. J. Firth. 1994. *Mathematical Games Made Easy for Primary Grades.* Barrie, Ont.: Exclusive Educational Products.

A. Craig Loewen, coeditor of delta-K, *is an associate professor of mathematics education, assistant dean of Student Program Services, and interim associate dean in the Faculty of Education at the University of Lethbridge.*