# The Abacus: The Oldest Calculator

*Sandra M Pulver*

The abacus—the world's oldest calculating machine—can be used for performing all basic arithmetic operations. It has taken many forms: the dust abacus, the line abacus, the grooved abacus and the bead abacus, which is still used today.

The first known abacus was the dust abacus, which was probably invented by the Babylonians. Lines were drawn as place holders with a finger or a sharp instrument, either on the ground or on a table covered with sand or dust. Pebbles were put on the lines as counters to indicate the numbers. The dust abacus was used until the end of the ninth century AD.

Before the fourth century AD, the Romans developed a grooved abacus from the line abacus. The Romans based their numbers on the decimal system, and their abacus had 19 grooves and 45 stones that slid in the grooves to count numbers. It was the first abacus in which the counters were fixed in their columns.

From the grooved abacus, the Chinese developed a bead abacus, which is still used in many forms in many Oriental countries. The bead abacus consists of a series of rods holding five or six beads each. It can be used for counting both very large and very small numbers, and it is a fast and efficient tool for daily arithmetic calculations. Of all the ancient calculating devices that have come down to us, the bead abacus is the only one on which all four arithmetical operations can be performed rapidly and relatively simply. For those trained in its use, it is an efficient means of adding, subtracting, multiplying and dividing—and even for extracting square and cube roots.

In China, the bead abacus (the *suan p'an*) is still used by peddlers, merchants, accountants, bankers and hotelkeepers—and even by astronomers and mathematicians. It is so deeply rooted in the Far Eastern tradition that the Westernized Chinese and Vietnamese in Bangkok, Singapore, Taiwan, Europe and North America often continue to do calculations on an abacus, even though they have easy access to modern calculators. Even more striking is the fact that in Japan, one of the world's foremost producers of pocket calculators, the abacus (the *soroban*) is still the main calculating instrument in everyday life, and every child learns how to use it in school. Even in the former Soviet Union, the abacus (the *s'choty*) appears alongside modern calculators and is often used for calculating what customers must pay in shops, supermarkets, hotels and department stores.

Today, the best-known form of the bead abacus is the Chinese suan p'an. The suan p'an has 10–12 rods, with two beads on the upper part of each rod and five beads on the lower part. Each of the five beads on the lower part represents 1 unit, and each of the two beads on the upper part represents 5 units. Numbers are set by moving beads toward the crossbar that separates the upper and lower parts of the rods. To set the number 3, for example, raise three beads on the lower part of the first rod on the right. For the number 9, lower one upper bead (worth 5) and raise four lower beads (worth 1 each).

The value represented by a bead depends on which rod the bead lies. A lower bead on the ones rod represents a value of 1; on the tens rod, it represents a value of 10; on the hundreds rod, it represents a value of 100; and so on. Similarly, an upper bead on the ones rod has a value of 5, an upper bead on the tens rod has a value of 50 and so on. Thus, relatively few beads are needed for representing large numbers.

The Japanese imported the abacus from China in the 16th century and used it in that form until the end of the 19th century, when they modified it by having one upper bead instead of two. In 1938, they further modified it, this time by having four lower beads instead of five. This is what became known as the soroban. The soroban is usually one foot long and two inches wide, and it usually has 21 rods (*keta* in Japanese).

On the soroban, as on the Chinese abacus, each bead on the lower part of a rod represents 1 unit, and each bead on the upper part represents 5 units. Again, each bead is pushed toward the horizontal crossbar in a calculation. The value represented depends on which rod the bead is on. Three lower beads on the ones rod represent the number 3, while three lower beads on the hundreds rod represent the number 300.

The horizontal bar on the soroban has a unit point marked on every third rod (the first rod, the fourth rod, the seventh rod and so on). These unit points are used either to represent a decimal point or to indicate where the ones place is. On the soroban, the first rod represents the thousandths place, the second rod represents the hundredths place and the third rod represents the tenths place. Whole numbers begin to be represented at the fourth rod.

## Adding on the Soroban

In performing addition, push beads toward the crossbar.

For 2 + 6, first set 2 on the ones rod, using two lower beads. To add 6, push one 5-unit (upper) bead and a 1-unit (lower) bead to the crossbar. A total value of 8 appears.

When there are not enough lower beads, use complements with respect to 5. The complements of 5 are 1 and 4, and 2 and 3. To add 1, 2, 3 or 4 to 4, add 5 and then subtract its complement, as follows:

$$4 + 1 \text{ becomes } 4 + (5 - 4),$$
$$4 + 2 \text{ becomes } 4 + (5 - 3),$$
$$4 + 3 \text{ becomes } 4 + (5 - 2).$$

The complements of 10 are 1 and 9, 2 and 8, 3 and 7, 4 and 6, and 5 and 5. To add the numbers 1 through 9 to 9, add 10 and then subtract its complement, as follows:

$$9 + 1 \text{ becomes } 9 + (10 - 9),$$
$$9 + 2 \text{ becomes } 9 + (10 - 8),$$
$$\vdots$$
$$9 + 9 \text{ becomes } 9 + (10 - 1).$$

In adding multidigit numbers on the soroban, work from left to right, not right to left as you would when adding with pencil and paper. For 22 + 12, first set 22. Then add 10 (using a lower bead on the tens rod) and 2 (using two lower beads on the ones rod). The result forms mechanically. In the last step, you do not need to use the complement of 5 to add 2, because there are enough beads on the ones rod.

For 374 + 918, which involves carrying, first set 374. There are not enough beads remaining on the hundreds rod to represent 900. Therefore, to add 900, use its complement (1,000 – 100). Add 10 (using a lower bead from the tens rod) and then add 8 using its complement (10 – 2). You will then have the correct result.

## Subtracting on the Soroban

In subtraction, beads are pushed away from the crossbar.

For 3 – 2, first set 3. To subtract 2, push down two 1-unit beads. One 1-unit bead remains against the crossbar, so you have a result of 1.

The soroban has only four 1-unit beads, so to subtract 1, 2, 3 or 4 from 5, you must subtract and add its complement. Think of the numbers –1 through –4 as follows:

$$-1 = -5 + 4,$$
$$-2 = -5 + 3,$$
$$-3 = -5 + 2,$$
$$-4 = -5 + 1.$$

To perform 5 – 4, first set 5 by pushing the 5-unit bead on the ones rod toward the crossbar. Then add –5 + 1 by pushing the 5-unit bead away from the crossbar and pushing a 1-unit bead toward the crossbar. Only the 1-unit bead remains at the crossbar, so you have a result of 1.

To subtract the numbers 1 through 9 from 10, subtract 10 and add its complement. Think of the numbers –1 through –9 as follows:

$$-1 = -10 + 9,$$
$$-2 = -10 + 8,$$
$$-3 = -10 + 7,$$
$$\vdots$$
$$-9 = -10 + 1.$$

For 781 – 377, first set 781. Then use the complement of –300 (–500 + 200) to subtract the 300. Then subtract the 70. To subtract 7, use the complement of –7 (–10 + 3).

## Conclusion

You need only know the addition and multiplication tables for the numbers 1 through 9 in order to do arithmetic operations on the bead abacus.

This ingenious calculating device has its drawbacks, however. It requires intensive training and a precise touch (though this becomes instinctive with practice). Also, the slightest error makes it necessary to do the whole calculation over, and intermediate results (such as partial products in multiplication and remainders in division) disappear in the course of operations.

Nevertheless, students who learn how to manipulate the abacus will get better at mental calculations and will obtain a deeper appreciation of mathematics while learning about other cultures.

## Further Reading

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*Sandra M Pulver is a professor of mathematics at Pace University, New York. Active in professional organizations at the city, state and national levels, she is coordinator and host of the annual Greater Metropolitan New York Math Fair, which serves as a forum for outstanding high school students to display original work in mathematics. She has also served as regional chairperson of the international Mathematical Association of America (MAA) contest. For the past 15 years, she has served as grand judge in the mathematics category of the International Science and Engineering Fair, which showcases the creative and innovative research projects of the world's brightest young scholars from the US and more than 25 other countries.*