

## Piano Keys

*Kevin Wang*

On a full-sized piano, there are 88 keys. Many different combinations of keys can be pressed, and hence many sounds can be made. For example, a one-keyed piano can make one sound. A piano with two keys can make three possible sounds from pressing the keys: two at the same time, key number one and key number two. A piano with three keys can make seven sounds. Thus, on a full-sized piano with 88 keys, how many different sounds can be made?

Answer: 309 485 009 821 345 068 724 781 055 combinations/sounds, or  $3.0948501 \times 10^{26}$ .

The least mind-blowing way to solve it would be to form a table of values:

Keys pressed	Combinations
1	1
2	3
3	7
4	15
5	31

Here we see a simple chart. After some careful looking, we can see that when a value of one is added to each combination, it forms:

Keys pressed	Combinations
1	2
2	4
3	8
4	16
5	32

Now it gets clearer. The combinations seem to be just one more than 2 to the exponent of the value of the keys. That would result in a formula  $2^x - 1$ , where  $x$  is the number of keys.

This solution gives the answer of 309 485 009 821 345 068 724 781 055 combinations on a full-sized piano.

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*Kevin Wang is a Grade 8 student at Grandview Heights School, in Edmonton. He has always loved to do math and has witnessed the wonders it can do. One of his recent inspirations for math was Mr Walsh, who taught him to learn math and not just do it. Kevin is extremely interested in computers as the world seems to revolve around them, along with math, of course.*