The New Curriculum: Will It Make Mathematics Meaningful for Students?

Werner Liedtke

Looking Back

The framework for the previous British Columbia mathematics curriculum (Ministry of Education 1995) included general goals related to use of the imagination, tolerance for ambiguity, positive attitude, the ability to communicate, sense making, enjoyment and effective problem solving. Developing or demonstrating number sense was listed at the top of every grade level. Similar goals were part of the previous curricula in western and northern Canada.

Does any concrete evidence exist that is indicative of

- positive behavioural changes regarding the major goals of the mathematics curriculum?
- changes in use of the imagination, willingness to take risks, growth in tolerance for ambiguity and/or achievement of high levels of development in mathematical thinking?
- changes in or improvement of development of number sense?

During a conversation, a mathematics educator who is interested in key aspects of the elementary mathematics curriculum responded to the question "Do you have any data that indicate that we reached these goals?" with, "Perhaps the goals were too high."

Hume (2009) suggests that "it's time to take a new approach to math." In part, his conclusion is based on the fact that about 30 per cent of Canadian parents hire tutors to assist their children with mathematics. This suggests that for many students the goals of the curriculum were not reached.

Hume also reports that "kids like and enjoy mathematics in the elementary grades," but then things worsen for many students. Is liking and enjoying mathematics sufficient? One Grade 7 BC teacher said that according to a survey in his school all Grade 7 students enjoyed problem solving, but test results showed that they could not solve problems. Enjoyment does not imply that curriculum goals are reached.

My data collection and years of teaching span several mathematics curriculum changes. The observations originate from data about conceptual understanding and number sense collected annually from interacting with K–7 students in many classrooms; conducting thorough diagnostic interviews with Grades 1–7 students; talking to teachers-to-be, students and parents in and around schools; and talking to adults who went through the school system.

Each group of students enrolled in courses about mathematics teaching, learning and assessment (including distance education courses with teachers from different areas of BC as well as from other provinces) completed at least one assignment that involved a conversation about mathematics learning and/or assessment with a student and an adult. Over the years many hundreds of transcripts of conversations with subjects from urban and rural areas, even with teachers on picket lines, were read and analyzed.

My data led me to conclude that many students did not achieve some of the key goals of the previous curriculum and that mathematics is not meaningful for them despite the time, energy and money spent on several revisions and additions to the curriculum. Could the goals be too high?

Looking Ahead

Is it possible that the Western and Northern Canadian Protocol (Alberta Education 2006), which provides the framework, topics and general goals for the new elementary mathematics (K–9) and was adopted, or adopted with minor revisions, by jurisdictions in western and northern Canada, can provide the new approach that Hume is looking for? This new curriculum includes the following list of critical components that students must encounter in a mathematics program and goals that can positively affect students' learning about mathematics. Ensuring that students encounter these components and reach the goals requires paying special attention to important ideas related to teaching and assessment.

The Critical Components— Selected Issues Related to Teaching

Communicating Mathematically

Students are expected to communicate in order to learn and express their understanding. The ability to talk and write in one's own words about mathematics and to connect familiar language to mathematical language is essential to the development of conceptual understanding, an important goal of the new curriculum. Writing tasks allow students to think about their thinking. As recordings are shared, ideas may be reinforced or modified. Discussions in cooperative settings and comparisons of strategies and reports by students can contribute to flexible as well as advanced thinking.

Writing was mentioned as part of communicating mathematically in the previous curriculum, but neither specific outcomes nor ideas were suggested for making it part of teaching. This curriculum included other oversights and errors, which indicate a need for final editing. As a result, it is not surprising that surveyed students did not believe that writing was an important part of learning about mathematics (Liedtke and Sales 2008). When asked to write, some students stated, "Writing is not what we do in math." The "must" condition in the new curriculum implies that talking and writing are an essential part of mathematics learning.

Connecting

Students are expected to connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines. Connecting can be thought of as a problem-solving strategy. Students should be able to connect everything they learn or have learned to previous learning and to experiences outside the classroom. It is discouraging when students are unable to respond to, "Who would want to multiply two two-digit numerals (for example, 32×25) or two decimals (for example, 4.2×3.6), and why would people want to do that?" During interviews, two students answered with, "A teacher" and "The smartest person in the classroom."

My data support the results collected from American teachers (Howe 1999). Graduates of our school system are unable to make up a meaningful word problem for the equation $1\frac{3}{4} \div \frac{1}{2} = [\ ]$. Many confuse it with $1\frac{3}{4} \div 2 = [\ ]$. (Satisfy your curiosity. Ask students from middle or high school to make up word problems for $8 \div \frac{1}{2} = [\ ]$ and $8 \div 2 = [\ ]$. The results may be surprising.)

The statement in the new curriculum about connecting can be used to illustrate an important issue: the need for specificity of language. Lack of specificity can be detrimental to reaching goals. Specific language or examples or even both are required to make it possible to translate statements like "connecting ideas to other concepts" into appropriate action.

Mental Mathematics and Estimation

Students are expected to demonstrate fluency with mental mathematics and estimation. How is it possible to foster the development of fluency and assess it without knowing the meaning of the term? Examples, criteria and a definition are required to make implementation possible.

Why is it that estimation has been and is very problematic? Students' responses to requests that involve estimating can be fascinating. Whenever the opportunity arises, I ask questions like:

- How many students do you think are in that part of the playground?
- How many bicycles do you think are in the bicycle stand?

Many students will give answers with a digit in the ones place or to the nearest five. Rarely is the word *about* used as part of a response nor do explanations of estimation strategies mention the use of a referent.

Many people are confused about *guessing* and *estimating* and the language that is used with these terms. Some authors of crossword puzzles give the same meaning to guess and estimate or they equate estimation with making a rough guess. Some teachers use descriptors like *good, logical* and *rough* when students are requested to make a guess or when guesses are categorized; for example, "That's a good guess." I have heard "guesstimate" used as part of a request (how confusing is that?), and student teachers ask young students to estimate without telling them what that means.

An article about lions in the May 15, 2006, *Vancouver Sun* stated, "Estimated number of those attacks that are fatal: 66." Air Canada's website (www.aircanada.com/en/about/fleet/a319-100xm.html) states that the range of one Airbus is 4,442 km. How many test pilots were lost to determine this result? How was the distance that smokers must keep between themselves and a gas pump determined and at what cost? The sign informs us that it is 7.5 m. Reporting results in this fashion is inappropriate, and the statements indicate a lack of number sense.

The learning outcomes related to estimation in the new curriculum can resolve the existing issues related to estimation and making statements about numbers.

Problem Solving

Students are expected to develop and apply new mathematical knowledge through problem solving. The previous curriculum (Ministry of Education 1995, 2) states: "Students must learn the skills of effective problem solving, including the ability to communicate solutions, so that they will become reasoning, thinking individuals able to contribute to society." An impressive sounding statement indeed, but what are the skills of effective problem solving? How is effective defined? How are solutions to be communicated and why? How does sharing solutions contribute to becoming reasoning, thinking people who are able to contribute to society? Does any data exist to indicate that the students who went through the system reached any of the ambitious goals included in the statement? This example is included to reiterate the need for specificity of language.

Teaching through problem solving represents a major change from the previous curriculum and requires special strategies, questioning techniques and assessment procedures. Open-ended questions and high-order thinking questions are essential. All the different types of student responses need to be accommodated (Liedtke 2010).

Mathematical Reasoning

Students are expected to develop mathematical reasoning. The goals included under Reasoning Mathematically in the previous curriculum (p 3) include "Mathematics instruction should help students develop confidence in their ability to reason and to justify their thinking" and "Mathematics should make sense, be logical, and be enjoyable." Without elaboration it is impossible to interpret logical and enjoyable as part of a goal.

At one time student teachers were asked to write specific learning outcomes for the cognitive and the affective domain. Some students tried to transfer this to my courses and on their lesson plans recorded as a goal, "The students will enjoy the lesson." After a smile, I completed these outcomes with "or else" (with an apology to the student and an explanation). The previous WNCP included the specific outcome, "Students will enjoy optical illusions." It is impossible to be serious about this outcome in attempting to reach it with a group of students and then assessing it!

The criteria for mathematical reasoning I have adopted (Greenwood 1993, 144) include "everything should make sense," "getting unstuck on one's own," "ability to identify errors in answers, use of materials and thinking," "using a minimum of counting, and rote pencil and paper computations," and "being willing to try another strategy." My data led me to conclude that students have not reached these goals. The strategies and questioning techniques that are required for teaching through problem solving will contribute to the development of mathematical reasoning.

Technology

Students are expected to select and use technology as tools for learning and solving problems. The new curriculum (Alberta Education 2006, 9) includes a list of possible uses for calculators and computers. The following are key entries of this list for the elementary grades:

- Develop number sense: Any activities that can contribute to the development of number sense will be of benefit to students.
- Develop personal strategies for mathematical operations: Questions can be posed that lead students to develop their own strategies and contribute to other aspects of learning as well; that is, number sense, flexible thinking.
- Explore and demonstrate mathematical relationships: Students can use the calculator to test and describe results of tests.

Visualization

Students are expected to develop visualization skills to help them process information, make connections and solve problems. This critical component requires elaboration. What are specific examples of visualization skills? How do these skills help students process information and what kinds of information? What are examples of the connections that are facilitated? How do the skills assist in problem solving? The discussion under Visualization in the WNCP (Alberta Education 2006) does not provide any answers. In fact, it includes other statements that require definitions, specific examples or both. For example, how many readers would visualize the following statement in the same way? "The use of visualization provides students with opportunities to understand concepts and make connections among them" (p 9).

Interactions with groups of students and with individuals during interviews or discussions have provided me with data that support the conclusion that many students are unable to visualize. This is not surprising, because visualization is not mentioned or explicitly dealt with in the previous curriculum.

However, what is surprising is that reference to developing or demonstrating number sense is part of every grade level though there are no references to ability to visualize number, a key aspect of number sense. Visualization is essential for making mathematics meaningful. Fostering students' ability to visualize requires special strategies.

The New Curriculum—Possible Impact on Students

Despite entries that lack the necessary specificity, the inclusion of undefined terms, and subjective statements, translating the new curriculum into action can make mathematics meaningful for students. However, this positive effect depends on several factors related to teaching and assessment.

Reaching the goals that are part of any major curriculum change depends on appropriate professional development for those whose task it is to translate the curriculum into action. Professional development is required that shows how to make the critical components a must part of a mathematics program.

Professional Development

The essential components of this development must deal with strategies that are new or may differ from those employed to translate the previous curriculum into action. That means the focus should be on "Developing ability to visualize, learning through problem solving, and 'fluency' with estimation and mental mathematics" (Alberta Education 2006, 6), on "Attaching meaning to what is learned" (p 2) that implies the acquisition of conceptual knowledge and on the "opportunity to develop personal strategies" (p 6). Since number sense is the key foundation of numeracy, the accommodation of key aspects of number sense as part of ongoing teaching must be isolated and addressed. The critical components and the goals of the new curriculum require that the appropriate assessment strategies be discussed.

Development of Visualization

Visualization is an important component of all aspects of sense making, especially number sense (the key foundation for numeracy), spatial sense (a requisite for the ability to solve problems) and problem-solving ability in general. The following examples illustrate the importance of the ability to visualize. As far as number sense is concerned, when students hear the names for numbers (whole numbers, fractions, decimals, integers) or see numerals, they should be able to "see" the numbers behind these names. This ability is illustrated by the Grade 5 student, who as part of her explanation that involved relating fractions included the phrase, "I see it in my brain." Without the ability to visualize numbers or "see the numbers in one's brain," and without being able to think flexibly about numbers and numerals, it is very difficult, if not impossible, to reach some of the major learning outcomes in the new curriculum.

The focus of a lesson with a group of Grade 1 students last year was on recognizing, visualizing and relating numbers. For one type of task, one student showed the age of a younger sibling (two) and another his age (seven). Sample questions that were posed to the group included: "Show and quietly name a number close to two," "Show and name a different number that is just as close," "Show and name a number between two and seven that is close to seven," "Show and name a different number between two and seven," "Show and name a number greater than seven that is not close to seven," and "Show an even number between two and seven" (I had watched the students talk about and illustrate even numbers as sharing numbers).

Another task involved an attempt to identify a mystery number. After each hint was presented—for example, "The number is between two and nine"—the students were requested to show what they thought the number or numbers could and could not be. During the procedure a boy in the front row looked up and asked, "How old are you anyway?" This was a legitimate question considering the age of his teacher and the age of the even younger student teacher. I asked him to guess. "81," he said. "How about 181," I answered to which he uttered a drawn out, "Oh." I am still curious about the discussion that might have taken place at the dinner table about the old teacher who taught about mathematics.

The ability to visualize has to be a focus for whole numbers, decimals, fractions and integers. The role of visualization as part of spatial sense, number sense and problem-solving ability is illustrated by such abilities as translating a problem or an equation into a diagram that indicates the meaning of the numbers, the intended order and/or the intended action or actions. A lack of ability to visualize the appropriate type of division accounts for the inability to create a meaningful word problem for equations like $1\frac{3}{4} \div \frac{1}{2} = [\ ]$.

Why do so many students lack the ability to visualize? Possible reasons include lack of specific goals for lessons, inappropriate practice and assessment, ineffective use and/or lack of use of materials, use of nonspecific and incorrect language, and insufficient introductions to operations and procedures (Leidtke 2007). The majority of these shortcomings are addressed in the new curriculum.

Opportunities to Learn Through Problem Solving

One assumption for learning through problem solving is that it is better to solve a problem in many ways than to solve many problems in the same way. Teaching through problem solving presents challenges because it requires special teaching strategies and questioning techniques, the ability to accommodate different types of responses from students and special assessment techniques. Questions must be asked that allow students to use what they know to calculate an answer in a new way. Learning new ideas through problem solving ensures that these ideas are meaningful.

Number Sense the Key for Numeracy

Why do so many responses from students and adults indicate a lack of number sense? An examination of entries related to number sense in the previous curriculum indicates the following problems: key aspects are not identified, possible strategies for fostering the development of number sense are not suggested, activity settings for the key aspects are not identified, practice appropriate for the key aspects is not illustrated, assessment strategies and suggestions for reporting to parents are missing, and settings that could be detrimental to a development of number sense are not identified. The main aspects of number sense include visualizing number, recognizing number without having to count, flexible thinking about number and numerals, estimating number, relating numbers or numerals, and connecting numbers or numerals. The accommodation of these aspects requires special classroom settings and appropriate practice and needs to become part of ongoing teaching.

Ability to visualize, learning through problem solving and number sense are interrelated and are essential for the acquisition and development of conceptual understanding, a major goal of mathematics teaching and learning. Conceptual understanding facilitates new learning. This understanding is fostered in settings where students are allowed to communicate their reasoning orally as well as in writing. Communication in the mathematics classroom is essential (Elliott and Garnett 2008). Research results (Hiebert 2000, 437) indicate "that instruction can emphasize conceptual understanding without sacrificing skill proficiency."

Number sense is essential because it is a requisite for major goals of the new curriculum that include mental mathematics strategies for the basic facts, estimation and mental mathematics strategies for the operations, personal algorithms or strategies for computational procedures, and "fluency" with estimation and mental mathematics.

Professional development needs to focus on strategies and settings that are conducive to the development of number sense and conceptual understanding since this development is vital to making mathematics meaningful.

Selected Issues Related to Assessment

A new curriculum framework that includes components that students must encounter requires discussions and reflections about specific learning outcomes, teaching strategies and assessment techniques as well as possible reporting procedures about the key goals and components. During a discussion among teachers, school boards and the ministry about the assessment data that is collected annually from Grades 4 and 7 BC students, one board member observed that present test results "are too narrow an assessment to measure the overall state of student achievement" (Sherlock 2010).

Assessment instruments are required that yield information about students' ability to think and to think mathematically. Data about how students think is essential not only for reports to caretakers but also for meaningful intervention. Some assessment instruments do not yield any meaningful data about the critical components and key goals for students; that is, multiple-choice tests.

Any attempt to collect assessment data about the critical components of the curriculum requires appropriate and fair assessment items. Many assessment items students face lack these characteristics. This is true for all types of tests (Liedtke 2005). Simple questions that identify one answer as the correct response or ask students to select a response from several choices do not yield any meaningful data. For example, Which does not belong? Which one is different? Which one comes next? Which one is incorrect (correct)? Which statement is true (false)? These simple questions can be very unfair because students must guess what an author was thinking. For example, consider any item that shows part of a repeating or growing pattern and the question, "What comes next?" Since a repeating pattern can be extended in many ways and can easily be changed to increasing patterns, identifying one correct response can punish many students who are flexible in their thinking. If it is an author's intent to look for one correct response, specific instructions are required.

At the end of presentations to parents about such ideas as confidence building, risk taking and flexible thinking, some will share concerns about their children. Anxiety that is caused by timed basic fact tests (for example, mad minute) is a frequent concern. Last year one mother told a story about her Grade 5 son, whose response to the request to extend a pattern was marked incorrect. After he explained his thinking to his mother, he shared the conclusion, "I think my teacher has lost her imagination."

The following further illustrates examples of inappropriate types of requests that identify one response as correct. Several friends forwarded an e-mail of a four-item one-minute test. They had failed the test and wondered whether or not I could pass it. Not one sender commented about the format of the items, nor did others who looked at them. People automatically assume that if items appear on a test they must be appropriate. Three of the items are mathematical:

Continue this sequence in a logical way: M T W T _ _ _

Correct this formula with a single stroke: 5 + 5 + 5 = 550

Draw a rectangle with three lines:

Some authors of test items assume that the one extension for sequences they have in mind is logical and others are not. Could it be, to quote the Grade 5 student, that these authors have lost their imagination? Students who are taught through problem solving, who are confident risk takers, and who persevere and exhibit curiosity (all goals of the curriculum) will be able to present different logical ways of extending this sequence other than thinking of what an author has in mind, in this case, days of the week. According to the author, the correct response to the second item involves drawing a segment to change the first plus sign to 4. It is just as easy to change the "equal" to a "is not equal" sign.

These examples illustrate a few concerns about some types of assessment. If only one response is identified as correct and is expected, appropriate detailed instructions are required.

The third example illustrates the use of incorrect terminology or language in the request as well as in the answer. Three segments are shown in the interior of a rectangular region. Is the use of inappropriate or incorrect language an unusual occurrence? The answer to the question might surprise a few people (Liedtke 2005).

A columnist of the local newspaper wrote the Foundations Skills Assessment (FSA) test for Grade 4 and included the following item in his report: "The students will take a bus 62 km to a nature park. The bus travels at 40 km/h. About how long will the trip take?" For many ESL students the answer would be 60 km or 62 km or none of the above, depending on the choices that are provided.

The framework of the new curriculum requires a careful look at the types of questions that appear on assessment instruments. Perhaps the critical components of the new curriculum require a focus on Foundations Ideas Assessment rather than on Foundations Skills. If such a test is administered at the beginning of the year, teachers could use the results to make instructional adjustments and plan effective IEPs for students who require them. Everyone could come out a winner!

Looking Back from the Future

Here are some questions that should be part of a backward glance five or ten years after the implementation of the new curriculum:

- Does observable and measurable data exist that show that students encountered the critical components of the curriculum?
- Is there evidence that the goals for students were or are reached?
- Did the new curriculum provide the "new approach to math" that columnist Hume suggested?
- Did the new curriculum reduce the need for tutors as was suggested by a former BC minister of education?
- Were there noticeable changes in assessment procedures and reporting to parents?
- Did commercial tutoring companies focus on the critical components of the curriculum or did they continue to present and assess procedural learning?
- Are there some who claim that perhaps the goals in the new curriculum are too high?

Speculations About Answers to the Questions

The new curriculum presents an opportunity to have a positive and lasting impact and to make mathematics meaningful for students. For many teachers the ability to make mathematics meaningful for students will depend on the professional development that is provided. This development must focus on characteristics of classroom settings, teaching strategies and assessment techniques that centre on development of visualization, learning through problem solving, and developing number sense and conceptual understanding.

It took a lot of time, effort and money to produce the new curriculum. However, it has been introduced without setting aside the monies required for support services during implementation and for final editing of the answers that are predictable for most students. As a result of a period of cutbacks, future researchers and columnists will likely conclude that nothing has changed and it's time to take a new approach to math.

Is there a possible solution? Perhaps monies need to be set aside at the outset of a revision for professional inservice. As well, textbook publishers must explain and illustrate the teaching and assessment strategies that are required for accommodating the critical components and general goals of the program.

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Werner Liedtke is a professor emeritus at the University of Victoria, British Columbia. He has taught elementary school and courses in mathematics education and assessment. His main areas of interest include curriculum development and assessment strategies related to key aspects of early numeracy.