Goals of the New Elementary Mathematics Curriculum: The Power of Open-Ended Questions and Tasks

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Introduction: The Problem and Assumptions

According to the *Common Curriculum Framework for K–9 Mathematics* (Ministries of Education 2006, 4), the main goals of mathematics education include preparing students to use mathematics confidently to solve problems and to communicate and reason mathematically. Students who have met the goals of instruction should, among other things, exhibit a positive attitude toward mathematics, engage and persevere in mathematical tasks, be confident and take risks, and exhibit curiosity.

How could these important goals be reached? Hiebert (2000, 437), a mathematics educator interested in the quality of teaching and learning in schools, suggests that "although research can rarely prove that a particular course of action is the best one for all people and for all time, it can help boost the level of confidence with which decisions are made." Based on his research, Hiebert has developed guidelines for designing mathematics classroom environments. Two of those guidelines have implications for the goals set out in the *Common Curriculum Framework* (CCF): students need opportunities to engage directly in the kind of mathematics that are stated in the goals, and instruction can emphasize the conceptual understanding inherent in these goals without sacrificing skill proficiency.

A project that involved Grades 1 and 2 students led Spungin (1996) to conclude: "Problems and tasks that have multiple solutions or methods of solution are best in stimulating discussion, debate, creativity and risk-taking." According to the author, open-ended tasks promote curiosity, provide opportunities for students to wrestle with difficult ideas to revise their thinking, and demonstrate respect for all ideas by encouraging contributions from all students. A replication of Spungin's project with Grade 1 students led to similar conclusions (Liedtke, Kallio and O'Brian 1998). These results clearly indicate that open-ended problems and tasks are conducive to reaching the goals set out in the CCF for students.

Two challenges exist. Because the majority of problems and questions that are included in references used by students lack open-endedness, they need to be modified. Presenting open-ended tasks to groups of students requires the orchestration of discussions that accommodate all types of responses, provide opportunities for learning to respect different ideas and allow students to revise their thinking.

The examples described in this article illustrate the challenges that can exist and show the power of open-ended problems. I will also show how tasks that lack open-endedness can be detrimental because they can discourage willingness to take risks.

Tasks and Settings with Multiple Solutions

Sorting Tasks

Sorting is a thinking strategy that is related to many aspects of mathematics learning. For example, elementary students might be asked to look at four different blocks, figures or animals and respond to these questions: Which of these is not like the other? Which of these is not the same? For many students this request implies that one choice is different, and they must determine which one the person who designed the task had in mind. Students who guess correctly are rewarded. Students who guess differently may believe that their choices are incorrect.

The following example is taken from a reference for students. One choice is identified as correct: Which of the following numbers does not belong in the set? 8 17 25 33. (Which number would you pick and why?)

How can this problem be changed to foster students' confidence, encourage them to take risks and develop their ability to communicate and reason mathematically? Possible options include the following:

- Questions like, Which of these do you think does not belong? Why do you think so? will result in having students verbalize the reason or reasons for their choice. As they listen to others state their reasons, new ways of thinking about what has been presented may be discovered. The ability to defend one's choice makes a response correct.
- Students could be told that someone selected one option; that is, 17 for the example above, and they are asked to think of possible reasons for choosing this number. Asking students to think of more than one reason has advantages. However, the question must be posed carefully.

For example, some questions try to encourage flexible thinking by asking, Can you think of another way? (Spungin 1996). This can be detrimental. Some students will opt out and respond quickly with a definite no, and any attempt to collect further diagnostic or assessment information has to stop.

During an observation in one classroom the students were using the benchmarks 0, ½ and 1, and attempting to place fractions on this part of the number line. When the teacher asked a student if he can place ⅔ on the number line, the student immediately responded with, "Yes, what else?" After a brief pause and a smile, the lesson continued. Questions that can be answered with one word; for example, Can you...? or Do you know of another way...? are of little value. Equally ineffective are questions like, In how many different ways can you sort these things? How many ways can you come up with? or How many solutions are there? Should full marks be assigned to such responses as none, zero or any number that students record?

The following types of requests encourage students to talk:

- Try to think of more than one possible reason or way for....
- Try to think of at least two ways to....
- How would or could you....
- Try to....
- Show me what you would do to....

Meanings of words and ideas may be internalized and valuable diagnostic and/or assessment information becomes available as explanations are shared.

One student's reason for selecting 17 was that it is the only one with straight lines, and he was unable to think of another possible reason for this choice. This response supports Peck, Jencks and Connell's (1998) findings that without a follow-up question or a brief interview, more than one-half of the time students may be misjudged. Data from my collection of interviews indicate that children may also state unpredictable logical reasons for choices that are identified as correct. Then there will always be justifications that are too good to be true. For example, a child was asked to sort objects from a box that somehow went together. When he was asked to explain his sorting technique, he pointed and stated: "These are interesting; these aren't." To foster flexible thinking students can be asked, working alone or with a partner, to think of how each member in a list of items might be different from the others.

If having students identify one odd member in a collection is the goal, the request must be specific. For example, as students face four different animals, the types of requests could be as follows:

- Think about what animals eat.
- Think about where animals live and show me the animal that does not belong.
- Tell me why that is the case.

However, a specific request may not guarantee that the desired response will be elicited. For the choices, S T 7 L, and the request, I am thinking of letters of the alphabet. Which one does not belong? yielded the following responses from two children: "This one (T) because it has a roof on it, and this one (S) because it is curvy."

Tasks with Patterns

I agree with those who suggest that some published programs for the primary grades devote too much time to activities with patterns. My observations lead me to conclude that too many tasks in the early grades are similar. They do not connect to any aspects of mathematics learning, and in some settings students spend too much time on the low-level activities of colouring or cutting and pasting. I agree with a former NCTM president's conclusion that, "Work with patterns is probably overemphasized in some quarters as the defining component of algebra for younger learners" (Fennell 2008).

During interviews I have asked students who have looked at patterns during their first three years of school what they think a pattern is. The majority of them were unable to make up their own definition. My collection includes the following responses from four students:

- A pattern is one shape after another.
- A pattern is different colours in a pattern.
- A pattern is very neat.
- A pattern is good and fun for you to practise. (Liedtke 2010)

Activity sheets and assessment items about patterns that are provided to students typically include the question: What comes next? or the request: Extend in a logical way. The majority of authors who design tasks for this question and request have one correct response in mind. Without instructions that are much more specific and detailed, many students will not produce the response the authors had in mind. A common misconception is that a sequence is determined by the first few terms. However, a sequence is not defined by the first several terms, but by a function or a context. The question, What comes next: 1, 3, 5, 7? allows for anything to be a correct answer, and thus the students' explanation of their thinking becomes essential.

Patterns can be extended in many ways. For example, it is easy to change a repeating pattern into a growing or increasing pattern and vice versa. An answer key that identifies one response as correct can be detrimental for students. Because different correct responses are possible, the need for a follow-up question or brief interview suggested by Peck, Jencks and Connell (1998) is even greater than for the sorting tasks that were described. The following illustrate a few of the possible dilemmas that can arise in response to sorting tasks:

- For the pattern of different two-dimensional figures: triangle–square–triangle–square–triangle–square, a five-year-old boy selected a circular shape in response to, What comes next? His reason for this choice was, "I want to see a circle." It might have been tempting to conclude that this boy lacks an understanding of repeating patterns. However, that was not the case. When the question, What would you put next? was posed several times, he created a repeating pattern that included the circle (Liedtke and Thom 2009).
- As part of an interview with a Grade 1 student the request, What comes next? was made for 1 2 3.... Without any hesitation he recorded 6. Was a high level of thinking involved in arriving at the response? His response to, Why did you choose six? reinforced the need for a follow-up question. The reason, "I am six years old," was logical according to him. The look on his face seemed to suggest, "Let's put down a number that is important to me and then go from there."
- The problem, Extend in a logical way: M T W T, was sent to me over the Internet, and it identified one response as being correct. I have presented this problem to a number of students and adults and have several responses that differ from what the author had in mind—the first letter of the days of the week. Each of these responses is logical, according to those who created it, and I agree.
- After a presentation to parents at a local school, one mother told me that her Grade 5 son had brought home a page of tasks about patterns. The question for the whole page was, What comes next? and every response he had recorded was marked by the teacher as being incorrect. After he explained his answers to his mother she asked, "So what do you think?" He responded, "I think my teacher has lost her imagination." This powerful scenario points to the need to dream up new ways to ask students to work with patterns.

What are some possible options for tasks?

1. The instructions can be specific and detailed. For example, As you decide what you think should come next in the shown sequence, think of a repeating pattern. However, this may not guarantee that the desired response is elicited.
2. To encourage students to be confident and take risks, an open-ended question can be posed: What do you think could come next and why? Explain your reason or reasons.
3. Students can be challenged to use their imagination by making the request to try and generate many different patterns for a given sequence.

During a workshop with teachers, the point was made that one of the most effective ways to probe understanding of patterns is to present a pattern, hide one or more of the members and ask students, What do you think is hidden? How do you know? In retrospect, it was unfortunate that the pattern that was displayed consisted of seven two-dimensional figures and the member in the middle was hidden. One very observant teacher was quick to point out that she thought there existed several possible responses to the question. She cited the example of the boy who wanted to see a circle, stated that she could insert a circle and then continue the pattern by flipping the members about the circle. She also suggested that two or more figures, one above the other, could be hidden. Unforeseen scenarios can turn into valuable learning experiences.

Guessing and Estimating

Making and sharing a guess requires confidence and involves some risk taking. Since that is the case, whenever an opportunity arises young students should be asked to guess. Students should realize that all guesses are equally valued. If that goal is reached, they will participate with confidence whenever they are asked for a response.

Let's assume a group of students is asked to guess how many jelly beans are in a jar. It must be assumed that every recorded number represents the quantity a student visualized while looking at the jar. If that assumption is made, all guesses have to be evaluated in the same way; that is, as "good guesses." It could be very discouraging for some students and it may affect their future willingness to take risks if one or two responses are identified as "very good" or as "excellent." It is more appropriate to use the descriptors "lucky" and/or "a little luckier" since there is little evidence of the students' thinking.

Young students learn the difference between guessing and estimating. The estimation strategies students develop involve the use of referents or benchmarks. During the early stages of developing estimation strategies, different degrees of familiarity with units will result in estimates that will vary from student to student. These differences should not result in different evaluation categories. All estimation results should be acknowledged in the same way.

In some student references the request is made to record estimates for calculations or measurements. For example, students may be asked to estimate the sum of 24 and 17 or to estimate the perimeter of a rectangle that is 35 cm × 3 cm. Sometimes students are asked to record estimates before answers are calculated or objects are measured. It is easy to guess why some students reverse the order of this request or change their estimates after their calculation or measurement. Neither of these types of tasks will give any insight into students' ability to estimate. Assessment about estimation requires an oral or a written explanation; that is, What would you do to estimate the number of jelly beans in the jar?

Teachable Moments

Whenever young children make mistakes or have difficulty expressing ideas in their own words, some teachers will take advantage of what they think is a teachable moment, correct the mistakes and/or tell children what to say and/or think. These teachers have the false belief that they are providing a shortcut to cognitive development. They forget that ideas grow slowly and, along with the meanings of words, are internalized when children have the opportunity to express them in their own words at their own level of ability. An opportunity to talk about what has been learned is essential for learning how to communicate and reason mathematically. Mistakes and unexpected responses should lead to an exchange of questions and a discussion of ideas.

The goals of the CCF can be reached by attending to the power of open-ended questions and tasks that encourage students to explain their thinking. I conclude by presenting a conversation as told to me by a teacher in one of my courses. After his Grade 2 son had looked at his digital watch and stated, "Dad, it's 5:41, 22 minutes to *Scooby Doo*" the following exchange took place:

"How many minutes is it from 41 to 50?"

"Nine minutes."

"How many minutes is it from 50 to 60?"

"Ten minutes."

"How much is nine plus ten?"

"Nineteen …, but dad, there are two minutes of commercials first."

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