WHAT MIGHT BE DONE IN MATHEMATICS

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Editor's Note: Professor Phibbs has shown interest in MCATA affairs, having served as Department of Mathematics' representative on our Council. Those who have taken courses from him attest to his ability to make what seems complex appear quite simple. We welcome his considered opinions about mathematics education.

New Views, New Language

The last few years have seen a revival of interest in the subject matter of the mathematics curriculum in the schools. The volume of mathematical research has increased enormously. The demand for mathematicians by our society is increasing rapidly; not only for defence purposes but also in the fields of sociology, economics, and business. This being so, it has become increasingly necessary to do all that is possible to train mathematicians quickly and efficiently.

To this end it is proposed that the less important subject matter be taken out of the curriculum which should concern itself with the basic concepts of mathematics. For example the solution of large numbers of numerical problems concerned with the calculation of the sides and angles of triangles is considered to be wasteful of time although of course a high degree of competence in the arithmetic involved is desirable. Again, while a familiarity with the idea of percentages is important, there seems to be no good reason why much time should be spent on its applications to business transactions. Instead it is considered to be of much more importance that the student should be presented with a picture of mathematical systems as instances of "sets" having "elements" which may be combined by certain "operations", the whole having a certain form or "structure". We find such words as "closure", "commutative", "distributive" appearing together with a symbolic notation which in some of the new texts is kept to a minimum, in others is allowed full play. Thus there will be not only a change in point of view but also a new language in which it is expressed.

Here certain difficulties may well appear. The new approach is more mature, mathematically, than the conventional one and it may be that the less able students will experience more difficulty with this approach. Perhaps the solution for this problem is to have some separation of the students at the Grade XI or XII levels; in one group to place those students who are evidently

much interested in mathematics and who are likely to go on to use it at university; in the other those students who for various reasons do not have the same deep interest in the subject or who are planning to pursue their studies in other fields.

Teachers' Equipment

Another difficulty is that by and large the instruction in our schools and the training of our teachers have not been patterned on the new ideas. If instruction in the new mathematics is to be really effective then the relevant ideas and notation must become the automatic equipment of teachers. This, of course, involves very difficult readjustment for teachers to make. The answer, here, is to give intending teachers an acquaintance with what is required before they graduate, and already at the university this is being done. For others who have been teaching for some time the problem of readjustment is very real and troublesome. Perhaps it could be met with types of inservice action possibly it could be minimized by the use of carefully chosen textbooks.

Besides the change in point of view it is also proposed to introduce new subject matter which so far has been generally presented at the university. In high school, mostly at the Grade XII level, some elementary treatment of groups, linear transformations, matrices, determinants and allied topics are likely to make their appearance. High school algebra will remain much as it is at present except that where it is proper to do so its concepts will be expressed in the new language. The algebra remains because it is the necessary prerequisite for calculus.

Structure or Manipulation

The new material, besides being interesting in itself, provides the opportunity for the investigation of new systems with accompanying structures, elements and operations. Non-commutative operations make their appearance. It is at this stage that the importance of the notion of "structure" begins to appear. It becomes evident that systems which at first sight appear to be very different are yet absolutely identical as far as their "structure" is concerned.

It seems a pity that for the greater part of his school life the student would be familiar only with the ordinary number system and that only at the Grade XII level is his outlook really broadened. Surely early knowledge of more than one type of mathematical system is desirable quite soon in the lower grades - if we wish to keep the student's interest alive. Many simple systems of a kind which can be readily appreciated by quite young children exist and could be used. There are the symmetries of the square, equilateral triangle, the rotation groups and the additive groups of the integers relative to a given modulus which come to mind. An elementary contact with geometry is necessary - this at present

seems to be first met with in junior high school. But the simple ideas needed for the above suggestions could easily be appreciated much earlier. The same applies to simple geometric constructions. The idea of loci could illustrate very effectively the ideas of set and function in a way which would, at an early age, drive home the full generality of these concepts, to be met at later stages of their training.