THE PRE-SCHOOL CHILD'S
CONCEPT OF NUMBER

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Mr. Holditch has abstracted the following article from a long and complicated masters thesis by Alec Brace under the direction of Dr. L. Doyal Nelson. In 1962-63, Mr. Brace, who has now left us for duties at Memorial University, St. John's, Newfoundland, conducted a research of elemental importance to all primary teachers of number. Administrators, principals, and supervisors, have traditionally followed practices of admittance and grouping of beginners which might well be re-examined in the light of this research.

## Rote or Concept?

In this research, the basic number knowledge of children was investigated. The researchers assume that rote memorization of symbols and facts forms a poor basis for early growth of number concepts. Based on this idea, the writers endeavored to find what school beginners know. In this way the most important factors would be found which influence early growth of number knowledge. The influence of sex, older brothers and sisters, social, economic and age factors on play school children in the Edmonton area were assessed in this study.

Accepting these as the influencing factors, the following questions were investigated:
(a) Was there a relationship between rote counting and number concept?
(b) Was there a difference in number knowledge of pre-school children coming from homes of high and low socio-economic levels?
(c) Was there a difference in number knowledge between boys and girls?
(d) Was there a difference in number knowledge attributable to older siblings in school?
(e) Was there a difference in number knowledge in children from age five and one-half to age six and children from age six to six and one-half?

## Background of the_study

What children know about numbers in early years has been
a controversial point amongst writers on the subject; ranging from the point of view that such knowledge is meagre to the opinion that it is extensive. Piaget set forth the idea that there were three stages in the development of number concept and accordingly made the following classifications.
(a) Pre-operational Stage (age 4 to 5) -- children have not developed an idea of invariance; i.e., objects in a group change when arrangements of the objects change.
(b) Intuitive (age 5 to 6) - children are incapable of reasoning logically with number, but vague ideas are evident.
(c) Operational Stage (age 6-1/2 to 8) - concrete reasoning is possible, but abstract reasoning develops at a later age.

Although Piaget was critized for the way he found his samples and some of the odd ways in which he handled his statistics, those who followed in this study of children's thoughts about numbers, such as Dodwell. Lunzer and Holmes still accepted Piaget's classification. Estes, however, took an opposite point of view about "number concept". Further investigations along this line by such investigators as Brownell, Gunderson and Mott do not apply since their work was carried out using verbal responses. Piaget, Dodwell and the others had always confined investigations strictly to observation of things done by the children they were investigating, as opposed to verbal responses.

Experimental Design
Play school children attend three two-hour sessions each week in Edmonton. No formal instruction is permitted in this program. The program included children of the required ages for the year 1962-63. Included in the sampling were forty-nine of age five and a half and under, forty between ages of five and a half and six years and thirty-five of age six and over. There were sixty-three boys and sixty-one girls. The sample was considered representative although admittedly urban. Pilot tests were prepared and used to develop a test of fifty-four items which was divided into six sub-tests:

1. Rational Counting - this was designed to test one-to-one correspondence of names and objects counted. Discs or sticks were recognized in ones, twos, fives and tens.
2. Comparisons - groups of colored blocks toy horses, dolls or rattles were compared for equalities and inequalities of numbers of related objects:- i.e., Is there a horse for each rider?
3. Conservation of Number - variance and invariance of totals were tested. This test was carried out as follows:

Blue and red blocks were first placed side by side in one-to-one correspondence; then the red blocks were equally spaced but with greater spacing between them than between the blue blocks. The child was then asked if the number was the same or different. In a similar manner the equalities of colored liquids in different sizes of containers were compared.
4. Cardinal Property of Number - recognition of objects in a group without counting was tested with cards of colored buttons varying in the number of buttons from two to nine. Both patterned and random groups were used for identification.
5. Ordinal Property of Number - ten toy horses arranged in single file (running a race) were employed: i.e., Which horse is in third place? Similarly firemen climbed to the fifth step of a ladder or a car was parked on a lot in the third place of the second row.
6. Place Value - concept of place value was tested with bundles of ten sticks. To these bundles additional single sticks were added to make groups of eleven, nineteen, twenty-three and so forth.

Administration, Scoring and Analysis
The tests were administered from March 15 to April 30, 1963, individually to children by one of the writers, with an assistant who recorded results. Of the fifty-five items tested the first four questions involving counting were recorded as logarithms of the highest number reached in each item. These logs were each multiplied by ten and then summed as a composite counting score. The remaining questions (5-55) were assigned a score of one or zero.

A factor analysis was made of the intercorrelations of items five to fifty-three, revealing the fact that seven separate factors were being tested by forty-nine items. Item 5 as tested consisted of two factors. Item 4 (cardinal numbers) also consisted of two separate factors. Only those with a factor loading of 4 or above were used in defining a factor. Below this loading they were discarded.

When the researchers had analyzed the factors to see which were the same in part or wholly, they found that they were really only testing about seven items:

1. Ordinal number
2. Conservation of number
3. Place value
4. Recognition of groups to five
5. Recognition of groups above five
6. Comparisons
7. Ordinal number

Omitting all the statistical details we can say certain things about these items as shown in this study. The business of counting "one", "two", "three", etc. $i s$ always easier for these beginners than counting "two", "four", "six" - or any other way. Indeed, with these beginners it was found that almost threequarters of them were unable to count by 5 's or lo's. Strangely enough it was also found that those who were able to spot the correct number of colored blocks, regardless of how they were arranged, could also recognize the number of buttons on a card without laboriously counting them out. If a little fellow could easily pick out numbers on a card, the chances are that he would also know which horse was in third place.

As well as being able to count "one", "two", "three", beginners could also tell, when looking at groups of buttons on cards, which cards held the largest number of buttons. This was done correctly more often by sight than by count. Older children did not exhibit this ability as well as the younger pre-schoolers. Four-fifths of the children were deceived when a quantity of liquid was poured from a thin narrow vessel to a wide mouthed one; they thought the amount of liquid had changed. Recognizing the numbers one to five on a card was easy; beyond five was difficult. Girls and boys could do all these things with equal ease; however, those children who came from families in the higher income brackets obtained the best scores.

Consequently, if you are trying to decide whether your pre-schooler is likely to do well at arithmetic don't decide on the basis of the highest number to which he can count. If a child says "nine" it does not necessarily follow that he knows what "nine" means or how it compares with "ten". It is significant if he knows whether he is on the seventh or even the fourth rung of a ladder. A more signigicant fact yet would be a proven ability to add three's to "ten" to make thirteen or two's to make twelve. Some of these skills which seem to be impossible for him at five-and-a-half might well develop when he reaches age six. It will still be very surprising if a little fellow, even after several years, can recognize the fact that when you pour liquid from a saucepan into a graduated beaker you have the same amount in each vessel.
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(2) Dodwell, P.C. "Children's Understanding of Number and Related Concepts". Canadian Journal of Psychology, XIV, 1960, pp. 191-205.
(3) Estes, B.W. "Some Mathematical and Logical Concepts in Children". Journal of Genetic Psychology, LXXXVIII, 1956, pp. 219-222.
(4) Gunderson, A.G. "Number Concepts Held by Seven-year-olds". The Mathematics Teacher, XXXIII, 1940, pp. 18-24.
(5) Holmes, E.E. "What do Pre-First Grade Children Know about Number". Elementary School Journal, April 1963, pp. 397-403.
(6) Lunzer, E.A. "Recent Studies in Britain Based on the Work of Jean Piaget". Occasional Publication No. 4. The National Foundation for Education Research in England and Wales, 1962.
(7) Mott, S.M. "Number Concepts of Small Children". The Mathematics Teacher, XXXVIII, 1945, pp. 291-301.
(8) Piaget, Jean. "The Child's Concept of Number". (Translated by C. Gattegno and F.M. Hodgson.) Routledge, 1952.
(9) Fisher, R.A. "Contributions to Mathematical Statistics". Article 32. New York: John Wiley \& Sons, 1950.
(10) Wohlwill, J.F. "A Study of the Development of the Number Concepts by Scalogram Analysis". Journal of Genetic Psychology, Vol. XCVI - XCVII, 1960, pp. 345-347.

## RESEARCH IN PROGRESS

At present the following research projects are in progress at the University of Alberta and you should hear of them in later publications.

Pelletier, Marcel - "The Measurement Concepts of Grade I Children".

Seward, Ronald -- "The Relationship between Mathematical and Verbal Analysis".

Harrison, Bruce - "An Attempt to Compare the Understanding of Mathematical Ideas of Grade VIII Students in Various 'Modern' and 'Traditional' Programs".

Chamchuk, Nick - "Comparison of Mathematics Achievement of Grade IX Students who had STM Training in Grade VII and Grade VIII with Students who were Taught the Traditional Program"; the criterion in this last study will be the Grade IX Departmental Examination.

