A DISCOVERY LESSON

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Discovery teaching is mentioned, if not condoned, in every textbook and almost every article we pick up. However, it is seldom defined, the reason being that it seems very difficult to pin it down precisely. In fact, when anyone refers to discovery, he could mean a number of things. In this article I plan to explain a lesson that I taught recently. I have no particular need to call it "discovery teaching", but I think it approaches some of the ideals we have in mind when we think of discovery. I have called it a discovery lesson, although this might be a poor title.

I will now attempt to explain how the lesson was set up and then proceed to draw some educational implication from the lesson. As a precautionary note, I should mention that we conceive that the basic attack on a lesson of this kind is to set up a mathematical situation and allow students to react to it. To do this, one must be prepared to allow students to discuss mathematical ideas with very loose terminology. Eventually, it is of the utmost importance that symbols and concepts be rigidly defined, for this is the essence of mathematics learning; but accurate terminology is out of place in an introductory mathematical lesson.

The concept we dealt with was linear relations in two variables. The objective of the lesson was not to try to break down the concept into very small parts to be analyzed thoroughly, but rather to provide a broad framework in which the students were able to work. One readiness concept which the class (a Grade V class in this case) possessed was that of locating points on a rectangular coordinate system, that is, given a point (4,6) the pupils would all agree on its location. They knew that a number on the horizontal axis was represented by $\square$ and the vertical axis was represented by $\triangle$. And they further knew when an open sentence was true or false.

As we thought of presenting the lesson, we decided that the linear relation in two variables could be examined in at least three ways. It is possible to (1) picture the relationship with a graph, (2) make up a rule which the relation (linear equation) follows, or (3) make up a table of corresponding ordered pairs of numbers. Upon examining these ideas, we decided that relating the rule to the table is fairly simple. Even going from the rule to the graph or from the table to the graph or vice versa is obvious. In fact, it appeared that the most general concept of all three was the graph, and that in order to give the students the broadest perspective of this situation, we should begin working with the graph to find the rule. If the graph and the rule could be explored sufficiently, the student would have a good idea of what was involved in a linear relationship.

The question arose what particular graph should come first if the lesson were to begin with a graph on a rectangular coordinate system. One possibility was for the instructor to present a sequence of graphs to the class in a certain predetermined order to make the rule more discoverable and to enable the students to detect patterns between rules and graphs. The other alternative was for the students to make up graphs; the rule, of course, would be unknown to them. The disadvantage of this method was that the students would be presenting the class with a random selection of examples and the patterns would be less obvious. However, this second method would provide for high motivation, that is, the students would really feel that they were responsible for the class themselves, and they would have more of a chance to direct their own learning.

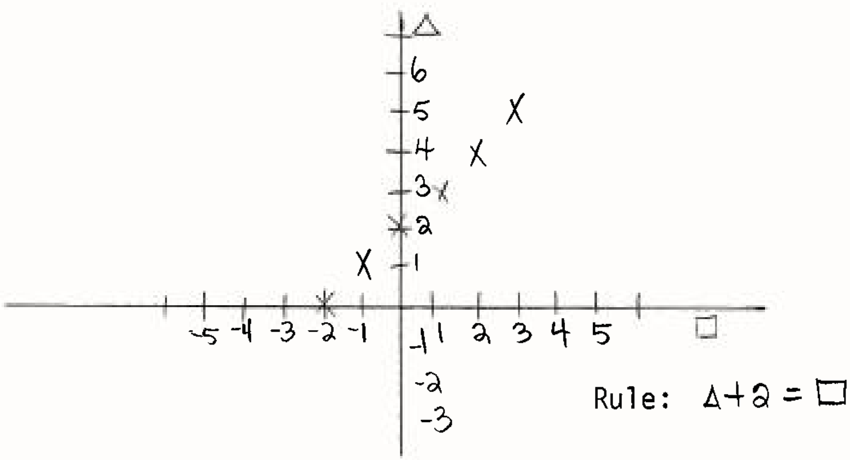
After analyzing the competence and character of the class, we decided to take the latter course of action with all its hazards. The only instructions which the children were given was that they were to make up straight line graphs in any direction they desired, but they were to try to make up what they thought would be interesting graphs. This last comment was made to try to get the children to make up graphs which were similar to the graphs previously presented or which illustrated a particular idea they had.

I will now report on the results of the activities in a group of average students just completing Grade V. I feel that these very same ideas could be used at least in Grade VII, if not in Grades VIII or IX. The class consisted of approximately 20 pupils. We limited the number in the class because of the results of other work we had done on discovery. We have found that large classes have difficulty in coping with discovery technique, especially where there is a considerable variability in mathematical ability among the class members and where the mathematical situations into which they are immersed are rather loosely structured.

I shall proceed to point out the highlights of two 40-minute periods of the discussion of graphs and rules. No mention was made of tables initially.

The first part of the activity was to have a student mark on a rectangular coordinate chart on the board a set of points which were in a straight line. The graph was put on the board:

Figure 1

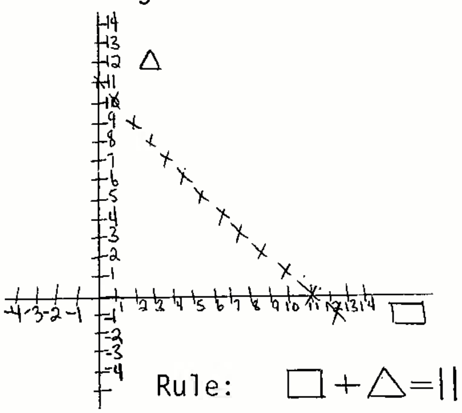


The students now understood that they were supposed to find the rule for this set of points, that is, they were supposed to find the relationship between the numbers that could go into the box (□) and the numbers that could go into the triangle (△). The first response was that they had to be equal. Five pupils spoke to this point. Someone suggested that if they were equal you would get a diagonal, although this was not clearly expressed. The discussion on the equal sign continued with a boy who said there always had to be an equal sign in the rule, but that didn't mean the two things were equal. Another comment emphasized that the box and the triangle could be equal, but then you would get something different from the thing represented by the graph. Finally, the boy who put the graph on the board noticed that his equation was two up from zero. The discussion on this problem ended here.

The most impressive thing about the comments by the students is their vagueness. The pupils had none (or little) of the terminology. They kept saying "they" have to be equal. Even Robert's comment was vague when he said his graph was two up from zero, but I feel the students all knew what he was talking about. The important aspect of this situation was that although the students knew very little of what to say and how to say it, they still reacted with a great deal of enthusiasm. The mathematical situation was structured very loosely; the students had much to learn, but still they were able to react strongly with little frustration and with a minimum amount of direction from the teacher. The problem was left when they felt (or perhaps it was when the instructor felt) they had discussed it enough.

A second graph was put on the board (see Figure 2). The instructor asked if they could extend the graph in either direction. The challenge was taken and points below the graph were found. This gave some assurance that they knew what the points of the graph were and in fact had a feeling for the relationship. Once they began discussing the rule, they suggested the statement □ + 1 = △ - 1. The instructor looked puzzled and really could not understand this rule. After a considerable discussion, the meaning of the statement was made clear. The equation meant that whenever the number in the box (□) went up one, the number in the triangle (△) went down one. They were asked to test the rule by substitution to see if it worked. Two or three examples which did not work were given. However, even after these examples many of the students knew in their hearts that this way of stating the rule made sense to them. No attempt was made by the instructor to correct this idea.

Figure 2

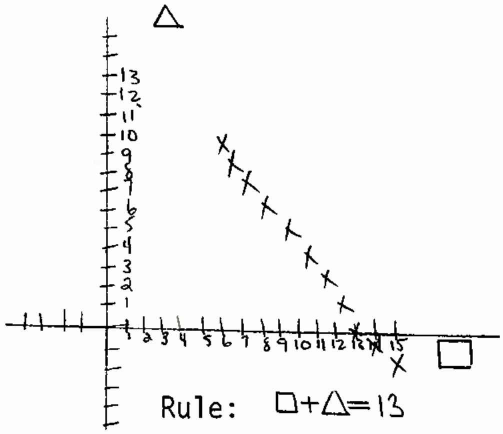


These two equations had taken 35 minutes to discuss; with five minutes left of the period, the instructor suggested trying to list the names of the points, something they had not done before on a systematic basis. After two ordered pairs were down, someone shouted "I got it!" In another minute every hand but one or two went up. The instructor asked the pupil who had his hand up first; unfortunately he did not have the correct rule. However, the rest of the class had it correct, and so the rule □ + △ = 11 was written down. The first class period ended here.

Upon discussing this lesson with an observer, the instructor was reprimanded strongly for structuring the situation to the point where he told the students to use a table. The instructor felt at this point that the students were motivated, very highly, perhaps, to the point of frustration. They were even motivated to the use of the table. He decided that the situation needed more structuring and so suggested the table. But, perhaps another kind of structuring would have been more appropriate. It is interesting to find two experienced teachers, both relatively well versed in discovery techniques, having similar mathematical backgrounds, and observing the same class, disagreeing on this point, as was the case of the instructor and the observer. It is also interesting to speculate on why the first pupil to have the answer was wrong. This is probably purely a coincidence. One conclusion was obvious: the students felt very, very cheerful and satisfied after this lesson. They had indeed discovered a mathematical relationship. I say "discovered", while the observer insisted they may as well have been told.

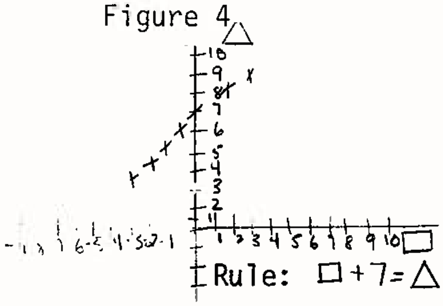
In the second period, one day later, another pupil put another graph on the board (see Figure 3). The instructor allowed a group of four students to discuss the solutions they had arrived at. He then had the students write the rule on the board since most of the pupils seem to have grasped the relationship. There was some disagreement within the group because one of the group insisted his equation was correct. The two equations presented were □ + △ = 13 and 13 - □ = △. After a lengthy discussion, it was decided that they were both right. However, at this point, a discussion ensued as to which was the better of the two. This discussion finally resolved itself into a question of which one came first and Mike insisted that 13-□ = △ came first to his mind. In opposition to Mike's stand, a student asked: "Where did you get 13 in the first place? You had to think that the numbers added up to 13." Although this latter student stated his argument very badly, Mike had to agree that he arrived at the number 13 by noticing that the box plus the triangle added up to 13. Poorly stated as it was, the argument convinced Mike that maybe his equation was an afterthought, but still correct.

Figure 3



This dicussion is interesting from two points of view. First, two different equations were presented and the children agreed that they were both correct. They were, in fact, discovering equivalent equations. It is interesting that the students at this point did not think of any other equations. It is very doubtful that they thought of these rules as being different names for the same relationship, but at least the idea that two different rules could be used to establish the same graph was brought out in the discussion. The second important point of this discussion was the idea of the students trying to decide which equation came first. They were, in fact, trying to decide which was the most natural way to solve the problem. The consensus was that you arrive at the rule by adding the number in the box (□) to the number in the triangle (△) and see what this sum is equal to. In a manner of speaking, they were discovering not only the rule but also the nature of discovering.

Figure 4



The second graph of the second period was put on the board by a student (see Figure 4). No discussion about the points was held. A group of students acting as judges were allowed to discuss the equation, and this group of five or six students ended up with the following four equations. $\square + 7 = \triangle$, $\triangle - 7 = \square$, $7 + \square = \triangle$, and $\triangle - \square = 7$.

Each could readily see that they all applied to the set of points. However, again a debate followed as to which equation was the best. No one tried to define "best". The students seemed to have little trouble talking about these rules as representing the same idea. In fact, mention was made of one of the rules simply being the reverse of the other and, therefore, not really different. The instructor terminated the lesson at this point.

Education Outcomes

One might argue that this lesson could be presented more efficiently by telling the students that there are such things as equations for relationships. Perhaps the only way to find out which is the most efficient method is to test with control classes. However, if we wish to carry out such tests, the problem is a difficult one except for the test of mathematical information. If we admit that we are interested in something more than information, we need instruments to measure this other thing. I would now like to discuss some of the aspects and educational outcomes of the lesson other than informational aspects.

The first aspect of the lesson was the opportunity for students to discuss mathematics. More important, they were discussing mathematics without precise terminology, but they could still communicate. It would appear that the development of the ability to discuss mathematics and mathematical problems is important. Along with this would be the development of confidence, in the students, of being able to create mathematics without the assistance of an authority.

A second educational outcome is related to the notion that the student had control of the symbols and not vice versa. An example of this was when a student wanted to write the rule "$\square = \triangle$" for one of the graphs; a second student said "You may write the equation '$\square = \triangle$' but the equation will not give you the points of the graph." The feeling was clearly aroused in these classroom situations that you may do anything you want to do, but you should only do those things that are going to get you some place. Another illustration of this idea of controlling symbols was brought into the discussion which took place in the class concerning which equation was better. In that particular instance the point of the discussion was missed because "better" or "best" was never defined. "Best for what?" was never asked. Again, the matter of picking the best is an example of using symbols in the way we want to use them.

An air of self-evaluation was developed in the class. The teacher played a very minor role and the students were evaluating, on their own, their progress. I am sure that self-evaluation is a legitimate aim of school education, but even more important, it is very legitimately a behavior pattern that must be exhibited by the mathematician. As such, this outcome would be classified under knowledge of mathematics as know-how rather than information.

The fourth outcome I wish to mention is motivation, which was very high. Motivation is an educational objective. A teacher should consider his task one of making the student like mathematics as well as making the student learn it, that is, motivation in education is an end in itself, not just a means. One of the obvious outcomes, although it may not have shown up on any attitudes scale, is that students enjoy discovery-learning situations.

Another aspect of the learning situation is that a different kind of learning appears to be taking place. An example of this is the student noticing two equations can be written for the same rule. This is especially a revelation for the student who has to defend his equation, and for the others who take part in the attack. I am suggesting here that there is a difference between knowing that there are such things as equivalent equations and having a "feeling" for them. In all these discussions the word "equivalent" was never mentioned. An estimated one half of the class had a very good feeling for the notion of naming a rule in more than one way, but none of them attempted to verbalize or name this idea. The kind of learning taking place, then, is where the idea is more important than its name or description.

Another outcome relates to the discussion about which answer came first, that is, which is more natural. The students at this point were touching on the idea of how we learn. Which is the easiest way of figuring something out? "Discovering how we learn" is a different undertaking from discovering relationships. The students were trying to discover how the process of finding a relationship came about, that is, which is the easiest and least contrived way of coming up with an answer.

A final question to be asked after all this may be: "This is all very well, but did they learn any mathematics?" This is a legitimate question only after you have placed certain qualifications on it. You would, first of all, have to agree that the first six points I have mentioned all concern mathematics and, in fact, are all of vital concern to mathematics and mathematics learning. So the only questions you are asking when you say "How much mathematics did they learn?" are, for example:

1. If I gave them a graph, could they find the equation?

2. If I gave them an equation, could they find the graph?

3. If I gave them a table of ordered pairs, could they find the graph or the equation?

In other words, you would be asking how much mathematical <u>information</u> did they acquire.

Before answering this question, I would want you to concede the importance of the first six outcomes mentioned. I am sure of one thing: if they did "learn any mathematics", they could not do it with any facility. But, again, the objective of the lesson never intended to develop facility in manipulations or tabulations or rule making. Consequently, one has to be careful when asking: "How much mathematics did the students learn?" In fact, the students were not tested on mathematical information.

Summary

The seven aspects and educational outcomes of the lesson give some idea of the things this lesson was designed to do. If your views on education disagree with the emphasis that I have placed on the various objectives, then you will not use the discovery method as I have suggested. If you think another objective is more important, you will set up the mathematical situation differently. There is no "one" discovery method of teaching, but any discovery method will emphasize most of these objectives. Of one thing I am sure: it is invalid to talk about the discovery method of teaching in the light of traditional objectives of mathematics.

The greatest need in the field of mathematics education today is for a closer look at discovery. I have implied that teachers who are interested in this area must indeed be creative. They must set up situations to which students can react, not situations of special patterns or of using analogies or some very special problems in number theory, but rather of honest mathematical situations, structured highly enough so that the particular students being taught can react to them. Indeed, this would be different for every student. And finally, I have tried to impress upon you my belief that if you disregard discovery, you more than disregard a method, you disregard a whole set of objectives of present-day mathematics education.