CHANGES IN HIGH SCHOOL MATHEMATICS

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I am very pleased to have been invited to speak at this initial conference of the Specialist Council in Mathematics. I wish to congratulate The Alberta Teachers' Association for the initiative and foresight which it has shown in the sponsorship of the various specialist councils. I am confident that before many years their establishment will prove to have been a very significant step in the professional growth of teachers of Alberta.

I am happy to have been asked to speak about changes in the mathematics program of the high school because in this way I can talk in generalities and probabilities. As you know, it is suggested by at least some advocates of modernization that the new mathematics will emphasize generalization and introduce probability and statistical inference. If I happen to repeat several things which havebeen said earlier to somewhat different context, I ask you to bear with me.

It is almost trite to say that there is a growing climate of opinion among teachers and other interested educators across the continent, yea, across the western world, that a rather fundamental revision of the mathematics program in our schools is not only necessary but urgent and inevitable.

This is simply for the reason that if mathematics teaching is to fulfill the aims and functions of public education it must adjust to the growing and developing nature of mathematics and to its changing role in our cultural and economic life. In simpler words, it must keep up to date with the changing, legitimate needs and demands of society. These needs and demands are expressed by such important pressure groups or segments of society as the universities, technical institutes, certain professional

organizations, business, and sometimes by a lay organization to which Mr. Joe Citizen belongs, and occasionally by a royal commission which has been charged with the task of assessing the demands of all these groups.

Just what changes in the mathematics program of the secondary schools should be made to meet legitimate needs is still uncertain in many respects. Many demands are being expressed by the various groups mentioned above and many suggestions offered. In Alberta, the Cameron Commission has made a few pointed recommendations which deserve the careful consideration of curriculum groups as well as of the University, the Department and The Alberta Teachers' Association. Experimentation is taking place in many parts of the continent, including a little in Canada. More is needed and more will be done. We in Alberta, as elsewhere in Canada, are inclined to wait until more affluent and venturesome educational systems carry out experiments and produce text books and other teaching materials. However, I believe that the general outlines of desirable changes in our mathematics education are beginning to emerge and that from these we in Alberta can begin to work at a high-school revision that represents more than tinkering a little here and there, and that will have pretty clear, long-term goals.

The revision which you and I envision is of such proportions that the task which the mathematics curriculum committee faces is not an all-or-nothing job. Rather it will be one of continuing revision as we become ready to take each successive step towards our eventual goal.

Questions which a curriculum committee will be asked are at least three. What specific changes do you propose? How are these changes to be effected? How soon will they be brought about? The answers to these questions will in no small measure depend, upon you, upon us. I refer, foremost, to the curriculum makers in the classroom - the teachers of mathematics - and to those educators who, less directly, guide the course of the curriculum in the schools - the principals, department heads, superintendents, departmental officials, and the staffs of the Faculty of Education and of related faculties. We must always recognize that the education which our boys and girls receive is, in the last analysis,

determined by the teacher in the classroom, not by curriculum committees, nor provincial courses of study, nor university faculties, nor The Alberta Teachers' Association. The latter four agencies are effective only insofar as they influence the teacher in the classroom.

The burden of curriculum change then rests upon all five groups - the Department of Education determines the curriculum, the appropriate curriculum subcommittee writes the guide, the Faculty of Education trains, The Alberta Teachers' Association motivates, and the teacher functioning as an individual implements. Hence the informed and cooperative efforts of all five groups are needed. I should have included a sixth because society represented by the non-educational public has a right to be heard, indeed, to be consulted.

Though a member of the current departmental curriculum subcommittee on secondary school mathematics, I am hardly in a position to tell you what changes are proposed, neither for the immediate future nor in the long-range view. I cannot speak for the committee because it has formulated no collective opinion as yet. If you are expecting from me some definite statement regarding changes which the Department of Education is proposing in the mathematics of the junior or senior high school, then you will be very disappointed for I have nothing to say. I can only express personal opinions about aspects of change and ask you to think along with me.

The Department, speaking through a curriculum guide prepared by a curriculum subcommittee, can express what it desires to be taught in the schools. The architects of the guide, i.e., the subcommittee and the Department which approves it, will hope that the Faculty of Education will so design and direct its training program that its graduates will in very substantial part give effect to the desired curriculum. They will also hope that present teachers, through their own efforts and with the help of The Alberta Teachers' Association, the university and other inservice training leadership personnel will become proficient in implementing the new or revised curriculum. I cannot emphasize too strongly my firm conviction that the rate of revision and the success of any revised programs in mathematics will depend, above all, on the

inservice education of teachers who are now in the classroom. This is not to say that those who direct the program of the school - principals, department heads, supervisors - may not also need in-service education.

Having thus far dealt in generalities, may I now venture on to some probabilities.

I am suggesting that the subcommittee might draft a set of guide-lines or basic assumptions for itself, somewhat as follows:

1. We must recognize that - to quote Robert Rourke - "There is a difference between what can be taught and what should be taught in secondary school mathematics."
2. Departmental guides or directives should move ahead only as fast as the teachers are willing to accept them and are or become qualified to carry them out.
3. For our long-term objectives we should draft in general out-line a tentative, again to quote Rourke, "ideal program as if we had the teachers, while recognizing that for many years we won't have them".
4. We should think in terms of advance along a broken front instead of waiting until the whole province can move ahead together.
5. We must be guided by consideration of the needs and capacities of the majority of students who need continuing mathematics in high school, not just of a select minority. This will not preclude the possible necessity of parallel programs.

It is my conviction that in the past, when confronted with the desirability of rather fundamental curriculum change or reform, we have prepared and authorized course outlines with insufficient thought as to whether or not our teachers were prepared and able to implement them in the classroom, or could in a short time be-come so prepared. It is my humble determination insofar as I have any influence that we do not do this in high school mathe-matics no matter how convinced we may be of the need for sweeping changes. I am equally anxious that we do not mark time when we could be moving ahead.

Therefore I suggest that directives which are to apply to all mathematics teachers should be realistic, with the odds well in

favor of the implementation of these directives. Hence the need for gradual advance despite the complications of possible frequent changes in texts or text materials. Hence the need, too, of knowing all along where it is we are heading. Our overall goals should be defined at the beginning.

Now, my postulate for advance along a broken front is a controversial one and may not win support. Let me try to explain and defend my point. It implies that schools and school systems which are ready to move ahead before the province as a whole is ready, should be permitted, enabled and encouraged to do so.

You see, I believe that there are weaknesses - as well as strengths in a closely controlled provincial system such as ours. It is commonly expected that the schools move along together, that any changes take place in all schools and at the same time. This often means that, either some schools and some teachers move ahead half-cocked or, that changes are long delayed while the process of preparation for the change goes on. If there is a long delay, some teachers and some schools, who are ready and enthusiastic about embarking on some new departures, lose their enthusiasm while waiting for permission to depart from the prevailing program and waiting for guidance or approval in the choice of temporary or experimental text material. We may be somewhat in this position now in Alberta with respect to secondary school mathematics. In the United States, and in England, with their much more flexible educational systems and much greater local autonomy, curriculum change does, I believe, move ahead on a broken front. Some urban school systems in a typical American state may be successfully offering modernized mathematics courses long before the high schools of the state as a whole. And, it may seem strange to us, how American college-bound students, prepared in many different states and in many different school systems, can write common examinations for university entrance. I refer to the examinations of the College Entrance Examinations Board. It indicates that entrants to university need not all have followed the same program in mathematics, for example.

We in Alberta should not close our eyes to the possibility of moving ahead on a broken front. To suggest one more argument for my postulate, just think of the assistance and incentive this would

provide for inservice education in adjoining schools and think of the enlarged opportunities this would provide for try-out and experimentation with course outlines and text materials before their general adoption in the province as a whole.

I would regret it very much if our university were to oppose such an approach on the grounds that it could not readily provide for students coming in with differing backgrounds. This would disappoint me very much in my Alma Mater.

My fifth point was that we should be guided by consideration of the needs of the majority of high school students. This is so obvious as to seem almost trite. Yet I have been asked the question: is not the kind of mathematics being proposed for the academic student too difficult for all but the very bright? Now it would seem to me that any revised programs in Mathematics 10 and 20, for example, should be as palatable as the present courses for about the same proportions of students. I don't believe that we should be expected to provide a strictly university entrance pattern of courses designed for the upper 25 percent of students in our schools. The regular sequence of academic courses must hold broader educational values that will satisfy twice that many students, if not more. But, we would hope that, given competent teachers, the modern favored program would be easier to adapt in degree and depth to the varying capacities and needs of students. I would hope that the committee will not have to consider three parallel programs in mathematics - namely academic, vocational, and general. Perhaps in the revision of the non-academic courses, which I believe to be as necessary as a revision of the academic program, we can come up with one flexible course which can be given a commercial bent in one class, a technical twist in another and a consumer slant in a third. I am not so optimistic as to think that the proposed "new mathematics" is so flexible and versatile in degrees of application that one type of program could serve all students. Some advocates are saying just that. (This doesn't mean identical content; streaming, yes.)

Now at last may I turn to speculate on what specific revisions should be contemplated.

In line with Recommendation 62 of the Cameron Commission and with

thinking in circles of mathematics teachers everywhere, aspects of so-called "new mathematics" should be introduced on a gradual basis.

The emphasis in current trends is upon concepts, upon understanding of mathematics rather than upon skill in manipulation of mathematical rules. The skills are not to be neglected but it is believed that with more meaningful teaching they can be acquired more easily. Now, meaningful mathematics depends more upon teacher preparation than upon curriculum preparation, upon how we teach than upon what we teach. To quote C. B. Read, writing in the March issue of School Science and Mathematics under "New Wine in Old Bottles!":

> "The important thing, if we are to have truly modern mathematics is to revise the way in which we approach the material which we now teach; it makes relatively little difference what topics we include."

The implications of this to the curriculum committee are two: (1) that course outlines be so designed as to encourage, yea, to insist, that attention at all times be given to the development of deeper understanding of relationships and processes, (for example, teaching of the linear function and the solution of linear equations) and (2) that text materials recommended be so arranged as to facilitate the development of meaning, or understanding, for example, of the linear function concept, and the solution of linear equation, before launching upon practice and drill in the mechanical process of solving equations or graphing linear functions. In the past we have all too often tended to short-circuit the development of a concept before introducing drill exercises.

Another change needed within the body of existing material is to cull out any obsolete and unimportant materials. In this the committee will have to be guided by instances where consensus has been reached by the university, or the technical institute, or business, or other important bodies which are vitally interested in the mathematical competence of the school product which comes to them. It appears that logarithms are becoming less and less important, that work in factoring and special products and in the solution of equationscould well be simplified and reduced, and

that less time should be devoted to the study of theorems and constructions in geometry and to social applications in arithmetic.

Following this, the relative emphases to be given to the topics retained need to be re-examined in the light of mathematics today. The shift in point of view to which I have hinted implies that we put more emphasis upon basic concepts and the structure of mathematics and less upon operational facility. This in turn suggests that in arithmetic and algebra we must develop a fuller understanding of our number system and its underlying laws, the laws of algebra. Hence we should teach something of number systems to other bases, something of modular systems like our clock numbers, bring out the characteristics of our system of natural numbers and how these are extended to the rationals, the real numbers and eventually to the complex numbers. Whether or not consideration of such generalized concepts as groups and fields has a necessary place in academic mathematics of the high school remains to be determined. However, an understanding of the basic laws of algebra would appear to be as essential to an intelligent grasp of algebraic processes as an appreciation of the foundations of geometry is to an intelligent study of geometry. For example, all factoring can be explained on the basis of the distributive law.

Let me illustrate by way of example what implications this point of view has to the teaching of indices, a section in our Mathematics 20 course. If you teach Mathematics 20, may I suggest that you ask yourself these questions: Do my pupils know that the figure "2" written with the index "5" means 2.2.2.2.2. by reason of a definition which we accept? Do they know <u>why</u> $N^0 = 1$? Can they explain why "$2^{\frac{1}{2}}$" is defined as square root of "2"? Have they come to appreciate that the definitions for fractional, negative and zero indices have been arbitrarily made in such a way that the laws derived for positive integral indices will still apply? If so, yours is meaningful teaching; you have the point of view which modern mathematics demands, which good teaching of mathematics has always favored. The section on indices provides an excellent opportunity for bringing out the arbitrary nature of definitions and their role in proof and illustrates the truth of Read's statement which I quoted earlier: "The important thing, if we are to have truly modern mathematics, is to revise the way in which we approach the material that we teach."

I have thus far approached the problem of changes in high school mathematics from the angle of change in point of view towards the mathematics we now teach. I have suggested that we need to teach less of some of the old and more of some of the old. Our approach will require the introduction of some new concepts, for example: the postulates of algebra and elementary ideas of sets, relations and ordered pairs; also the extension of concepts now neglected, such as: inequalities, number as an abstraction with many names, variable as a place holder or as a symbol for a set of numbers, and function as a set of ordered pairs obeying certain conditions.

There is of course another angle to the change in high school mathematics, namely change in content through the introduction of new mathematics. A strong case can perhaps be made, for example, for including some elementary statistics and probability in the fund of useful mathematical knowledge of today's citizens.

To summarize, I have not told you what changes in any courses in high school mathematics are envisioned by the senior curriculum subcommittee because it has not yet begun to work nor is it even fully constituted. For example, I would like to see a representative of the Mathematics Council on the subcommittee. Two meetings of a committee representing both junior and senior high schools were held last year during which time possible objectives for curriculum change were discussed without reaching any definite conclusions. It was then decided to form two subcommittees but with some joint membership in order that close liaison could be maintained. The Junior High School Subcommittee with Bob Plaxton of Viscount Bennett Junior High School, Calgary as chairman, met in June at which time it was decided to attempt some experimentation in Grade VII next fall or winter with the experimental materials prepared by Gage and Company. Some other materials may also be tried out in a small way at junior high school level.

I anticipate that the senior subcommittee will get down to some earnest study this fall. I would hope that some experimental units can be found which the subcommittee would be prepared to see in Grade X or XI mathematics in lieu of less work in some existing chapters. Also, I have some personal ideas concerning our geometry.

May I conclude by enumerating what obstacles must be overcome

before a new program in mathematics for Grades VII to XII can be introduced and implemented. This specialist council can and will, I trust, provide much help in meeting these problems.

Problem No. 1 - Agreement must be reached as to content - old materials to be discarded, or to be given new emphasis; new materials to be introduced, and at what grade levels, and for whom.

Problem No. 2 - Suitable text materials must be secured. It hardly appears likely that Alberta can write its own. We could however adapt materials prepared elsewhere. It would seem to me that wherever schools are ready to do so, they should be encouraged to try out modifications of existing courses or to experiment with such new units as may be considered worthy, or to follow an alternate textbook upon approval. Some departmental control could be exercised.

Problem No. 3 - General teacher acceptance of new ideas and new programs in mathematics must be secured - enthusiastic acceptance insofar as possible. This may or may not be easy.

Problem No. 4 - Teachers must be helped to become prepared to implement the changes as they are introduced. It is unlikely that guidebooks will be sufficient. Many teachers must somehow obtain a richer background in pertinent mathematics. This will not be easy to assure but it is crucial to the success of what we envision. Here lies an important role of The Alberta Teachers' Association and of the Mathematics Council.

In what order should we seek to meet these problems? Assault on all fronts!

Let's make the revision in secondary school mathematics a real team effort. Let's move ahead as we are ready, not in one big jump with the imminent danger of falling into the middle of a whirlpool. Let's accept that our greatest need is to revise the manner in which we approach the material we now teach. For many teachers - outside this meeting - this implies acquiring a deeper and fuller appreciation and knowledge of mathematical concepts and understandings which underlie the mathematics we teach. Then the task of provincial curriculum revision will become a practical one.