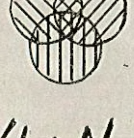
Mathematics Council

The Alberta Teachers' Association

Inaugural Conference

August 16-19, 1961



Vol. 1, No. 1

PREFACE

The subject matter of this publication consists of some of the addresses and summaries of other addresses that were delivered at the inaugural conference of the Mathematics Council of The Alberta Teachers' Association, held August 16 - 19, 1961 at the University of Alberta in Edmonton and attended by a representation of mathematics teachers at all grade levels from various parts of the province.

An effort was made to include the addresses in their entirety wherever possible. The editor wishes to express his gratitude to those speakers who did not have their addresses in written form, but upon request took the time and trouble to submit summaries of the ideas they presented at the conference so that these could appear here.

The intention of this volume is to circulate more widely the ideas discussed at the conference. It is the hope of the executive committee that a volume such as this will become a yearly project. Newsletters published four times a year - January, April, June, and October - will keep the members of the council abreast of the latest developments.

- John Cherniwchan
Editor

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MATHEMATICS IN TRANSITION

Julius H. Hlavaty*

*Dr. Hlavaty is chairman of the Department of Mathematics, DeWitt Clinton High School, New York City. The list of his professional activities in the field of curriculum reform is impressive. One activity should be mentioned here. He is director of the Commission on Mathematics Program, College Entrance Examinations Board, which just completed its work recently.

I - Fifty Years of Curriculum Reform in Mathematics

The golden age of mathematics, the nineteenth century, saw the making of more mathematics than all previous history. In the first half of the twentieth century, again more mathematics was produced than in all previous history, including the golden age. That means that the tons (literally) of clay tablets of the Babylonians, the archives of the Egyptians, the august work of the Greeks, the splendid achievements of the Hindus and Arabs, and the revolutionary advances of the age of Newton have been more than matched by relatively recent forward steps in mathematics.

In all the great ages, the growth of mathematics was accompanied by - causing and being caused by - expanding uses and applications. A demand for people to maintain the development of mathematics and for those who would utilize it also confronted these ages with the problems of mathematical education and specifically the problems of elementary and secondary education.

Editor's Note - Dr. Hlavaty does not ordinarily write out his addresses or make digests of them. However, he has submitted to us a copy of an article he recently wrote which contains essentially the same ideas as those developed at our conference and has given us freedom to use any portion of it. What follows is from this article.

Our century and our country have in particular wrestled with the problem of mathematics in secondary education. A brief review of this story is in order, if only to put into proper perspective the current feverish activity in mathematical education.

A fruitful collaboration between mathematicians and school people culminated in 1923, in the publication by the Mathematical Association of America of the report, The Reorganization of Mathematics in Secondary Education.

Hardly was the ink dry on this report than a new phenomenon interfered radically with the nascent implementation of its recommendations. This was the explosive expansion of the secondary schools. Though the report became (and even is today) the major guide in curriculum construction and textbook writing, it soon became evident that its recommendations were neither desirable nor feasible for large sections of the new secondary school population.

During the 1930's an increasing awareness of the discrepancy between the various needs and drives of the high school pupil and the largely academically oriented point of view of the report led teachers to the reconsideration of the whole problem of the high school program in mathematics. It is significant and a mark of the times that the mathematicians and the educators found it impossible to formulate a universally acceptable program. In fact, two basically different reports on the problem emerged: one, Mathematics in General Education (1) and the other, The Place of Mathematics in Secondary Education, which was the Report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics (2).

World War II prevented any real testing of the comparative values of the two sets of recommendations and the post-war period placed the whole problem of mathematical education in such a new dimension that the whole problem called once more for a fundamental re-examination of all the premises. There was increasing dissatisfaction with the many attempts at partial and local solutions. These various attempts found a focus in the work of the Commission on Mathematics. The genesis and the operations of the work of the Commission on Mathematics were reported by Albert E. Meder, Jr., then executive director of the Commission, in the winter of 1958.(3)

The Commission, consisting of university mathematicians, leaders in the training of teachers of mathematics, and secondary school teachers, was organized in 1955. This group spent four busy years formulating tentative proposals, discussing them, tearing them apart, rejecting some, reformulating others, and elaborating still others. Not only did each proposal have to survive the gamut of the criticisms and reactions of the individual members of the Commission, but each was made the subject of careful and critical review by the profession at large. Representatives of the Commission presented the developing program of the Commission to innumerable local, regional, state, and national conferences of teachers of mathematics, mathematicians, and educators. The reactions of the profession at large had a marked influence on the final report.

References

(1) Progressive Education Association on Commission on the Secondary Curriculum, Committee on the Function of Mathematics in General Education, New York: D. Appleton Century, 1940.

(2) New York: Bureau of Publications, Teachers College, Columbia University, 1940.

(3) "Mathematics For Today", College Board Review, No. 34, pp.7-10.

II - What Mathematics is of Most Worth?

The Commission succeeded in formulating and proposing a nine-point program for college-capable students:

1. Strong preparation, both in concepts and in skills for college mathematics at the level of calculus and analytic geometry.
2. Understanding of the nature and role of deductive reasoning - in algebra, as well as in geometry.

3. Appreciation of mathematical structure ("patterns") - for example, properties of natural, rational, real, and complex numbers.
4. Judicious use of unifying ideas - sets, variables, functions, and relations.
5. Treatment of inequalities along with equations.
6. Incorporation with plane geometry of some coordinate geometry, and essentials of solid geometry and space perception.
7. Introduction in Grade XI of fundamental trigonometry - centered on coordinates, vectors, and complex numbers.
8. Emphasis in Grade XII on elementary functions (polynomial, exponential, circular).
9. Recommendation of additional, alternative units for Grade XII: <u>either</u> introductory probability with statistical applications, <u>or</u> an introduction to modern algebra.

This nine-point program was elaborated in the first volume of the Report of the Commission on Mathematics, <u>Program for College Preparatory Mathematics</u>, which was published in the spring of 1959.

To give a concrete illustration of the point of view that should guide the creation of a new curriculum in mathematics, and to provide teachers with some of the new subject-matter material which is proposed in the program, the Commission found it necessary during the four years of its work to issue a number of small publications in the form of pamphlets. These materials were improved, others of a similar nature were written, and the whole was incorporated in the second volume of the Report, the <u>Appendices</u>.

<u>III - Implementing Curriculum Proposals</u>

An increasing segment of persons responding to the Report is made up of tough-minded administrators who are working on the implementation of a new program in mathematics. Local school boards and state departments of education have strengthened, or in some cases initiated, programs of supervision in mathematics. They have been brought face to face with the concrete problems of teacher-training and retraining in service: organizing and subsidizing inservice programs and finding people to conduct these training courses. Often, they ask the Commission for speakers and instructors. State departments of education are reviewing their certification requirements and they also ask for advice. It is clear that the fifth chapter of the Commission's Report ("Implementation: the vital role of teacher education") is becoming increasingly pertinent for all concerned, from individual teachers to state-wide systems. The schools of education are undertaking serious reviews of their programs for future teachers and for teachers now in service. The growing program of the National Science Foundation is a part of this major activity in teacher education, and recently, the American Association for the Advancement of Science decided to use Chapter 5 of the Report in its regional conferences on the Teacher Preparation-Certification Study.

In the beginning stages of this experiment in mathematical education it was easy to distinguish the role of the Commission from that of the quite different but contemporaneous Advanced Placement Program and the Illinois Plan. The Commission assumed an important position at the yearly summer and inservice training institutes through the use of its pamphlets and particularly of its Introductory Probability and Statistical Inference, an experimental text used and reviewed very extensively, perhaps because they were the only materials available.

Today the role of the Commission is still quite distinctive and an important source of the swelling stream fed also by the massive work of the School Mathematics Study Group, the growing influence of the Illinois Plan, and the innumerable attempts sponsored by local, state, and federal bodies. The Report gave explicit and concrete form to what has emerged as a consensus of the many seemingly different approaches to the problem of a new curriculum in mathematics. This conclusion was underlined by the publication of The Secondary Mathematics Curriculum, the Report of the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics, in May 1959. While the National Council's recommendations do not agree in every detail with those of the Commission, they do agree in major outline and emphasis.

The School Mathematics Study Group (Yale Project) accepted in essence the recommendations of the Commission. In its tremendously productive writing project in the summer of 1959, the SMSG produced sample textbooks for Grades VII through XII. In only one instance - geometry - did it significantly depart from the recommendations of the Commission. In this case it produced an updated and relatively rigorous textbook in Euclidean geometry, which however includes a treatment of solid geometry and a unit on coordinate geometry. In all other areas it strove deliberately to implement Commission recommendations.

CURRICULUM CHANGES IN ELEMENTARY MATHEMATICS

Neil M. Purvis*

*Mr. Purvis is associate director of curriculum, Department of Education. What follows here is the summary he prepared of his address to our conference.

I appreciate the opportunity of reporting to the Mathematics Council of The Alberta Teachers' Association recent developments in elementary mathematics curriculum in this province. While it is probably correct to assume that the majority of representatives here are directly concerned with secondary school mathematics I offer no apology for discussing with you recent activities and future plans relative to mathematics programs in Grades I-VI. Any changes anticipated in mathematics in the elementary grades should be known and understood by teachers of the junior and senior high school grades if we are to develop a well articulated and continuous program for all children in Alberta.

It may be of interest to this specialist council to know why a revision of the elementary arithmetic program was undertaken within the last four years. During the past ten years there has been a steadily growing interest in mathematics on this continent and elsewhere. While it is true that the majority of mathematics experimental groups began their work at the college and university level, many of them subsequently showed an increasing interest in secondary school mathematics and still later an interest in elementary school arithmetic. I am sure the work of the School Mathematics Study Group, as well as the work of such people as Dr. M. Bieberman of Illinois, Dr. F. Weaver of Boston, and many others is well known to most of you. In addition to this influence for change in elementary arithmetic curriculum coming from outside the province, there were a number of factors of a purely local nature which also contributed to the revisions which we are considering today. The two series of arithmetics, Making Sure of Arithmetic in Grades I-II, and Study Arithmetic in Grades III-VI, which have been in use in Alberta schools for some time seem to have created problems of articulation between

Grades II and III. There has also been dissatisfaction with the success children achieved in their ability to solve problems and to work with the number system. The present texts in Grades III-VI were authorized in 1947, which makes them out of date with respect to the data used in problems. Although the revision had actually been undertaken previously, the report of the Royal Commission on Education in Alberta, which was presented to the Legislature late in 1959, recommended a special study of modern mathematics and its gradual introduction into the school system.

After a careful consideration of the developments in experimental mathematics programs elsewhere and the conditions which existed in this province, the Elementary Curriculum Committee appointed an Elementary Arithmetic Subcommittee late in 1957 to study the present program, to become acquainted with the professional literature in the field and to evaluate textbooks which were available. The membership of this subcommittee included three classroom teachers, two supervisors, one Faculty of Education representative, one school superintendent and a member of the Department of Education.

The subcommittee spent considerable time becoming familiar with recent developments in elementary arithmetic. Because arithmetic has been traditionally a textbook course in this province, the committee saw the selection of new textbook materials as one of its chief responsibilities. Three criteria were established which textbooks had to meet before they would be considered by the committee for careful evaluation. The three criteria were:

1. the series had to have been published since 1956,
2. it had to be complete from Grades I-VI, and
3. it had to have teacher guidebooks.

The publication date of 1956 was chosen to insure that the new authorizations would not be more than five or six years old before they were introduced into the classrooms. The committee insisted on a complete series from Grades I-VI in an attempt to overcome the problem of articulation which had been evident in the present program. Teacher guidebooks were required because of the beneficial effect which similar publications had exerted on the reading program in elementary school. The following six series of arithmetic texts met the criteria and were selected by the subcommittee for careful evaluation and teacher reaction:

Exploring Arithmetic--Osborne et al
Understanding Arithmetic--McSwain et al
Arithmetic in my World--Stokes et al
New Winston's Arithmetic--Deverell et al
Arithmetic we Need--Buswell et al
Seeing Through Arithmetic--Hartung et al

Evaluation of the six series took the form of a three-pronged program. Fourteen study groups, each consisting of 6-18 teachers, evaluated one or more series through all the grades in order to obtain a picture of the total program of teaching the number system, fundamentals, problem-solving and measurement. About 70 other teachers experimented with one series in their own classrooms. Books for pupils and teachers were supplied in sufficient quantities to equip both the study groups and the experimental classrooms for the school year 1959-60. An open-ended type of questionnaire-study guide, developed by the subcommittee, was supplied to study group teachers and experimental classroom teachers. At the end of the year these questionnaires were returned to the subcommittee for compilation and analysis. In addition to the evaluations by study groups and experimental classrooms, a testing program was administered. All pupils in the experimental program were tested in September and again in May of the 1959-60 school year, as were an equal number of pupils still using the Study Arithmetic Series. While the subcommittee fully realized the many weaknesses which exist in such a testing program, it was felt that some useful information could be obtained in this way. In view of the lack of familiarity of experimental teachers with the new materials and the presence of many uncontrolled factors, the committee agreed that any information obtained could not be considered as research evidence but rather the findings of action research which would demand cautious and reserved interpretation.

Following a careful study of the reactions of teachers in both study groups and experimental classrooms and an evaluation of test results, the subcommittee eliminated four series of textbooks from further consideration and recommended that two series:

Seeing Through Arithmetic, W. J. Gage, Ltd., and
Arithmetic We Need, Ginn and Company,

be authorized for use in Grades I-VI in Alberta schools. In June 1961, the Minister of Education gave his approval to this recommendation for dual authorization. The _Seeing Through Arithmetic_ Series is the newest and most up-to-date set of elementary arithmetic texts now available. The series employs a somewhat new and different approach to the teaching of the number system, the fundamental processes, problem-solving and measurement. The _Arithmetic We Need_ Series, while more traditional in its approach, places a greater emphasis on understanding of the number system than does the present series, _Study Arithmetic_. The main difference in content covered in the two series results from the ratio approach to percentage in the _Seeing Through Arithmetic_ Series which means that the study of this topic is pretty well completed in Grade VI.

The new approach to teaching arithmetic in elementary school, which is sometimes referred to as modern mathematics, could possibly be said to have the following characteristics:

1. a shift of emphasis from mechanics to understanding,
2. an emphasis on helping pupils see mathematical patterns, relationships and principles,
3. an emphasis on a systematic approach to the solution of problems employing the use of equations and ratio,
4. a shift in emphasis from the stimulus-response psychology of learning to the Gestalt,
5. an increased emphasis on preciseness of mathematics language and mathematical processes.

The anticipated changes in elementary arithmetic in 1962 are not sufficiently different to cause any great concern. They do, however, involve certain changes in both method and content with which teachers will need to become familiar. In view of these changes at the elementary level in the immediate future and the probability of changes at the secondary level in the not too distant future, this Mathematics Council has come into existence at a most favorable time. Here is an opportunity for your organization to render an extremely valuable professional service to mathematics teachers in this province as well as an important educational service to the children.

ADEQUATE PREPARATION IN MATHEMATICS FOR STUDENTS ENTERING UNIVERSITY

R. C. Jacka*

*Mr. Jacka is associate professor of mathematics at the University of Alberta. This is a summary of his address at the conference.

Students should have some knowledge of the development of the number system beginning with the properties of the natural numbers and a logical construction of the integers, rational numbers, real numbers, and complex numbers from the foundation of natural numbers. The laws of algebra such as commutative law, associative law and distributive law should be known by name.

In trigonometry, less emphasis should be placed on numerical solution of triangles and more emphasis on graphs of the trigonometric functions, identities, and solution of trigonometric equations.

I would strongly recommend the addition of a course in Analytical Geometry and Elementary Calculus to the high school program even if it were only an option given in the large schools. For students entering the Faculty of Engineering or taking any course which requires physics in the first year, some prior knowledge of the calculus is very helpful.

TEACHER PREPARATION IN MATHEMATICS

W. F. Coulson*

*Mr. Coulson is a member of the Faculty of Education of the University of Alberta and a member of the subcommittee on secondary school mathematics. What follows is a summary of his address at the conference.

What criteria should we use to develop the program of mathematical preparation to be followed by the prospective teachers of mathematics in the schools of Alberta? Does the present program meet or attempt to meet these criteria? How should the present program be modified to meet these criteria?

We will assume that a single set of criteria can be listed which would apply to all teachers of mathematics, elementary and secondary, and proceed to discuss these criteria. First, our teachers should know secondary mathematics. It is necessary to know more about your subject than the little area being taught. The interrelationships must be known. Knowledge of theory is as essential as knowledge of the fundamental operations.

The second criterion is that requiring a breadth of mathematical preparation. Daily classwork will not be simply textbook learning or following the recipe. Stimulation and inspiration will be a part of each teacher's teaching.

A third criterion that might be mentioned is that of the proper attitude of the teacher toward the subject area of mathematics. Mathematics of today is a living, growing subject. It is not stagnant. The attitude of the student will be influenced by that of the teacher.

Another aspect of the attitude of the mathematics teacher is that of self-improvement, self-study, or the simple keeping abreast of the everyday happenings in his field. Each teacher should be reasonably familiar with the current thinking in mathematics and mathematics education.

Experience in dealing with the interrelation of the various parts of the field of mathematics might be listed as the fourth criterion Are the areas into which the study of mathematics is traditionally divided - arithmetic, geometry, algebra, trigonometry - related, or are they again as tradition seems to imply, unrelated? What are some of the unifying concepts which could be used to interrelate the various areas of mathematics? Study and applications of these topics are essential.

The fifth, and for this discussion, the last of the criteria which might be used to design a program of mathematics preparation for prospective teachers, is the necessity of knowledge of the relationship of mathematics to other disciplines and to various fields of work. At one time mathematics was related only to the physical sciences and technology. Not so in this modern, fast moving world. A teacher must be aware of some of the uses to which the subject may be put.

Now considering the four basic certificates, does the holder of one of these certificates meet each of the five above mentioned criteria? The holders of the Standard E and Junior E certificates receive practically no formal instruction in mathematics at the university level. For the teacher in the elementary route, there is no provision for completion of a major in mathematics on the bachelor's degree. This might imply that mathematics is a relatively unimportant part of the elementary school curriculum. Experts in mathematics education at the elementary school level must come from two sources: (1) people who have been educated as secondary school teachers and have entered the elementary school field, and (2) individuals who studied mathematics outside any formal pattern. Do we want to leave the preparation of experts in elementary school mathematics to such chance?

For individuals following the secondary education route as majors in mathematics, the program does satisfy these criteria in a general way. In order that these people be even better prepared for a position in the teaching of mathematics, some revisions in course content or course sequence might be indicated. At any rate, a study of what is best for mathematics education in the Province of Alberta might be considered.

An attempt has been made to list the criteria of the program of preparation of prospective mathematics teachers. As can be seen, one group of teachers has a program which very nearly satisfies all of these criteria while another group has a program which does not meet any of the criteria. As a result of looking at the preparation of prospective mathematics teachers, some recommendations may be forthcoming from the ATA Mathematics Council, which will have an effect on the preparation of these individuals to teach mathematics.

CHANGES IN HIGH SCHOOL MATHEMATICS

L. W. Kunelius*

*Mr. Kunelius is high school inspector for the Calgary area. He is also chairman of the subcommittee on secondary school mathematics. His address to the conference is reproduced here in its entirety.

I am very pleased to have been invited to speak at this initial conference of the Specialist Council in Mathematics. I wish to congratulate The Alberta Teachers' Association for the initiative and foresight which it has shown in the sponsorship of the various specialist councils. I am confident that before many years their establishment will prove to have been a very significant step in the professional growth of teachers of Alberta.

I am happy to have been asked to speak about changes in the mathematics program of the high school because in this way I can talk in generalities and probabilities. As you know, it is suggested by at least some advocates of modernization that the new mathematics will emphasize generalization and introduce probability and statistical inference. If I happen to repeat several things which havebeen said earlier to somewhat different context, I ask you to bear with me.

It is almost trite to say that there is a growing climate of opinion among teachers and other interested educators across the continent, yea, across the western world, that a rather fundamental revision of the mathematics program in our schools is not only necessary but urgent and inevitable.

This is simply for the reason that if mathematics teaching is to fulfill the aims and functions of public education it must adjust to the growing and developing nature of mathematics and to its changing role in our cultural and economic life. In simpler words, it must keep up to date with the changing, legitimate needs and demands of society. These needs and demands are expressed by such important pressure groups or segments of society as the universities, technical institutes, certain professional organizations, business, and sometimes by a lay organization to which Mr. Joe Citizen belongs, and occasionally by a royal commission which has been charged with the task of assessing the demands of all these groups.

Just what changes in the mathematics program of the secondary schools should be made to meet legitimate needs is still uncertain in many respects. Many demands are being expressed by the various groups mentioned above and many suggestions offered. In Alberta, the Cameron Commission has made a few pointed recommendations which deserve the careful consideration of curriculum groups as well as of the University, the Department and The Alberta Teachers' Association. Experimentation is taking place in many parts of the continent, including a little in Canada. More is needed and more will be done. We in Alberta, as elsewhere in Canada, are inclined to wait until more affluent and venturesome educational systems carry out experiments and produce text books and other teaching materials. However, I believe that the general outlines of desirable changes in our mathematics education are beginning to emerge and that from these we in Alberta can begin to work at a high-school revision that represents more than tinkering a little here and there, and that will have pretty clear, long-term goals.

The revision which you and I envision is of such proportions that the task which the mathematics curriculum committee faces is not an all-or-nothing job. Rather it will be one of continuing revision as we become ready to take each successive step towards our eventual goal.

Questions which a curriculum committee will be asked are at least three. What specific changes do you propose? How are these changes to be effected? How soon will they be brought about? The answers to these questions will in no small measure depend, upon you, upon us. I refer, foremost, to the curriculum makers in the classroom - the teachers of mathematics - and to those educators who, less directly, guide the course of the curriculum in the schools - the principals, department heads, superintendents, departmental officials, and the staffs of the Faculty of Education and of related faculties. We must always recognize that the education which our boys and girls receive is, in the last analysis, determined by the teacher in the classroom, not by curriculum committees, nor provincial courses of study, nor university faculties, nor The Alberta Teachers' Association. The latter four agencies are effective only insofar as they influence the teacher in the classroom.

The burden of curriculum change then rests upon all five groups - the Department of Education determines the curriculum, the appropriate curriculum subcommittee writes the guide, the Faculty of Education trains, The Alberta Teachers' Association motivates, and the teacher functioning as an individual implements. Hence the informed and cooperative efforts of all five groups are needed. I should have included a sixth because society represented by the non-educational public has a right to be heard, indeed, to be consulted.

Though a member of the current departmental curriculum subcommittee on secondary school mathematics, I am hardly in a position to tell you what changes are proposed, neither for the immediate future nor in the long-range view. I cannot speak for the committee because it has formulated no collective opinion as yet. If you are expecting from me some definite statement regarding changes which the Department of Education is proposing in the mathematics of the junior or senior high school, then you will be very disappointed for I have nothing to say. I can only express personal opinions about aspects of change and ask you to think along with me.

The Department, speaking through a curriculum guide prepared by a curriculum subcommittee, can express what it desires to be taught in the schools. The architects of the guide, i.e., the subcommittee and the Department which approves it, will hope that the Faculty of Education will so design and direct its training program that its graduates will in very substantial part give effect to the desired curriculum. They will also hope that present teachers, through their own efforts and with the help of The Alberta Teachers' Association, the university and other inservice training leadership personnel will become proficient in implementing the new or revised curriculum. I cannot emphasize too strongly my firm conviction that the <u>rate</u> of revision and the <u>success</u> of any revised programs in mathematics will depend, above all, on the inservice education of teachers who are now in the classroom. This is not to say that those who direct the program of the school - principals, department heads, supervisors - may not also need inservice education.

Having thus far dealt in generalities, may I now venture on to some probabilities.

I am suggesting that the subcommittee might draft a set of guidelines or basic assumptions for itself, somewhat as follows:

1. We must recognize that - to quote Robert Rourke - "There is a difference between what can be taught and what should be taught in secondary school mathematics."
2. Departmental guides or directives should move ahead only as fast as the teachers are willing to accept them and are or become qualified to carry them out.
3. For our long-term objectives we should draft in general outline a tentative, again to quote Rourke, "ideal program as if we had the teachers, while recognizing that for many years we won't have them".
4. We should think in terms of advance along a broken front instead of waiting until the whole province can move ahead together.
5. We must be guided by consideration of the needs and capacities of the majority of students who need continuing mathematics in high school, not just of a select minority. This will not preclude the possible necessity of parallel programs.

It is my conviction that in the past, when confronted with the desirability of rather fundamental curriculum change or reform, we have prepared and authorized course outlines with insufficient thought as to whether or not our teachers were prepared and able to implement them in the classroom, or could in a short time become so prepared. It is my humble determination insofar as I have any influence that we do not do this in high school mathematics no matter how convinced we may be of the need for sweeping changes. I am equally anxious that we do not mark time when we could be moving ahead.

Therefore I suggest that directives which are to apply to all mathematics teachers should be realistic, with the odds well in favor of the implementation of these directives. Hence the need for gradual advance despite the complications of possible frequent changes in texts or text materials. Hence the need, too, of knowing all along where it is we are heading. Our overall goals should be defined at the beginning.

Now, my postulate for advance along a broken front is a controversial one and may not win support. Let me try to explain and defend my point.. It implies that schools and school systems which are ready to move ahead before the province as a whole is ready, should be permitted, enabled and encouraged to do so.

You see, I believe that there are weaknesses - as well as strengths in a closely controlled provincial system such as ours. It is commonly expected that the schools move along together, that any changes take place in all schools and at the same time. This often means that, either some schools and some teachers move ahead half-cocked or, that changes are long delayed while the process of preparation for the change goes on. If there is a long delay, some teachers and some schools, who are ready and enthusiastic about embarking on some new departures, lose their enthusiasm while waiting for permission to depart from the prevailing program and waiting for guidance or approval in the choice of temporary or experimental text material. We may be somewhat in this position now in Alberta with respect to secondary school mathematics. In the United States, and in England, with their much more flexible educational systems and much greater local autonomy, curriculum change does, I believe, move ahead on a broken front. Some urban school systems in a typical American state may be successfully offering modernized mathematics courses long before the high schools of the state as a whole. And, it may seem strange to us, how American college-bound students, prepared in many different states and in many different school systems, can write common examinations for university entrance. I refer to the examinations of the College Entrance Examinations Board. It indicates that entrants to university need not all have followed the same program in mathematics, for example.

We in Alberta should not close our eyes to the possibility of moving ahead on a broken front. To suggest one more argument for my postulate, just think of the assistance and incentive this would provide for inservice education in adjoining schools and think of the enlarged opportunities this would provide for try-out and experimentation with course outlines and text materials before their general adoption in the province as a whole.

I would regret it very much if our university were to oppose such an approach on the grounds that it could not readily provide for students coming in with differing backgrounds. This would disappoint me very much in my Alma Mater.

My fifth point was that we should be guided by consideration of the needs of the majority of high school students. This is so obvious as to seem almost trite. Yet I have been asked the question: is not the kind of mathematics being proposed for the academic student too difficult for all but the very bright? Now it would seem to me that any revised programs in Mathematics 10 and 20, for example, should be as palatable as the present courses for about the same proportions of students. I don't believe that we should be expected to provide a strictly university entrance pattern of courses designed for the upper 25 percent of students in our schools. The regular sequence of academic courses must hold broader educational values that will satisfy twice that many students, if not more. But, we would hope that, given competent teachers, the modern favored program would be easier to adapt in <u>degree</u> and depth to the varying capacities and needs of students. I would hope that the committee will not have to consider three parallel programs in mathematics - namely academic, vocational, and general. Perhaps in the revision of the non-academic courses, which I believe to be as necessary as a revision of the academic program, we can come up with <u>one</u> flexible course which can be given a commercial bent in one class, a technical twist in another and a consumer slant in a third. I am not so optimistic as to think that the proposed "new mathematics" is so flexible and versatile in degrees of application that one type of program could serve all students. Some advocates are saying just that. (This doesn't mean identical content; streaming, yes.)

Now at last may I turn to speculate on what specific revisions should be contemplated.

In line with Recommendation 62 of the Cameron Commission and with thinking in circles of mathematics teachers everywhere, aspects of so-called "new mathematics" should be introduced on a gradual basis.

The emphasis in current trends is upon concepts, upon understanding of mathematics rather than upon skill in manipulation of mathematical rules. The skills are not to be neglected but it is believed that with more meaningful teaching they can be acquired more easily. Now, meaningful mathematics depends more upon teacher preparation than upon curriculum preparation, upon how we teach than upon what we teach. To quote C. B. Read, writing in the March issue of School Science and Mathematics under "New Wine in Old Bottles!":

> "The important thing, if we are to have truly modern mathematics is to revise the way in which we approach the material which we now teach; it makes relatively little difference what topics we include."

The implications of this to the curriculum committee are two: (1) that course outlines be so designed as to encourage, yea, to insist, that attention at all times be given to the development of deeper understanding of relationships and processes, (for example, teaching of the linear function and the solution of linear equations) and (2) that text materials recommended be so arranged as to facilitate the development of meaning, or understanding, for example, of the linear function concept, and the solution of linear equation, before launching upon practice and drill in the mechanical process of solving equations or graphing linear functions. In the past we have all too often tended to short-circuit the development of a concept before introducing drill exercises.

Another change needed within the body of existing material is to cull out any obsolete and unimportant materials. In this the committee will have to be guided by instances where consensus has been reached by the university, or the technical institute, or business, or other important bodies which are vitally interested in the mathematical competence of the school product which comes to them. It appears that logarithms are becoming less and less important, that work in factoring and special products and in the solution of equationscould well be simplified and reduced, and that less time should be devoted to the study of theorems and constructions in geometry and to social applications in arithmetic.

Following this, the relative emphases to be given to the topics retained need to be re-examined in the light of mathematics today. The shift in point of view to which I have hinted implies that we put more emphasis upon basic concepts and the structure of mathematics and less upon operational facility. This in turn suggests that in arithmetic and algebra we must develop a fuller understanding of our number system and its underlying laws, the laws of algebra. Hence we should teach something of number systems to other bases, something of modular systems like our clock numbers, bring out the characteristics of our system of natural numbers and how these are extended to the rationals, the real numbers and eventually to the complex numbers. Whether or not consideration of such generalized concepts as groups and fields has a necessary place in academic mathematics of the high school remains to be determined. However, an understanding of the basic laws of algebra would appear to be as essential to an intelligent grasp of algebraic processes as an appreciation of the foundations of geometry is to an intelligent study of geometry. For example, all factoring can be explained on the basis of the distributive law.

Let me illustrate by way of example what implications this point of view has to the teaching of indices, a section in our Mathematics 20 course. If you teach Mathematics 20, may I suggest that you ask yourself these questions: Do my pupils know that the figure "2" written with the index "5" means 2.2.2.2.2. by reason of a definition which we accept? Do they know <u>why</u> $N^{o} = 1$? Can they explain why "$2^{\frac{1}{2}}$" is defined as square root of "2"? Have they come to appreciate that the definitions for fractional, negative and zero indices have been arbitrarily made in such a way that the laws derived for positive integral indices will still apply? If so, yours is meaningful teaching; you have the point of view which modern mathematics demands, which good teaching of mathematics has always favored. The section on indices provides an excellent opportunity for bringing out the arbitrary nature of definitions and their role in proof and illustrates the truth of Read's statement which I quoted earlier: "The important thing, if we are to have truly modern mathematics, is to revise the way in which we approach the material that we teach."

I have thus far approached the problem of changes in high school mathematics from the angle of change in point of view towards the mathematics we now teach. I have suggested that we need to teach less of some of the old and more of some of the old. Our approach will require the introduction of some new concepts, for example: the postulates of algebra and elementary ideas of sets, relations and ordered pairs; also the extension of concepts now neglected, such as: inequalities, number as an abstraction with many names, variable as a place holder or as a symbol for a set of numbers, and function as a set of ordered pairs obeying certain conditions.

There is of course another angle to the change in high school mathematics, namely change in content through the introduction of new mathematics. A strong case can perhaps be made, for example, for including some elementary statistics and probability in the fund of useful mathematical knowledge of today's citizens.

To summarize, I have not told you what changes in any courses in high school mathematics are envisioned by the senior curriculum subcommittee because it has not yet begun to work nor is it even fully constituted. For example, I would like to see a representative of the Mathematics Council on the subcommittee. Two meetings of a committee representing both junior and senior high schools were held last year during which time possible objectives for curriculum change were discussed without reaching any definite conclusions. It was then decided to form two subcommittees but with some joint membership in order that close liaison could be maintained. The Junior High School Subcommittee with Bob Plaxton of Viscount Bennett Junior High School, Calgary as chairman, met in June at which time it was decided to attempt some experimentation in Grade VII next fall or winter with the experimental materials prepared by Gage and Company. Some other materials may also be tried out in a small way at junior high school level.

I anticipate that the senior subcommittee will get down to some earnest study this fall. I would hope that some experimental units can be found which the subcommittee would be prepared to see in Grade X or XI mathematics in lieu of less work in some existing chapters. Also, I have some personal ideas concerning our geometry.

May I conclude by enumerating what obstacles must be overcome before a new program in mathematics for Grades VII to XII can be introduced and implemented. This specialist council can and will, I trust, provide much help in meeting these problems.

Problem No. 1 - Agreement must be reached as to content - old materials to be discarded, or to be given new emphasis; new materials to be introduced, and at what grade levels, and for whom.

Problem No. 2 - Suitable text materials must be secured. It hardly appears likely that Alberta can write its own. We could however adapt materials prepared elsewhere. It would seem to me that wherever schools are ready to do so, they should be encouraged to try out modifications of existing courses or to experiment with such new units as may be considered worthy, or to follow an alternate textbook upon approval. Some departmental control could be exercised.

Problem No. 3 - General teacher acceptance of new ideas and new programs in mathematics must be secured - enthusiastic acceptance insofar as possible. This may or may not be easy.

Problem No. 4 - Teachers must be helped to become prepared to implement the changes as they are introduced. It is unlikely that guidebooks will be sufficient. Many teachers must somehow obtain a richer background in pertinent mathematics. This will not be easy to assure but it is crucial to the success of what we envision. Here lies an important role of The Alberta Teachers' Association and of the Mathematics Council.

In what order should we seek to meet these problems? Assault on all fronts!

Let's make the revision in secondary school mathematics a real team effort. Let's move ahead as we are ready, not in one big jump with the imminent danger of falling into the middle of a whirlpool. Let's accept that our greatest need is to revise the manner in which we approach the material we now teach. For many teachers - outside this meeting - this implies acquiring a deeper and fuller appreciation and knowledge of mathematical concepts and understandings which underlie the mathematics we teach. Then the task of provincial curriculum revision will become a practical one.

THE IMPACT OF MODERN MATHEMATICS ON THE DEVELOPING PROGRAM IN HIGH SCHOOL MATHEMATICS

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*Mr. Lindstedt is in the Faculty of Education, University of Alberta. Mr. Lindstedt's talk to the conference was not in written form but he has submitted a paper which covers the same ideas and from which the last three sections are reproduced here.

I - Pressures for Changes in the Mathematics Curriculum

Dyer, Kalin and Lord(1) state:

> "All this suggests that the mathematics curriculum must grow with the times. The fact is that it has changed very little during the past century. The elementary school has offered arithmetic and some intuitive geometry; the college preparatory curriculum in high school has generally consisted of algebra in Grade IX, demonstrative geometry in Grade X, more algebra in Grade XI, and trigonometry, solid geometry, and occasionally advanced algebra in Grade XII. There have, of course, been some variations on this pattern and changes in emphasis in some schools from time to time, but by and large the mathematics curriculum in general use today looks remarkably like the 1890 model."

That the 1890 model is not adequate for the twentieth century is obvious from the characteristics of this century. Moving from the machine age into the atomic age and into the space age, this century has produced pressures which can no longer be ignored with reference to the high school mathematics program. We now look at four main kinds of pressure.

The Impact of Mathematical Research

Beginning at the graduate level, an explosion in mathematical knowledge has taken place during the twentieth century. This growth of pure mathematics has been the handmaiden to the explosion in scientific knowledge. In general, with deference to a time lag, the universities affect the curricula of the high schools (and below) by pushing back the frontiers of knowledge. This new mathematical (and scientific) frontier has produced a wide gap between the high school courses and the university courses. A great pressure has been built up therefore to change the mathematics curriculum of the college-bound student.

J. Dieudonne(2) states:

> "In the last 50 years, mathematicians have been led to introduce, not only new concepts, but a new language, which grew empirically from the needs of mathematical research, and whose aptitude for expressing mathematical statements concisely and precisely has repeatedly been tested and has won universal approval. But up to now the introduction of this new terminology has been steadfastly resisted by the secondary schools, who desperately cling to an obsolete and inadequate language: so that when a student enters the university, he will most probably never have heard such common mathematical words as set, mapping, group, vector space, etc. No wonder if he is baffled and discouraged by his contact with higher mathematics."

Professor Dieudonne goes on to evaluate the situation and comes up with a startling remedy:*

> "I think the day of such patchwork is over, and we are now committed to a much deeper reform, unless we are willing to let the situation deteriorate to the point where it will seriously impede further scientific progress. And if the whole program I have in mind had to be summarized in one slogan it would be: Euclid must go!"

Realizing that this would create a great vacancy, he goes on to suggest what should be put in its place:

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*Dieudonne was speaking of the situation in France: there seems to be an internationality about mathematical problems - he could have been talking about the United States or Canada.

"In more detail, I would list the following:

(i) Matrices and determinants of order 2 and 3.
(ii) Elementary calculus (functions of one variable).
(iii) Construction of the graph of a function and of a curve given in parametric form.
(iv) Elementary properties of complex numbers.
(v) Polar coordinates."

Growth in Scientific Research

To understand, to participate in, to contribute to the new age of science, students must have a mathematics program geared to the scientific knowledge of the twentieth century.

Einstein's relativity theory and his equation of energy are mathematical concepts. Nuclear physics is grounded in mathematics, much of it of the "modern" variety such as matrices, topology, group theory. The science of cosmology has demanded the revival and creation of mathematical theories to supply the answers to new problems. In the world of new scientific frontiers, the mathematically uneducated become illiterate.

New Mathematical Applications in Other Fields

The National Council of Teachers of Mathematics in its 1959 report remarked on the "stochastic" nature of the world. This nature has asked mathematics to sharpen the theories of probability and statistics to give patterns and relationships to a multitude of factors relevant to a multitude of problems.

Many of these problems occur in the field of social sciences. Investigators are saying that factors which exist must exist in some amount; these amounts can be measured, providing mathematics can produce the relevant measures. For example, the theory of matrices enables the research workers in many fields to consider, simultaneously, a multitude of diverse factors that impinge upon any situation. Mathematical theories of patterns and structures are applied to patterns and structures in the social, or economic or industrial world. Logical reasoning and the very nature of proof is manifest in mathematical deduction and implication. The law student may carry a book of torts in one hand supported by a mathematical set of "truth tables" in the other.

Automation

The greatest image of twentieth century automation is the so-called electronic brain, the digital computer. Descriptions in the popular media of communication say that it can "think". Actually, the thinking is done by (a) the designer, (b) the programmer - both mathematicians. And this is an application of the most fundamental principle in mathematics - the principle of counting (base 2!) and of determining equality. Behind all the wonders of automation stand the mathematicians. An understanding of fundamental mathematical principles is necessary to appreciate these machines; their operators must be mathematical technicians; their designers must be creative mathematicians.

References

(1) Dyer, Henry S., Kalin, Robert, and Lord, Frederick M., "Problems in Mathematical Education," Educational Testing Service, Princeton, N. J., 1956, pp. 18-19.

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II - Nature of the Changes in Secondary School Mathematics

Reselection of Ideas for the Details of the Courses

Some of the changes in the mathematics curriculum will be the normal, gradual changes in details which occur in any discipline as civilization marches onward and the environmental world changes about the student.

Part of the mathematics courses will become obsolete because of technological changes. Here is a specific example. In trigonometry, the solution of a triangle is important; if, say, three sides are known, it is important to know how to determine the measures of the angles. There are many relationships that could be used. Here are two:

The Cosine Law:

In A, B, C,

$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Half-Angle Tangent Law:

$$\text{Tan}\ \frac{A}{2} = \frac{s(s-b)\quad(s-c)}{(s-a)}$$

$$\text{Where } s = \frac{a+b+c}{2}$$

The development of the Cosine Law is elementary; its validity is readily proven and it is easily understood. However, because it involves the operations of addition and subtraction this relationship is not suited to the use of logarithms nor of a slide-rule. Twenty years ago, logarithms and slide-rules were the prize tools of the engineer. Therefore the more clumsy half-angle Tangent Law was emphasized in high school trigonometry because it involved multiplication, division and extracting square roots which could be done by logarithms or on a slide-rule. But the half-angle Tangent Law was difficult to develop and many high school pupils resorted to memorization in lieu of understanding. Today, fast and efficient computers are available which add, subtract, multiply, or divide with equal facility. Therefore the simpler Cosine Law can replace the more complicated half-angle Tangent Law.

Many other parts of mathematics become obsolete because of a change in treatment, in sequence or approach. It is particularly easy to spot the deadwood which persists from the days of mental discipline because of a traditional lag. B. E. Meserve(1) suggests that the following ideas are obsolete or at least should receive less emphasis: Horner's method for solving equations, formulas for solving general cubic and quartic equations, trigonometric identities for multiple angles, obscure types of factoring, use of logarithms.

Introduction of New Material

(a) Theory of Sets: B. E. Meserve states:

> "The theory of sets is of fundamental importance in all branches of mathematics. It should permeate the thinking of teachers at all levels - kindergarten through graduate school."

Most educators, working in the field of modern mathematics, feel that set theory is a great unifying idea for many parts of the discipline. The set idea carries with it new terms - elements, disjoint sets, subsets, null sets, complementary sets, finite and infinite sets, union and intersection. The theory of sets may be applied to such diverse "elements" as numbers, points, vectors, atoms, members of a woman's club, baseball leagues, electors, polynomials, switching circuits, mathematical operators.

(b) Mathematical Systems: In the introduction to the twenty-third yearbook of The National Council of Teachers of Mathematics, C. V. Newson says:

> "As already implied, the same system may occur frequently as the underlying pattern in many diverse real and ideal situations. Thus there is validity in the assertion that the mathematician is less concerned with development of general patterns that have widespread applicability in the study of particular situations."

For example, much of mathematics deals with the properties and operations of a set of numbers called the integers. We learn how to add, subtract, multiply, divide, factor, find powers, extract roots. But modern mathematics looks beyond these computational skills to discover underlying structures that may have wider applications. In this respect we find that the integers form a number system because they have defined for them two binary operations (addition and multiplication) which have closure, are commutative, associative, and one operation is distributive over the other.*

But integers are not the only number system. The structure of a number system is found with rational numbers and with real numbers. Indeed, there are number systems without "numbers" in them at all - for example, the collection of all subsets of any set (note that the elements of this set may be anything - you name it!) with the operations of "union" and "intersection" form a number system.

*This sentence summarizes the axiomatic definition of "a number system".

Getting back to integers, we might look at them as an example of a <u>group</u>. (A group is a mathematical system with clearly defined properties.) Group theory has many applications. For example, the whole area of <u>permutations</u> belongs to this mathematical structure.

Integers are also <u>rings.</u> This is another structure with certain properties, which contains many systems besides the integers (for example, polynomials). That is, if we study <u>rings</u>, rather than <u>integers,</u> we study a mathematical model with more generality and hence with greater application.

Integers are <u>not</u> "fields". (A field is yet another mathematical system). But the <u>rational</u> numbers (fractions) are <u>fields</u>. In other words, as we expand our number domains through the common stages - natural to integer to rational to real - we find new patterns and structures - groups, rings, and fields - that have different and wider applications. We have begun to nibble on a great rug that has a beautiful but intricate design woven into it.

<u>(c) Probability and Statistical Thinking</u>: The demand from the social sciences for mathematical models to deal with many factors that impinge simultaneously upon some situation, has brought forth the importance of the study of probability, variability, sampling, normal and hypergeometric distributions, etc. As the general standard of education is raised with the impact of a complex society, new ideas such as "norms", "means", "standard deviation", "statistic", "correlation", find their way into the literature of the day. Mathematical statistical models are a part of "modern mathematics". Indeed, this is a real frontier area with considerable growing pains due to its rapid crash development. Some of these ideas will undoubtedly be added to the secondary school mathematics program.

<u>(d) Other Specific Examples of New Material</u>:
(i) <u>Numeration Systems with Bases Other than 10</u>. - It is important to differentiate between the "mechanics" of mathematics and the "ideas" of mathematics. Symbolization, notation, numeration systems, algorisms all belong to the mechanics - they are used for the manipulation of symbols. If some of these "mechanics" are involved, then it is important to know and to understand the

rationale that has been used to design the symbolization. This is the case with the numeration system that is used to symbolize our numbers. One way to sharpen the understanding of the Hindu-Arabic numeration system is to consider a different system with a base other than ten. This immediately focuses attention on the essential characteristics of the system, such as its base, the use of zero, place value, grouping and regrouping.

For example, let us devise the numberal system that would be used to "count" the elements of this set,

X X X X X X X X X X X X X X X X X X X X X X X X X X X

first in base 10 (with usual word symbols as well), then in base 4 (with semi-creative word symbols with a canine slant!), and finally in base 2.

Base 10		Base 4		Base 2
Numerals:	Word Symbols:	Numerals:	Word Symbols:	Numerals
1	one	1	one	1
2	two	2	two	10
3	three	3	three	11
4	four	10	doggy	100
5	five	11	doggy-one	101
6	six	12	doggy-two	110
7	seven	13	doggy-three	111
8	eight	20	twoggy	1000
9	nine	21	twoggy-one	1001
10	ten	22	twoggy-two	1010
11	eleven	23	twoggy-three	1011
12	twelve	30	throggy	1100
13	thirteen	31	throggy-one	1101
14	fourteen	32	throggy-two	1110
15	fifteen	33	throggy-three	1111
16	sixteen	100	a houndred	10000
17	seventeen	101	houndred & one	10001
18	eighteen	102	houndred & two	10010

What is the sum of throggy-three and twoggy-two? Using our common algorism we compute:

$$\begin{array}{r} 33 \\ \underline{22} \\ 121 \end{array}$$

The answer is one houndred and twoggy-one!

What is the product of these two numbers? Obviously, it is

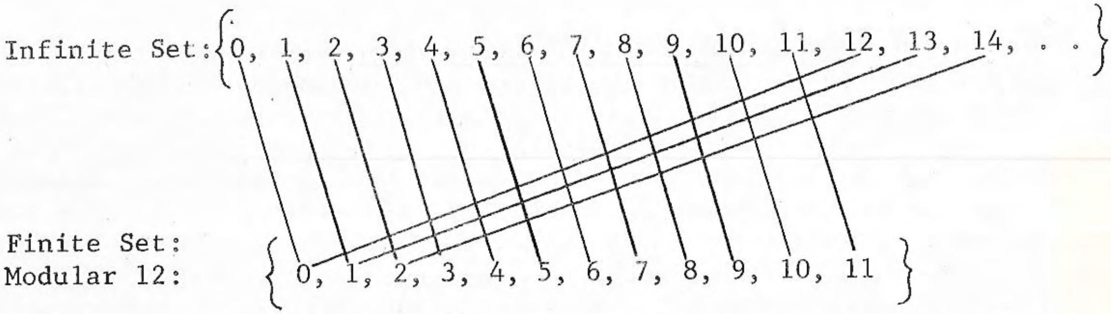
$$\begin{array}{r} 33 \\ \underline{22} \\ 132 \\ \underline{132} \\ 2112 \end{array}$$

or twoggy-one houndred and doggy-two!

Note that it is necessary to know the "basic facts" for addition and multiplication in base 4 in order to do this computation. Also note that the task of remembering these facts has been reduced (compared to base 10) to 16 from 100. In the above computations attention has been given to place value and to grouping by fours.

Of course, this study of other bases is more than a novel, intellectual work-out. There <u>are</u> practical applications. The digital computer, for example, is designed to "count" with a base 2 numeration system, the 2 digits (1 and 0) being an open and a closed circuit respectively. This means that the programmer must convert all numerals into a base 2 system.

(ii) <u>Modular Systems</u>--Infinite sets, for example, natural numbers, 1, 2, 3, 4, 5, 6, ... may be "mapped" into finite sets (sets with a definite number of elements) by means of a modular system. For example, all the hours of time, beginning at some arbitrary point of time could be represented by the infinite set 0, 1, 2, 3, 4, ... These "hours" could be "mapped" (and indeed, are) into the 12 hours on the face of a clock as follows:



We have used this finite set, modular 12, for centuries to tell the "time of day". We can "add" with this finite set, mod. 12, for example, 11 + 3 = 2; 5 + 7 = 0; 6 + 9 = 3. We can also multiply: 3 x 7 = 9; 2 x 6 = 0. Indeed, these two operations have closure, are commutative, associative, and multiplication is distributive over addition. This means that this finite set, of 12 elements only, is a number system. (Refer to

The idea of a modular number system carries us amazingly far into abstract algebra and has many used and applications both in the mathematical world and in the physical world of real events.

Some Changes in Emphasis

(a) Logical Reasoning: Euclidean geometry has long had a reputation as a discipline based upon deductive reasoning. Often the reasoning has been overlooked and students have merely memorized the "proofs". However, the development of reasoning ability is claimed to be an objective of high school mathematics, and certainly "modern" mathematics will emphasize this objective much more than it has been.

B. E. Meserve(2) states:

> "We lose many excellent opportunities for emphasizing deductive thinking when we restrict that topic to geometry. Often it is easier in algebra. We should make deductive thinking a part of all formal work in mathematics."

Mathematical systems are axiomatic systems; students become familiar with "undefined terms", "definitions", "axioms or postulates", "implications or theorems", "converses", "inverses", and "contrapositives", and so understand better the nature of truth.

(b) The Language of Mathematics: Modern mathematics suggests not only careful attention to a precise terminology, but presents the thesis that mathematics is a language.

E. A. Krug, in his book, The Secondary School Curriculum, begins a chapter on mathematics by saying:(3)

> "Mathematics is language, a more specialized, exact, precise, and refined form of language than any of those to which the term 'language' is ordinarily applied. It has become a powerful and indispensable tool in the description, analysis, and control of phenomena that can neither be accurately expressed nor understood in ordinary language alone. It is the most truly international of all languages. It is one of the most remarkable of all human achievements and as such forms part of the cultural heritage that schools seek to transmit and on which they build for the future.",

and he ends the same chapter:

> "And this is another way of saying that the mathematics curriculum is moving toward a fuller realization of mathematics not as a body of mechanical manipulations and routines, but as a language of unparalleled beauty, precision, and power."

Modern mathematics is concerned with ideas, with concepts. These are symbolized by many kinds of notation, which carry with them the sense of their referents. Some of the symbols are "nouns" - 5, x, AB; some of the symbols are "transitive verbs" - +, x, /; an important one is the verb"to be" - =. The mathematical sentence - the equation or inequality - expresses an idea.

This implies that modern mathematics is concerned with thinking, with comprehension, rather than just with manipulation. It is more than a tool or a skill, it is a structured body of related ideas extending the frontiers of knowledge.

References

(1) Meserve, Bruce E.;"Implications for the Mathematics Curriculum,"

Insights into Modern Mathematics, Twenty-Third Yearbook, NCTM, Washington, D.C., 1957.

(2) Ibid: page 415.

(3) Krug, E. A.; The Secondary School Curriculum, Harper and Brothers, New York: 1960, page 278.

III. Implications for Curriculum Planning

Aims and Objectives

H. L. Stein(1) states:

"In summary, then, a search of the literature on objectives of mathematics in the secondary school reveals mild swings from one point of view to another. Roughly, until the early twenties, the emphasis was upon mathematics as a discipline. From the twenties to the early fifties there appeared to be a swing to utilitarian emphasis. Presently, the shift is towards a thoroughly modernized program which emphasizes, in the main, mathematical concepts - at least for the college-capable students."

When educators consider the objectives of high school mathematics, they often mention a dualism: (1) those objectives for college-bound students, and (2) those objectives so far as a "general education" is concerned. The Commission on Mathematics of the College Entrance Examination Board in its report, "Program for College Preparatory Mathematics" (2) have, in spite of their main interest, recognized the obligation of the schools to serve "all the children of all the people" and suggest the following specific objectives for "general" students:

1. An understanding of and competence in, the processes of arithmetic and the use of formulas in elementary algebra. A basic knowledge of graphical methods and simple statistics is also important.
2. An understanding of the general properties of geometrical figures and the relationships among them.

3. An understanding of the deductive method as a method of thought. This includes the ideas of axioms, rules of inference, and methods of proof.
4. An understanding of mathematics as a continuing creative endeavor with aesthetic values similar to those found in art and music,...

Their objectives for the college-bound student are more challenging "to know something of the intellectual excitement and deep satisfaction that mathematics can offer", and at the same time help to prepare him for the tasks demanded by a technological age. As the report points out,

> "Mathematics is no longer reserved for the use of engineers and physical scientists, even though a great many of its applications are still in their hands. ... Today, mathematics exerts actual leadership in natural science, social science, business, industry, and other fields.
>
> ... the work in the classroom must be constantly related to other fields of knowledge; to the pupil's immediate experience, and to other activities (e.g., science and geography) in which he is engaged. The ultimate aim is the recognition on the part of the pupil that mathematics is a dynamic system, constantly developing, and that creative self-activity, is a fascinating realm of endeavor."

A close examination of the objectives for "college-bound" and for "general" students as stated by different groups, produces not a dualism, but, in fact, a synthesis of objectives. Statements of objectives for both groups of students include (a) the utilitarian value of mathematics and (b) the cultural value of mathematics. The difference between the statements is not one of <u>kind</u> but of <u>degree</u>. This idea is elaborated further in the section below on individual differences.

<u>The Need for Unification</u>
First, there is a need to find some unifying ideas that will synthesize and fuse the various threads of mathematical concepts into some definite, organized and sequential pattern. We do not naively imply that the field can be simplified into an easy sequence from

A to Z, and that our only problem would then be to put our mathematics students through the alphabet. But the warp and the woof of the mathematics tapestry should all contribute to a unified pattern held together by relationships and rational associations. Especially should "new" topics be so interwoven. The Secondary School Curriculum Committee of the National Council of Teachers of Mathematics list certain new topics, "not to be isolated, but woven into the curriculum".

One such "unifying idea" is the Theory of Sets. H. F. Fehr, in a talk at a mathematics institute in Madison, July, 1960 emphasized that the greatest hope for finding a synthesizer of the various new ideas of modern mathematics was the "set idea". Most experimental programs(3) at the elementary level include a foundational consideration of sets. At the high school level, the Theory of Sets is a popular and promising addition and approach and is included in most of the modern mathematics textbooks written for secondary students.(4)

The Commission on Mathematics lists the following as "unifying ideas": sets, variables, functions, and relations. These will undoubtedly form an important part of the outline of any new high school mathematics program.

A Team Approach to Curriculum Planning

Van Engen(5) says:

> "The reform of the mathematics program rests ultimately on three pillars - a tripod of sound principles. The first leg of the tripod rests upon sound mathematics... The second leg of the tripod rests upon sound philosophical principles... The third leg of the tripod rests upon sound psychological principles."

We have suggested here, a team of three. The mathematician will produce an organized, sequential body of mathematical content that will rigorously develop this discipline as a science; the educational philosopher will relate this content to the aims and objectives of twentieth century education, and the school psychologist will suggest methods and approaches consistent with known theories of learning and of child development. Such a team, working together, may produce a dynamic and vital mathematics program to replace our "1890 model".

The Provision for Individual Differences

The Secondary School Curriculum Committee of the National Council of Teachers of Mathematics appointed ten subcommittees to study various phases of the "modern" program in mathematics; one of these prepared a report on the provision for individual differences. They suggested:

(a) Teach the same mathematical structures and concepts to all, but vary the amount, complexity, depth, and manner of organization and presentation.
(b) For pupils of below average ability, have a program of minimum essentials. Research is needed to provide teachers with a detailed outline for below average pupils.
(c) Program of enrichment for the gifted should provide real challenge and basic information.
(d) Ability grouping is a desired practice.

Here is a recommendation that realistically recognizes that individual differences do exist and that ability grouping is "a desired practice". However, the provision is not one of dualism, but rather to "teach the same mathematical structures and concepts to all". The courses will vary from "minimum essentials" to enrichment with a "real challenge".

Traditional mathematics could not be organized in this way. Based primarily upon the memorization of facts, and the development of computational skills, the "1890 model" does not possess enough versatility to make provision for wide individual differences. That is why "dualism" has been persistently advocated.

Modern mathematics, however, based upon ideas, and organized with unifying structures may be flexible enough to meet the demands of both a college-preparatory course and a general course. We may find that this program will point to the possibility of a high school marriage of the two objectives, to prepare for life and to prepare for higher education.

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(4) e.g. Freund, J. E. A Modern Introduction to Mathematics, Prentice-Hall, 1956.

(5) Van Engen, Henry, The Theory of Arithmetic and Its Teaching, Mimeographed manuscript, 1960.

At the August Conference there were several requests from members for a bibliography on modern mathematics, experimental programs, and curriculum reform. Dr. J. Hlavaty recommended the list that appears below. Representation was made to the A.T.A. Library Committee, and this list of publications was approved for placement in the library. Members will be advised either through The ATA Magazine or through the MCATA Newsletter when these are available on loan.

Commission on Mathematics, College Entrance Examinations Board, 475 Riverside Drive, New York 27, New York

Report of Commission:
Vol. 1: Program for College Preparatory Mathematics
Vol. 2: Appendices

National Council of Teachers of Mathematics
1201 Sixteenth Street, N.W., Washington, D.C.

Insights into Modern Mathematics, Twenty-third Yearbook
The Growth of Mathematical Ideas, Twenty-fourth Yearbook
Instruction in Arithmetic, Twenty-fifth Yearbook
The Secondary Mathematics Curriculum

School Mathematics Study Group
School of Education, Stanford University, Stanford, California

Study Guide in Modern Algebra
Mathematics for Junior High School
Mathematics for the Elementary School

Studies in Mathematics
Vol. 3: The Structure of Elementary Algebra
Vol. 4: Concepts of Informal Geometry
Vol. 6: Number Systems

Mathematics for High Schools
(a) First Course
(b) Intermediate Course
(c) Elementary Functions
(d) Geometry
(e) Introduction to Matrix Algebra

California Association of Secondary School Administration
1705 Murchison Drive, Burlingame, California

New Developments in Secondary Mathematics

Scott Foresman Co.
Chicago, Atlanta, Dallas, Palo Alto, Fair Lawn (New York)

Studies in Mathematics Education

Ball State Teachers College Experimental Program
Muncie, Indiana

Eighth Grade Mathematics
Beginning Algebra
Intermediate Algebra

Boston College Mathematical Series
Boston College, Chestnut Hill 67, Massachusetts

Sets, Operations, Patterns
by Father Stanley Bezouszke

The Madison Project
Mathematics Department, Syracuse University
Syracuse, New York

An Experimental Program for Algebra in Grades IV-VII
by Robert B. Davis

New York State Mathematics Syllabus Committee
State Education Department, Albany, New York

Mathematics 10, 11, 12 (Syllabus)
Tenth Year Mathematics - Coordinate Geometry
Handbook for Eleventh Year Mathematics
An Experimental Course in Mathematics for Twelfth Year

University of Maryland Mathematics Project
1515 Massachusetts Ave. N.W., Washington 5, D.C.

Seventh Grade Text
Eighth Grade Text

ADDITIONAL REFERENCES

Adler, I., The Magic House of Numbers (New American Library)

Adler, I., The New Mathematics (New American Library)

Andree, R. V., Modern Abstract Algebra (Henry Holt and Co., New York)

Dantzig, J., Number, The Language of Science (Doubleday & Company, New York)

Dubisch, R., The Nature of Numbers (Ronald Press, New York)

Felix, L., The Modern Aspect of Mathematics (Basic Books, New York)

Levi, H., Elements of Algebra (Chelsea Publishing Co., New York)

Levi, H., Foundations of Geometry and Trigonometry (Prentice-Hall)

Kelly, J. L., Introduction to Modern Algebra (D. Van Nostrand Co.)

Nichols and Collins, Modern Elementary Algebra (Holt-Rinehart-Winston)

Sawyer, W. W., A Concrete Approach to Abstract Algebra (W. H. Freeman, San Francisco)

FILMS ON MATHEMATICS

McGraw-Hill has developed five films in a series called "Teacher Education in Modern Mathematics". The five are:

Concept of Function,
Irrational Numbers,
Number Fields,
Patterns in Mathematics, and
Sentences and Solution Sets.

One of the above was shown at the conference. The rental fee varies from $15 to $20 per week of use and these films are available from: McGraw-Hill Company of Canada, 253 Spadina Road, Toronto 4.

The other film shown at the conference was "Donald Duck in Mathmagic Land". This is available free of charge from the Audio-Visual Branch of the Department of Education.