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Contents

"POLYAN" MATHEMATICS, by H. F. McCall

REPORT ON THE 40TH ANNUAL NCTM CONVENTION, by W. F. Coulson and E. E. Andrews

TWO ALBERTANS RECEIVE NSF GRANTS

MATHEMATICS TOURNAMENT, CRESCENT HEIGHTS HIGH SCHOOL, CALGARY, by Sharon Brown

BOOK REVIEW - Introduction to Mathematics, by H. S. Hrabi

MCATA NOTES

"POLYAN" MATHEMATICS, by H. F. McCall

Editor's Note: Dr. McCall, principal of Seba Beach School, was awarded the Shell Merit Fellowship last year. The article below was intended to follow a discussion of the shortcomings of the mathematics that has already been introduced in many American schools and is being introduced to a degree in Canadian schools.

There is indication of the adoption of a "newer" mathematics, much more openly based upon induction, the reasoning of science, than traditional mathematics has ever been. The "newer" mathematics I have termed "Polyan", for it has been taught by the renowned Dr. G. Polya in many European and American universities for a good number of years. Besides the general mathematics of Professor Polya, there is considerable indication that a great deal of geometry will be placed in the primary grades. For those who would like to see the geometry books for primary grades, write for -

<u>Geometry for the Primary Grades, Books 1 and 11, and Teachers' Manuals</u>
Hawley and Suppes, Holden Day Inc., 728 Montgomery Street, San Francisco, California.

However, this is a mere detail. The really significant aspect of the "newer" mathematics is definitely Polyan, and, if you wish to acquaint yourself with something that is really interesting in mathematics, purchase -

<u>How To Solve It - A New Aspect of Mathematical Method</u>
Polya, G., Doubleday Anchor Books, Doubleday and Co., Inc., Garden City, New York, $1.10.

<u>Induction and Analogy in Mathematics</u>
Polya, G., Princeton University Press, Princeton, New Jersey, $5.50.

The keynote to Polyan mathematics is the solving of problems. The rigorous, systematic, deductive science of mathematics is not scorned and discarded as useless, but a different experimental, inductive science of mathematics which should play just as important a part in the world is introduced.

The main purpose for mathematics should be solving of problems, not philosophic contemplation of the wonders of our number system or even the wonders of flawless deductive reasoning. In Dr. Polya's "Preface" to the first printing of <u>How To Solve It</u>, he says, "If (the teacher) challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking."

It seems advisable to give here a fairly extensive quotation from the "Preface" to Induction and Analogy in Mathematics.

> Strictly speaking, all our knowledge outside mathematics and demonstrative logic (which is, in fact, a branch of mathematics) consists of conjectures. There are, of course, conjectures and conjectures. There are highly respectable and reliable conjectures as those expressed in certain general laws of physical science. There are other conjectures, neither reliable nor respectable, some of which may make you angry when you read them in a newspaper. And in between there are all sorts of conjectures, hunches, and guesses.
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> We secure our mathematical knowledge by demonstrative reasoning, but we support our conjectures by plausible reasoning. A mathematical proof is demonstrative reasoning, but the inductive evidence of the physicist, the circumstantial evidence of the lawyer, the documentary evidence of the historian, and the statistical evidence of the economist belong to plausible reasoning.
>
> The difference between the two kinds of reasoning is great and manifold. Demonstrative reasoning is safe, beyond controversy, and final. Plausible reasoning is hazardous, controversial, and provisional. Demonstrative reasoning penetrates the sciences just as far as mathematics does, but it is in itself (as mathematics is in itself) incapable of yielding essentially new knowledge about the world around us. Anything new that we learn about the world involves plausible reasoning, which is the only kind of reasoning for which we care in everyday affairs. Demonstrative reasoning has rigid standards, codified and clarified by logic (formal or demonstrative logic), which is the theory of demonstrative reasoning. The standards of plausible reasoning are fluid, and there is no theory of such reasoning that could be compared to demonstrative logic in clarity or would command comparable consensus.
>
> Another point concerning the two kinds of reasoning deserves our attention. Everyone knows that mathematics offers an excellent opportunity to learn demonstrative reasoning, but I contend also that there is no subject in the usual curricula of the schools that affords a comparable opportunity to learn plausible

reasoning. I address myself to all interested students of mathematics of all grades and I say: "Certainly, let us learn proving, but also let us learn guessing." This sounds a little paradoxical and I must emphasize a few points to avoid possible misunderstandings.

Mathematics is regarded as a demonstrative science. Yet this is only one of its aspects. Finished mathematics presented in a finished form appears as purely demonstrative, consisting of proofs only. Yet mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to guess the idea of the proof before you carry through the details. You have to combine observations and follow analogies; you have to try and try again. The result of the mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible reasoning, by guessing. If the learning of mathematics reflects to any degree the invention of mathematics, it must have a place for guessing, for plausible inference.

There are two kinds of reasoning, as we said: demonstrative reasoning and plausible reasoning. Let me observe that they do not contradict each other; on the contrary, they complete each other. In strict reasoning the principal thing is to distinguish a proof from a guess, a valid demonstration from an invalid attempt. In plausible reasoning the principal thing is to distinguish a guess from a guess, a more reasonable guess from a less reasonable guess. If you direct your attention to both distinctions, both may become clearer.

A serious student of mathematics, intending to make it his life's work, must learn demonstrative reasoning; it is his profession and the distinctive mark of his science. Yet for real success he must also learn plausible reasoning; this is the kind of reasoning on which his creative work will depend. The general or amateur student should also get a taste of demonstrative reasoning: he may have little opportunity to use it directly, but he should acquire a standard with which he can compare alleged evidence of all sorts aimed at him in modern life. But in all his endeavors he will need plausible reasoning. At any rate, an ambitious student of mathematics, whatever his further interests may be, should try to learn both kinds of reasoning, demonstrative and plausible.

I do not believe that there is a foolproof method to learn guessing. At any rate, if there is such a method, I do not know it, and quite certainly I do not pretend to offer it on the following pages. The efficient use of plausible reasoning is a practical skill and it is learned, as any other practical skill, by imitation and practice. I shall try to do my best for the reader who is anxious to learn plausible reasoning, but what I can offer are only examples for imitation and opportunity for practice.

The examples of plausible reasoning collected in this book may be put to another use: they may throw some light upon a much agitated philosophical problem: the problem of induction. The crucial question is: Are there rules for induction? Some philosophers say Yes, most scientists think No. In order to be discussed profitably, the question should be put differently. It should be treated differently, too, with less reliance on traditional verbalisms, or on new-fangled formalisms, but in closer touch with the practice of scientists. Now, observe that inductive reasoning is a particular case of plausible reasoning. Observe also (what modern writers almost forgot, but some older writers, such as Euler and Laplace, clearly perceived) that the role of inductive evidence in mathematical investigation is similar to its role in physical research. Then you may notice the possibility of obtaining some information about inductive reasoning by observing and comparing examples of plausible reasoning in mathematical matters. And so the door opens to investigating induction inductively.

I shall here "solve" a problem using these less rigorous, inductive methods and proofs suggested by Dr. Polya.

Find: the lengths of three mutually perpendicular edges x, y and z, of a box.

Given: the volume, V, of the box.
Condition: the surface S of the box is a minimum.

The first step in solving might be to change the problem to a simpler but analogous problem and solve it. Thus, let us find the lengths of the sides of a rectangle, being given the area (a) of the rectangle and the condition being that the perimeter (P) of the rectangle be a minimum.

Let the sides of the rectangle be X and Y units in length.

$2X + 2Y = P$

$X + Y = P/2$

At this critical juncture we well might make an educated guess, namely, that this rectangle will have to be a square if we are to obtain maximum area for minimum dimensions. (No fault should be found with this latter statement of slight aberration from the original postulates.)

Each side of a square with perimeter P is equal to P/4. Each side will also be one-half the sum of the two adjacent sides, i.e., $\frac{X + Y}{2}$

The area of this square would be $\left(\frac{X + Y}{2}\right)^2$ square units.

The area of the original rectangle would, in any case, be XY square units.

Is XY as large as $\left(\frac{X + Y}{2}\right)^2$?

We are, of course, presuming that it is not, unless X and Y are equal.

The difference in area will, in any case, amount to

$$\left(\frac{X + Y}{2}\right)^2 - (XY) = \frac{X^2 + 2XY + Y^2}{4} - \frac{4XY}{4} =$$

$$\frac{X^2 - 2XY + Y^2}{4} = \left(\frac{X - Y}{2}\right)^2 \text{ square units.}$$

Now, there is only one way to make this difference amount to nothing, or, in other words, there is only one way to make the area of the rectangle as large as the area of the square. This would be to have X = Y. Then, of course, the rectangle is a square.

The way to keep a large rectangle with minimum dimensions, then, is to have those dimensions equal - making the rectangle into a square.

By means of our intuitive recognition of patterns we now may form a conclusion from this simpler problem, that what happens when we deal with two dimensions might happen in an analogous fashion when dealing with three dimensions. If this is true, then, to keep a constant volume for a box with the minimum surface area for the faces of this box, we would want each face to be a square, that is, the box would be a cube.

However, we should test this theory.

To do this, let us once more simplify matters by supposing that one dimension, Z, of the box is fixed, so that only the other two dimensions, X and Y, may vary. Now, if we are to maintain the large volume with minimum dimensions, and one of these dimensions is fixed, the problem becomes one of maintaining a large product of the other two dimensions while they are at minimum magnitude. But this is the equivalent of finding the minimum dimensions of a rectangle of constant area, which we already discovered. This means that X and Y must be equal, to make the faces of the box, to which Z is perpendicular, both squares.

But since X, Y, and Z are equal members of a democracy with no special privileges to either, all representing "a dimension" of the box, we would obtain exactly the same results by holding X constant in our imagination and having Y and Z vary - also by holding Y constant and letting X and Z vary. Therefore, all sides of the box are squares and the box is a cube.

The general ideas underlying this method I have tried to make apparent throughout the discussion. Use of analogy was made in this inductive process, simplifying cases, observing regularity of patterns, making tentative generalizations and testing the guesses - utilizing the concept of keeping our brains clear by despotically holding one variable constant when we are bothered by too many variables at a time.

I think I have said enoughto give a general idea of this "mathematics for the scientific world" and to show its great difference from the "new mathematics" now appearing in Alberta, which is more like "mathematics for the mathematics philosopher".

It is my own personal hope that every mathematics teacher in Alberta can see that mathematics in any grade has value only to the extent to which it may be used to solve problems. The final test of the value of a mathematics course in, let us say, Grade Four, is in discovering how many kinds of problems each individual pupil can solve after having completed the course.

REPORT ON THE 40TH ANNUAL CONVENTION OF THE NCTM, by W. F. Coulson and E. E. Andrews

> Editor's Note: Messrs. Coulson and Andrews of the Faculty of Education, University of Alberta, attended the NCTM Convention held in San Francisco, April 16-18, 1962.

The organization for each day's activities consisted of general sessions in the morning and evening, and special sessions for elementary, junior high and senior high teachers during the remainder of the day. There were also special activities for supervisors of mathematics curricula, and for those concerned with teacher training in mathematics.

Two areas seemed to receive major attention throughout the various sessions.

First, concern with the modern mathematics curriculum is still very much in evidence. This concern is not only with the content of the curriculum but with the grade placement of specific topics. Several nationally known experimental courses have now been in use for five years or more. Speakers such as Herbert F. Spitzer, Max Beberman and J. Fred Weaver are taking a critical look at many of these courses. Some significant points made were.

Over-emphasis on such features of "higher" mathematics as the axiomatic approach, rigorous development, and precision of language,