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"POLYAN" MATHEMATICS, by H. F. McCall

> Editor's Note: Dr. McCall, principal of Seba Beach School, was awarded the Shell Merit Fellowship last year. The article below was intended to follow a discussion of the shortcomings of the mathematics that has already been introduced in many American schools and is being introduced to a degree in Canadian schools.

There is indication of the adoption of a "newer" mathematics, much more openly based upon induction, the reasoning of science, than traditional mathematics has ever been. The "newer" mathematics I have termed "Polyan", for it has been taught by the renowned Dr. G. Polya in many European and American universities for a good number of years. Besides the general mathematics of Professor Polya, there is considerable indication that a great deal of geometry will be placed in the primary grades. For those who would like to see the geometry books for primary grades, write for -

Geometry for the Primary Grades, Books 1 and 11, and Teachers' Manuals
Hawley and Suppes, Holden Day Inc., 728 Montgomery Street, San Francisco, California.

However, this is a mere detail. The really significant aspect of the "newer" mathematics is definitely Polyan, and, if you wish to acquaint yourself with something that is really interesting in mathematics, purchase -

How To Solve It - A New Aspect of Mathematical Method
Polya, G., Doubleday Anchor Books, Doubleday and Co., Inc., Garden City, New York, $1.10.

Induction and Analogy in Mathematics
Polya, G., Princeton University Press, Princeton, New Jersey, $5.50.

The keynote to Polyan mathematics is the solving of problems. The rigorous, systematic, deductive science of mathematics is not scorned and discarded as useless, but a different experimental, inductive science of mathematics which should play just as important a part in the world is introduced.

The main purpose for mathematics should be solving of problems, not philosophic contemplation of the wonders of our number system or even the wonders of flawless deductive reasoning. In Dr. Polya's "Preface" to the first printing of How To Solve It, he says, "If (the teacher) challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking."

It seems advisable to give here a fairly extensive quotation from the "Preface" to Induction and Analogy in Mathematics.

> Strictly speaking, all our knowledge outside mathematics and demonstrative logic (which is, in fact, a branch of mathematics) consists of conjectures. There are, of course, conjectures and conjectures. There are highly respectable and reliable conjectures as those expressed in certain general laws of physical science. There are other conjectures, neither reliable nor respectable, some of which may make you angry when you read them in a newspaper. And in between there are all sorts of conjectures, hunches, and guesses.
>
> We secure our mathematical knowledge by demonstrative reasoning, but we support our conjectures by plausible reasoning. A mathematical proof is demonstrative reasoning, but the inductive evidence of the physicist, the circumstantial evidence of the lawyer, the documentary evidence of the historian, and the statistical evidence of the economist belong to plausible reasoning.
>
> The difference between the two kinds of reasoning is great and manifold. Demonstrative reasoning is safe, beyond controversy, and final. Plausible reasoning is hazardous, controversial, and provisional. Demonstrative reasoning penetrates the sciences just as far as mathematics does, but it is in itself (as mathematics is in itself) incapable of yielding essentially new knowledge about the world around us. Anything new that we learn about the world involves plausible reasoning, which is the only kind of reasoning for which we care in everyday affairs. Demonstrative reasoning has rigid standards, codified and clarified by logic (formal or demonstrative logic), which is the theory of demonstrative reasoning. The standards of plausible reasoning are fluid, and there is no theory of such reasoning that could be compared to demonstrative logic in clarity or would command comparable consensus.
>
> Another point concerning the two kinds of reasoning deserves our attention. Everyone knows that mathematics offers an excellent opportunity to learn demonstrative reasoning, but I contend also that there is no subject in the usual curricula of the schools that affords a comparable opportunity to learn plausible

reasoning. I address myself to all interested students of mathematics of all grades and I say: "Certainly, let us learn proving, but also let us learn guessing." This sounds a little paradoxical and I must emphasize a few points to avoid possible misunderstandings.

Mathematics is regarded as a demonstrative science. Yet this is only one of its aspects. Finished mathematics presented in a finished form appears as purely demonstrative, consisting of proofs only. Yet mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to guess the idea of the proof before you carry through the details. You have to combine observations and follow analogies; you have to try and try again. The result of the mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible reasoning, by guessing. If the learning of mathematics reflects to any degree the invention of mathematics, it must have a place for guessing, for plausible inference.

There are two kinds of reasoning, as we said: demonstrative reasoning and plausible reasoning. Let me observe that they do not contradict each other; on the contrary, they complete each other. In strict reasoning the principal thing is to distinguish a proof from a guess, a valid demonstration from an invalid attempt. In plausible reasoning the principal thing is to distinguish a guess from a guess, a more reasonable guess from a less reasonable guess. If you direct your attention to both distinctions, both may become clearer.

A serious student of mathematics, intending to make it his life's work, must learn demonstrative reasoning; it is his profession and the distinctive mark of his science. Yet for real success he must also learn plausible reasoning; this is the kind of reasoning on which his creative work will depend. The general or amateur student should also get a taste of demonstrative reasoning: he may have little opportunity to use it directly, but he should acquire a standard with which he can compare alleged evidence of all sorts aimed at him in modern life. But in all his endeavors he will need plausible reasoning. At any rate, an ambitious student of mathematics, whatever his further interests may be, should try to learn both kinds of reasoning, demonstrative and plausible.

I do not believe that there is a foolproof method to learn guessing. At any rate, if there is such a method, I do not know it, and quite certainly I do not pretend to offer it on the following pages. The efficient use of plausible reasoning is a practical skill and it is learned, as any other practical skill, by imitation and practice. I shall try to do my best for the reader who is anxious to learn plausible reasoning, but what I can offer are only examples for imitation and opportunity for practice.

The examples of plausible reasoning collected in this book may be put to another use: they may throw some light upon a much agitated philosophical problem: the problem of induction. The crucial question is: Are there rules for induction? Some philosophers say Yes, most scientists think No. In order to be discussed profitably, the question should be put differently. It should be treated differently, too, with less reliance on traditional verbalisms, or on new-fangled formalisms, but in closer touch with the practice of scientists. Now, observe that inductive reasoning is a particular case of plausible reasoning. Observe also (what modern writers almost forgot, but some older writers, such as Euler and Laplace, clearly perceived) that the role of inductive evidence in mathematical investigation is similar to its role in physical research. Then you may notice the possibility of obtaining some information about inductive reasoning by observing and comparing examples of plausible reasoning in mathematical matters. And so the door opens to investigating induction inductively.

I shall here "solve" a problem using these less rigorous, inductive methods and proofs suggested by Dr. Polya.

Find: the lengths of three mutually perpendicular edges x, y and z, of a box.

Given: the volume, V, of the box.
Condition: the surface S of the box is a minimum.

The first step in solving might be to change the problem to a simpler but analogous problem and solve it. Thus, let us find the lengths of the sides of a rectangle, being given the area (a) of the rectangle and the condition being that the perimeter (P) of the rectangle be a minimum.

Let the sides of the rectangle be X and Y units in length.
$2X + 2Y = P$
$X + Y = P/2$

At this critical juncture we well might make an educated guess, namely, that this rectangle will have to be a square if we are to obtain maximum area for minimum dimensions. (No fault should be found with this latter statement of slight aberration from the original postulates.)

Each side of a square with perimeter P is equal to P/4. Each side will also be one-half the sum of the two adjacent sides, i.e., $\frac{X + Y}{2}$

The area of this square would be $\left(\frac{X + Y}{2}\right)^2$ square units.

The area of the original rectangle would, in any case, be XY square units.

Is XY as large as $\left(\frac{X + Y}{2}\right)^2$?

We are, of course, presuming that it is not, unless X and Y are equal.

The difference in area will, in any case, amount to

$$\left(\frac{X + Y}{2}\right)^2 - (XY) = \frac{X^2 + 2XY + Y^2}{4} - \frac{4XY}{4} =$$

$$\frac{X^2 - 2XY + Y^2}{4} = \left(\frac{X - Y}{2}\right)^2 \text{ square units.}$$

Now, there is only one way to make this difference amount to nothing, or, in other words, there is only one way to make the area of the rectangle as large as the area of the square. This would be to have $X = Y$. Then, of course, the rectangle is a square.

The way to keep a large rectangle with minimum dimensions, then, is to have those dimensions equal - making the rectangle into a square.

By means of our intuitive recognition of <u>patterns</u> we now may form a conclusion from this simpler problem, that what happens when we deal with two dimensions might happen in an analogous fashion when dealing with three dimensions. If this is true, then, to keep a constant volume for a box with the minimum surface area for the faces of this box, we would want each face to be a square, that is, the box would be a cube.

However, we should test this theory.

To do this, let us once more simplify matters by supposing that one dimension, Z, of the box is fixed, so that only the other two dimensions, X and Y, may vary. Now, if we are to maintain the large volume with minimum dimensions, and one of these dimensions is fixed, the problem becomes one of maintaining a large product of the other two dimensions while they are at minimum magnitude. But this is the equivalent of finding the minimum dimensions of a rectangle of constant area, which we already discovered. This means that X and Y must be equal, to make the faces of the box, to which Z is perpendicular, both squares.

But since X, Y, and Z are equal members of a democracy with no special privileges to either, all representing "a dimension" of the box, we would obtain exactly the same results by holding X constant in our imagination and having Y and Z vary - also by holding Y constant and letting X and Z vary. Therefore, all sides of the box are squares and the box is a cube.

The general ideas underlying this method I have tried to make apparent throughout the discussion. Use of analogy was made in this inductive process, simplifying cases, observing regularity of patterns, making tentative generalizations and testing the guesses - utilizing the concept of keeping our brains clear by despotically holding one variable constant when we are bothered by too many variables at a time.

I think I have said enoughto give a general idea of this "mathematics for the scientific world" and to show its great difference from the "new mathematics" now appearing in Alberta, which is more like "mathematics for the mathematics philosopher".

It is my own personal hope that every mathematics teacher in Alberta can see that mathematics in any grade has value only to the extent to which it may be used to solve problems. The final test of the value of a mathematics course in, let us say, Grade Four, is in discovering how many kinds of problems each individual pupil can solve after having completed the course.

REPORT ON THE 40TH ANNUAL CONVENTION OF THE NCTM, by W. F. Coulson and E. E. Andrews

> Editor's Note: Messrs. Coulson and Andrews of the Faculty of Education, University of Alberta, attended the NCTM Convention held in San Francisco, April 16-18, 1962.

The organization for each day's activities consisted of general sessions in the morning and evening, and special sessions for elementary, junior high and senior high teachers during the remainder of the day. There were also special activities for supervisors of mathematics curricula, and for those concerned with teacher training in mathematics.

Two areas seemed to receive major attention throughout the various sessions.

First, concern with the modern mathematics curriculum is still very much in evidence. This concern is not only with the content of the curriculum but with the grade placement of specific topics. Several nationally known experimental courses have now been in use for five years or more. Speakers such as Herbert F. Spitzer, Max Beberman and J. Fred Weaver are taking a critical look at many of these courses. Some significant points made were.

Over-emphasis on such features of "higher" mathematics as the axiomatic approach, rigorous development, and precision of language, can lead to a new kind of sterile formalism in school mathematics. The need for and interest in mathematics must still come from familiar aspects of the child's environment.

Many new courses attempt to introduce topics from geometry and algebra at a much earlier grade level than has been traditional. Topics completely new to school mathematics are also appearing. It was felt very strongly by some speakers that the question of grade placement is not primarily "how early may a topic be introduced" but "when is the optimum time for introducing it". Introduction at this optimum time will lead both to the furthering of mathematical insight and its application in meaningful problem situations.

Secondly, much interest was shown in the role that programmed learning can take in the teaching of mathematics.

J. E. Forbes of the Britannica Centre stressed the following points: (a) programs are not just a new form of textbooks, they cannot do the whole job; (b) there is no doubt that many programs are dull, repetitive and make no provision for abler students to leave out unnecessary repetition; and (c) there is no simple "yes" or "no" regarding the use of programs, each teacher must decide the best use to which they can be put in a given classroom situation.

J. Fred Weaver expressed concern as to what extent programs will foster or repress creativity in mathematics. In particular, he questioned the ability of these programs to provide for flexibility of approach, divergent thinking and "tolerance for ambiguity".

TWO ALBERTANS RECEIVE NSF GRANTS

Each year the National Science Foundation in Washington, D.C., through its Academic Year Institute program, provides opportunities for teachers of science and mathematics to study fulltime for an entire academic year. This year about 1,700 experienced secondary school teachers and supervisors, and 100 experienced college teachers will be supported as participants. In addition, about 50 recent college graduates who are fully certified to teach, but who have had no teaching experience, will be granted support.

Under this latter category, two Albertans will study at Washington University, St. Louis, Missouri, from June 1962 to June 1963. One of these students, Halia Boychuk, received her bachelor of education degree in the May, 1962 convocation. Miss Boychuk is a native Albertan, having received all of her education in this province, at Cork and at Ashmont. She received a Governor-General's award in Grade IX and a Hotelman's Association scholarship in Grade XII. She served as secretary-treasurer of the students' union at Ashmont. Miss Boychuk has received a Queen Elizabeth scholarship during her years at the University of Alberta.

The other participant is Alexander J. Dawson. Mr. Dawson received all of his education in Edmonton. He took Grades VIII and IX in one year and completed high school at Victoria Composite High School in 1958. Then he entered the three-year general program, obtaining a bachelor of science degree, majoring in mathematics, in 1961. During the 1961-62 academic year, Mr. Dawson attended the Faculty of Education in the program leading to certification following an approved degree.

MATHEMATICS TOURNAMENT, CRESCENT HEIGHTS HIGH SCHOOL, CALGARY, by Sharon Brown

> Editor's Note: Miss Brown is a member of the Eleven A class at Crescent Heights.

The second Annual Mathematics Tournament between Crescent Heights High School and Viscount Bennett High School was held on Tuesday, April 17. It was won again by Crescent, by a score of 77 to 68. The highest individual score was a tie between Marlene Warren of Crescent and Michael Smith of Viscount with 23 points each.

Teams, composed of four members from each school, were selected by means of elimination contests held several days prior to April 17. The judges, three in number, were Miss Eva Jagoe (Viscount Bennett), Miss Olive Jagoe (Crescent Heights), and D. Dack (Central).

The contest consisted of a series of three ten-minute tests with a five-minute break between each. The spectators were also given copies of the tests to work and were given the solutions during the breaks.

This year's tournament was arranged by the Eleven A Mathematics 20 class of Crescent Heights under the leadership of Mrs. M. Melech and Miss H. Morrison. Chairman, scorekeepers, timekeepers, pages, and ushers were all volunteers from Eleven A.

BOOK REVIEW, by H. S. Hrabi

Editor's Note: Mr. Hrabi is mathematics consultant for the Department of Education.

INTRODUCTION TO MATHEMATICS

Brumfiel, Eicholz, and Shanks; Reading, Massachusetts, (Addison-Wesley Publishing Company, Inc., 1961) xi + 323 pp., $4.

This book is the first in a series intended to serve the mathematical needs of students from Grades VIII to XI, and results from an experimental program carried on at Ball State Teachers' College, Muncie, Indiana. The experimental program was initiated in 1955, with most of the materials being tested at the Ball State Laboratory School. However, as the experiment broadened in scope, several other high schools in Indiana became involved. Other books in the series include ALGEBRA I (Grade IX), GEOMETRY (Grade X), and ALGEBRA II (Grade XI). The latter text is still in press, but should be available shortly.

The content of Introduction to Mathematics represents a sharp contrast with conventional topics taught at the Grade VIII level. Unit I, entitled "Symbols and Numerals", includes the history of numeration systems and a study of the base 10 system, as well as systems to other bases. Unit II is entitled "Rational Numbers". Some of the very simple concepts of the real number system are discussed in the four chapters of Unit III. Unit IV, Algebra, begins with a discussion of simple notions regarding sets. These notions form the basis for the presentation of algebraic concepts. Operations with negative numbers are discussed and used in the solution of story problems. If the very small section on trigonometric relations is disregarded, the geometry content of Unit V approximates that in the present Grade IX mathematics course in Alberta. The approach is somewhat different though, in that this text uses point-set notions in defining geometric figures. For example, a line segment is defined as a set of points consisting of two points, A and B, and all the points between these points.

The conventional topics of taxation, insurance, percent, and percent as applied to business, are not included in this text. There is no reference to the construction and reading of various types of graphs (bar, line and circle). Though there are sections for drill on basic computation with whole numbers and common and decimal fractions, these sections arise from a consideration of types of number systems and operations in these systems rather than from strict computational point of view. This procedure is in line with this statement of the authors in the preface of the teachers' manual: "You will notice that this text is an idea-oriented one rather than an answer-oriented one". The authors feel that the material in the text would constitute a very heavy course; a statement in the teachers' manual outlines a minimum course and a suggested time schedule to finish this minimum course in 34 weeks. Apparently, practice has indicated that this minimum course is still too heavy for slow students because the publishers have put out a mimeographed brochure outlining a course less difficult than the minimum course mentioned in the teachers' manual.

The teachers' manual and answer book outlines the objectives for each chapter and contains suggestions. Some answers are included. A more detailed guide book would be an invaluable aid, especially to the teacher who is handling this course for the first time.

There is less emphasis on social applications of mathematical concepts, especially percent. There is more emphasis on mathematical content - numeration systems, number systems, informal geometry. As concepts are presented, their application to inequalities as well as to equations is discussed. The notion of sets alluded to by many authors as an important unifying concept in mathematics is introduced and used as a basis for developing simple algebraic and geometric concepts. For those who are sympathetic to these broad outlines for curriculum change, this book merits consideration as a suitable text.

Alberta teachers will be interested in knowing that Introduction to Mathematics (British Columbia Edition) will be the only text authorize at the Grade VIII level in the Province of British Columbia beginning in the school year, 1962-63. Aside from the provision of more review exercises at the close of each chapter, the British Columbia Edition is not significantly different from the original publication.

MCATA NOTES

1. Membership

The term for membership in the MCATA coincides with the school year, September 1 to August 31. New members paying their fees after May 1, 1962, and participants in the MCATA Seminar who paid a $5 registration fee will be members in good standing until August 31, 1963.

2. Second Annual MCATA Conference

The MCATA conference will be held July 11-13, 1962 in the Mathematics-Physics Building of the University of Alberta. Members will register for the conference in MP132. The main conference room is MP126. A registration fee of $1 will be collected at the time of registration. Programs will be mailed to members before the end of June.

The following is a brief outline of the sessions:

	Wednesday, July 11
Morning	Registration
	Addresses of Welcome
	"Structure, Problem Solving and Stomach Thinking" - Eric MacPherson, UBC Discussion Groups
Afternoon	"Elementary Arithmetic 1961-62: A Year of Preparation" - N. M. Purvis, associate director of Curriculum
	"Programmed Learning" - J. A. McDonald, ATA Past President
	Thursday, July 12
Morning	"Open Sentences and Closed Minds" - Eric MacPherson Discussion Groups Answers by Mr. MacPherson to groups' questions

Afternoon "The Experimental Program in Grade VIII" - R. Plaxton, Viscount Bennett Junior and Senior High School, Calgary

"Programming for Computers" - R. S. Julius, U. of A.

Evening Business Meeting (Agricultural Building, Room 345) - Regional councils, fees, elections, and future activities.

Friday, July 13

Morning "The Role of the Statistician" - Dr. J. R. McGregor, U. of A.

"The Emerging Mathematics Curriculum" - A. Evenson, General Supervisor of Senior High Schools, Edmonton

Afternoon "I Come No More to Make You Laugh" - Eric MacPherson

Closing Remarks - President J. M. Cherniwchan

Adjournment

Instead of a question period following the first two addresses of our guest consultant, Eric MacPherson, a different approach will be used. Immediately following his address the conference will break up into small groups and discuss his talk. Questions from groups will be submitted to Mr. MacPherson to be answered by him at a special session provided on the second day of the conference.

Coffee will be provided during the breaks in Room MP132. A display of recent textbooks, and a selection of books for a mathematics library will appear in this room.

3. MCATA Elementary Mathematics Seminar, Alberta College, July 3-10

Further to the report in the April Newsletter, the following may be of interest.

Our original plan was to provide for a workshop consisting of from 50 to 100 teachers. The interest shown in the proposed seminar has been most gratifying. The April MCATA executive meeting in Calgary authorized the seminar planning committee to accept the first 100 applications as they were received, and to accept the others on the basis of geographical distribution, sponsorship, and date of receipt of application. The committee regrets that, due to accommodation limitations and difficulties in arranging for extra staff at this late date, a limit of 150 participants in this seminar had to be set. It is hoped that a workshop of this nature may be organized again another summer.

Arrangements have been made for two guest consultants, experts in the elementary mathematics field.

Raymond Cleveland, supervisor of mathematics for Rahway Public Schools, (New Jersey) has for several years been acting as a consultant for Scott, Foresman and Company on the Seeing Through Arithmetic series and is classified as a group author of this series. A recipient of several fellowships from the National Science Foundation, Mr. Cleveland has a very special interest in the teaching of mathematics. A field editor of mathematics for Scott, Foresman and Company, Merrill B. Hill has been responsible for organizing and directing many mathematics workshops in the United States. He has made a study of projects of the School Mathematics Study Group and other experimental groups.

Mr. Cleveland will be offering in the mornings a course in modern mathematics at the University of Alberta and will be available to the seminar in the afternoons only. Mr. Hill will be available both for morning and afternoon sessions.

N. M. Purvis, associate director of Curriculum, Department of Education; Mrs. J. Oldham, Holyrood School; Mrs. M. Palmeter, Inglewood School; Mr. R. Quail, principal, Scott Robertson School, all of Edmonton, have consented to help out with the group sessions at the seminar. It is hoped that arrangements can be made for additional people that have had extensive experience with the STA series to help out at the seminar.

The following is the outline of what is proposed at the seminar. There may be minor changes; the seminar planning committee has not had an opportunity to make a detailed study of it as yet.

Tuesday, July 3 - Background Information: History of arithmetics to present materials, Influence of psychological learning theory, Current trends

Wednesday, July 4 - Early Foundations: One-to-one correspondence and meaning of number; Recognition of numerousness, number sense; Grouping; Ordinal use of number; Grouping by tens, tallying, place value; Measurement ideas; Actions with sets of objects; Problem Solving - systematic approach; Difference between basic facts and computation

Thursday, July 5 - Computation: Basic facts; Grouping and regrouping; Numeration; Experience prerequisite to computation; Develop computation with representative example; Addition, subtraction, multiplication; Division; Fractions and decimals; Generalizations

Friday, July 6 - Problem Solving: Use of the equation; Additive and Subtractive type problems; Multiplicative type of problems; Quatative division problems; Partitive division problems; Comparison type problems; Systematic approach from the very beginning

Monday, July 9 - Problem Solving: Problems involving equal rates; Problems involving comparisons; Percent; Times as many; Fractions as components in rate problems; Multiple step problem; Averages; Area of rectangles

Tuesday, July 10 - Techniques for Using the Materials: Visual approach to learning; Individual Differences

4. The Newsletter

This will be the last newsletter until fall. Any suggestions that would improve the newsletter would be of help to the editorial committee. Constructive criticisms will be appreciated. Destructive criticisms will be received but not appreciated.

WE WILL SEE YOU AT THE SECOND ANNUAL CONFERENCE

University of Alberta, July 11-13, 1962

Editor - J. M. Cherniwchan, 276 Evergreen Street, Sherwood Park