THE ALBERTA TEACHERS' ASSOCIATION

MATHEMATICS COUNCIL

NEWSLETTER

Volume X, Number I, December, 1970

The Current Status of High School Calculus

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An article by Daly[1] outlines the development of mathematics curricula in Alberta. He reports that in 1934, calculus was included as a part of the Grade XII course in analytical geometry. In 1951, he continues, "Calculus and analytic geometry were discontinued, but trigonometry was included as a separate three-credit course."[2]

During the 17-year period following 1951, therefore, it was not possible for Alberta students to take a course in calculus at the Grade XII level. Two choices were available to them. They could take Mathematics 30 only, which was essentially an academic five-credit course in algebra, plane geometry and combinatorics, or they could take Mathematics 30 and Mathematics 31. The latter course was a course in trigonometry, carrying three credits. Most students who wanted to be admitted to a sciences program at a university in Alberta would elect both Mathematics 30 and Mathematics 31, as the former was a compulsory requirement, and the latter was "strongly recommended" by the universities.[3]

Although changes in textbook authorizations between 1951 and 1968 wrought a modernization of the approach to teaching Mathematics 30 and 31, the essential content of the courses remained substantially unchanged.

in this issue

During 1967 and 1968, reports circulated which indicated that the present high school mathematics courses had been examined, evaluated and were to be revised. By February, 1968, the Senior High School Curriculum Committee of the Alberta Department of Education had accepted *in principle* the recommendation that separate sequences of high school mathematics courses be adopted for students who were university-bound, non-university-bound, and business-oriented.[4] For the approximately 35 percent of Alberta high school students who intended to go to university, the sequence consisted of Mathematics 10 (Grade X), Mathematics 20 (Grade XI), and Mathematics 30 and 31 (Grade XII).

The Curriculum Guide for Mathematics 31, which became prescriptive in September, 1968, outlined the revised Mathematics 31 course:

> Mathematics 31 is made up of two parts: (i) Trigonometry, and (ii) Calculus OR Vector AND Matrices *(sic)*[5]

Each of the two parts was to consist of 80 periods (the periods 40 minutes in length). The choice as to which alternative to offer during the second half was left to the teacher or to the school, although some guidelines were given to assist in making the choice.[6] For the calculus alternative, the text *Calculus* by H. A. Elliott and others[7] was prescribed.

So far as the writer has been able to ascertain, only one Math 31 class in Calgary public schools was taking the non-calculus alternative during the 1968-69 school year.

One assumes that the recent inclusion of calculus as a Grade XII level course in Alberta has been precipitated by similar action in the United States. The increase in offerings of high school calculus in that country dates back to 1959 when the recommendations of the Commission on Mathematics of the College Entrance Examination Board were published. The Commission held that calculus is a college-level subject, and high schools should prepare students to begin calculus when they enter college; nevertheless, it recommended that well-staffed high schools offer their ablest students a year of college-level calculus and analytic geometry.[8] Short courses in calculus at the end of Grade XII were not recommended.[9] Woodby[10] concurred with this recommendation in 1965.

During the next few years, calculus was offered in more and more American high schools. Many of the students who completed it successfully were able to receive college credit through the Advanced Placement Program.

The Cambridge Conference on School Mathematics published its report in 1963.[11] Two topical outlines were suggested for Grades VII-XII Mathematics. One included a logically complete course in calculus in Grades XI and XII, after the limit concept is understood. The second introduced calculus briefly in Grade IX and then returned to a complete and rigorous study in Grades XI and XII. Justification for both approaches was given.[12]

There has been general agreement that calculus *can* be learned by Grade XII students.[13] Many authorities quote Bruner:[14]

> ... any subject can be taught to anybody at any age in some form that is honest.

Moise[15] puts it this way:

> ... a student who can ever learn calculus can learn it in the 12th grade.

But the controversy over whether or not calculus *should* be taught to Grade XII students has continued.

Ferguson[16] believes that many more schools should offer calculus than do now, but that certain prior conditions must first of all be met. The school must have a curriculum in which elementary and intermediate algebra, plan and solid geometry, trigonometry, and coordinate geometry are completed by the end of Grade XII. A college-level teacher must be available, students must be willing to spend eight to 10 hours per week on calculus homework, and must be adequately motivated. Ferguson cites figures from his own school to show that its program has been most successful.

Others would agree with Ferguson's assessment. Moise[17] writes as follows:

> Many schools teach calculus in the 12th grade; and the experience of the last few years shows that this is quite workable if qualified teachers are available. But the *if* is important.

Blank[18] advocates a full year of essentially intuitive calculus in Grade XII, giving the student insight into certain proofs without detailing them. Some of his reasons for favoring calculus are worth restating here:

1. Calculus is close to the spirit of secondary school mathematics in its manipulative and problem-solving aspects.
2. The concepts, methods and skills developed earlier in the high school mathematics program have applications to calculus.
3. Calculus provides the broadest possible base for students' later development.
4. Calculus plays a profound role in the intellectual and practical affairs of men.
5. Teachers feel more qualified in calculus than elsewhere, as present teacher education makes their having taken calculus more likely than any of the alternatives.

While the above endorsements of a Grade XII calculus course are not made without some reservations, the reader should be aware that there is another side to the coin.

Beninati[19] favors a Grade XII course in probability and statistical inference or analytical geometry over calculus. He contends that a calculus program offered by inadequately prepared teachers may lead to subsequent failure in college calculus courses, due to the student's inadequate preparation and his meaningless manipulation of symbols.

Allendoerfer[20] maintains that both the secondary schools and the colleges are heading for serious trouble in their teaching of calculus. While calculus is an essential part of a mathematical education, its importance in and relation to algebra, geometry and other subjects has been over-emphasized. Calculus is being taught too early to students who should be studying other mathematical topics such as theory of polynomials, rational functions, algebraic functions, trigonometric functions, inverse trigonometric functions, exponential functions, logarithmic functions, and a full treatment of analytic geometry.

The only conditions under which Allendoerfer would endorse a Grade XII calculus course are stringent:

1. The topics (listed above) must have been covered *prior* to Grade XII.
2. The teacher must be superior.
3. The course must emphasize an intuitive treatment of the limit concept and infinite series prior to calculus as such.
4. Integration, approached through sums of infinite series, must be taught before differentiation.

Brown[21] reported the results of a survey of 49 mathematics department heads at colleges and universities in Ohio. Thirty-two questionnaires were returned. To the question "For students who have already taken two years algebra, one year plane and solid geometry and a semester of trigonometry, what should be the emphasis for the remaining semester?", only one respondent (three percent) favored calculus. The topics favored over calculus were Analytic Geometry (53 percent), Probability and Statistics (16 percent), Function Concepts, Mathematical Structure and Analytic Geometry (nine percent), and Analytical Trigonometry and College Algebra (six percent).

When asked whether calculus should be included in the college-preparatory program, 69 percent responded "no", 25 percent answered "yes" (but in most cases there were conditions attached), and six percent were uncertain. Similar percentages were reported for answers to the question: "Should a short introduction to calculus be given during the senior year of high school?".

Buchanan[22] reported a wider survey. Two hundred thirty-three questionnaires were distributed to college and university mathematics department heads throughout the United States. They were asked to rank seven topics for Grade XII in order of their preference, responding to the questions in accordance with the consensus of opinion of their staff members. Seventy-three percent of the questionnaires were returned. Calculus ranked sixth in order of preference, with only 18 percent choosing it first or second. A unit on limits was ranked above calculus on every questionnaire for which calculus was not ranked first.

A number of researchers have studied the effects of various high school courses on performance in college calculus courses.

Chaney[23] found that University of Kansas students who had studied limits of sequences, limits of functions and continuity in Grade XII performed significantly better in first-year university calculus than students who had only studied analytical geometry.

Tillotson[24] studied 192 students in first-year calculus and analytic geometry at the University of Kansas. He found no significant difference in achievement between students who had taken an introductory course in calculus in Grade XII and those who had not.

McKillip[25] used the multiple regression technique to predict students' scores in the first year of college calculus at the University of Virginia. Scores predicted were derived on the presupposition that students had had no calculus in high school. Actual scores were subsequently compared with the predicted scores. Scores of students who had one to two semesters of calculus were not significantly different from predicted scores, whereas students who had taken two or more semesters of calculus in Grade XII attained scores that were significantly better than those predicted. McKillip's conclusion is that a Grade XII calculus course of less than two semesters must be justified on other grounds.

Maclay[26] concluded that non-accelerated students should not be placed in a high school calculus course, even when their pre-training has been in a new mathematics program. Specifically, the SMSG High School Mathematics Program did not provide adequate background for Advanced Placement in calculus.

Wick[27] found no significant differences in preparation for first-year college calculus as between SMSG and traditional high school programs. He noted a low but positive correlation between Grade XII mathematics average and success in first-year calculus. The conclusions were based on a study of 1962 students in six colleges and universities in Minnesota and Wisconsin.

Coon[28] found that 60 SMSG students were superior to 122 conventional students in university calculus. The difference was significant at the .05 level for success and at the .01 level for achievement in a first-year course. However, he urges caution in interpreting the results of his study attributing the differences to "continued mathematical maturity based on different high-school preparation, not (necessarily) to learning in calculus."

Anderson and Whittemore[29] found that students whose high school mathematics courses were 'modern' scored significantly better than students with a 'traditional' background, on a pre-university mathematics test (American College Testing Program: Mathematics [ACT]). The correlation between high school grade in modern mathematics and grade point averages in university algebra and trigonometry and university calculus was found to be significant, while that between traditional high school mathematics score and the above GPA's was not found to be significant.

Prouse and Turner[30] used the technique of multiple regression analysis to determine what combinations of 15 independent high school and university variables contributed to success in second-year university calculus. The high school variables which contributed most highly to success in Calculus 2 were Plane Geometry, Trigonometry, Algebra 2, rank in graduating class, Algebra 1 and ACT Mathematics standard score.

Woodby's conclusions and recommendations[31] provide the best possible summary of the opinions and findings reviewed above.

CONCLUSIONS

1. The individual teacher is the most important factor in the development of a strong mathematics program.

2. There is a lack of agreement on the mathematics that should be taught in Grade XII courses for college-bound students.

3. Acceleration and enrichment have generally accompanied the development of a strong mathematics program.

4. Concern for calculus prevails.

5. There is little acceptance of a course in probability and statistics as the fourth- or fifth-year mathematics offering in the college preparatory program. There is even less acceptance of courses in linear algebra, matrices and computer mathematics.

6. Analytic Geometry as a separate course has achieved only slight popularity.

7. It has been demonstrated that college-level calculus courses can be successfully taught at the high school level provided the following conditions are met: (a) that there are enough capable students; (b) that the teacher is adequately prepared.

RECOMMENDATIONS

1. Experimentation with various Grade XII courses in mathematics should continue.

2. Colleges and universities should provide guidance in the development of Grade XII programs.

3. Courses in calculus and analytic geometry of the warm-up variety are not recommended.

4. Grade XII courses in probability and statistics, linear algebra, or analytic geometry should be considered in preference to calculus for many high school students.

FOOTNOTES

[1]Daly, Richard R., "Alberta School Mathematics, 1867-1967", *Annual*. The Mathematics Council of The Alberta Teachers' Association, 1967, pp.4-14.

[2]*Ibid*, p.13.

[3]See, for example, The University of Calgary Calendar 1969-70, p.51.

[4]*MCATA Newsletter*, 7:2 (February 1968), p.9.

[5]Province of Alberta, Department of Education, *Curriculum Guide for Mathematics 31*. Edmonton: Department of Education, Province of Alberta, September 1968, p.2.

[6]*Ibid*, p.3.

[7]Elliott, H. A., Kenneth D. Fryer, James C. Gardner, and Norman J. Hill, *Calculus*. Toronto: Holt, Rinehart and Winston of Canada Ltd., 1966.

[8]Commission on Mathematics, *Program for College Preparatory Mathematics*. New York: College Entrance Examination Board, 1959, p.14.

[9] *Ibid*, p.15.

[10] Woodby, Lauren G., *Emerging Twelfth-Grade Mathematics Programs*. U.S. Department of Health, Education and Welfare, Office of Education, Washington, D.C., 1965, p.36.

[11] Cambridge Conference on School Mathematics, *Goals for School Mathematics*. Boston: Houghton Mifflin Co., 1963.

[12] *Ibid*, p.49.

[13] For example, see Woodby, *op. cit.*, p.35.

[14] Bruner, Jerome S., "On Learning Mathematics", *The Mathematics Teacher*, 53:8 (December 1960), p.610.

[15] Moise, Edwin, "The New Mathematics Programs", *The School Review*, 70 (Spring 1962), p.88.

[16] Ferguson, W. Eugene, "Calculus in the High School", *The Mathematics Teacher*, 53:6 (October 1960), pp.451-3.

[17] Moise, *op. cit.*, p.86.

[18] Blank, A.A. "Remarks on the Teaching of Calculus in the Secondary School", *The Mathematics Teacher*, 53:7 (November 1960), pp.537ff.

[19] Beninati, Albert, "It's Time to Take a Closer Look at High School Calculus", *The Mathematics Teacher*, 59:1 (January 1966), pp.29-30.

[20] Allendoerfer, Carl B., "The Case Against Calculus", *The Mathematics Teacher*, 56:7 (November 1963), pp.482ff.

[21] Brown, Robert S., "Survey of Ohio College Opinions with Reference to High School Mathematics Programs", *The Mathematics Teacher*, 56:4 (April 1963), pp.245ff.

[22] Buchanan, O. Lexton, Jr., "Opinions of College Teachers of Mathematics Regarding Content of the Twelfth Year Course in Mathematics", *The Mathematics Teacher*, 58:3 (March 1965), pp.223ff.

[23] Chaney, G.L., "The Effect of Formal Study of the Mathematical Concept of Limit in High School on Achievement in a First Course in University Calculus", *Dissertation Abstracts*, Vol. 28, p.2884A.

[24] Tillotson, Donald Bearse, "The Relationship of an Introductory Study in High School to Achievement in a University Calculus Course", *Dissertation Abstracts*, Vol. 24, p.577.

[25] McKillip, William D., "The Effects of High School Calculus on Students' First-Semester Grades at the University of Virginia", *The Mathematics Teacher*, 59:5 (May 1966), pp.470ff.

[26] Maclay, Charles W., Jr., "The Influence of Two Prerequisite Programs on Achievement in the High School Advanced Placement Calculus Course", *Dissertation Abstracts*, Vol. 29, pp.3917-8A.

[27] Wick, Marshall E., "A Study of the Factors Associated with Success in First-Year College Mathematics", *The Mathematics Teacher*, 58:7 (November 1965), pp.642ff.

[28] Coon, Lewis Hulbert, "S.M.S.G. Mathematics as a Factor Influencing Success in Freshman Calculus", *Dissertation Abstracts*, Vol. 25, pp.4475.

[29] Anderson, James, and Whittemore, Robert G., "Predictive Utility of Certain Criteria for Advanced Freshman Mathematics Courses", *The Mathematics Teacher*, 60:6 (October 1967), pp.619ff.

[30] Prouse, Howard, and Turner, V. Dean, "Factors Contributing to Success in Calculus 2", *Journal of Educational Research*, 62:10 (July-August 1969), pp.439ff.

[31] Woodby, *op. cit.*, pp.34-36.