THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

delta-k

(Formerly Mathematics Council Newsletter)

VOLUME X, NUMBER 3, MAY 1971

About Our New Name

In November, the MCATA executive gave the Editor permission to change the name of the "MCATA Newsletter" to something more appropriate for a journal of this type. It was felt that the publication was no longer a newsletter in the general sense of that word and that, therefore, it should not be called such.

The name "Delta - K" was chosen. Delta, as you probably know, is the fourth letter of the Greek alphabet (Δ), and is used in Mathematics to represent an increment, or increase. K stands for knowledge: knowledge of mathematics, knowledge of teaching mathematics, knowledge of new methods and developments in our discipline. Thus "Delta-K" represents the increase in knowledge which the journal constantly strives to produce.

We welcome reader reaction to the new name.

In This Issue

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From the Editor's Desk

MATHEMATICS MATERIALS

1. *Catalog of Lesson Aids*, available from B.C. Teachers' Federation Lesson Aids Service, 105 - 2235 Burrard Street, Vancouver 9, contains several items of interest to mathematics teachers, at very low cost. We suggest that you order this catalog.

2. *Mathematics Materials,* available from Western Educational Activities Ltd., 10577 - 97 Street, Edmonton 17, is an excellent source of books, games and other aids. The prices are competitive.

3. FREE, upon request to MCATA members, $1.00 to non-members: *An Active Learning Unit on Real Numbers,* by Dale N. Fisher - 158 pages of activities especially designed to cover all of the mathematical concepts outlined in the unit "Introduction to Real Numbers" in the Program of Studies for junior high schools of Alberta. This document was prepared especially for junior high, but both elementary and senior high school mathematics teachers will find some of the activities appropriate. Address your request to Mr. T. F. Rieger, The Alberta Teachers' Association, 11010 - 142 Street, Edmonton 50.

*   *   *

A PROBLEM

The Editor is interested in the properties of the "number" i^i. Is it real? Complex? What about i^{-i}? Which of the two is larger? Perhaps some mathematician who receives this publication can shed some light on the questions suggested. The Editor will welcome papers on the subject.

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Readers are again reminded that letters to the Editor are welcome anytime.

*   *   *

Your President, Mr. Jim Kean, attended the NCTM Annual Meeting in Anaheim, California, on your behalf, during the Easter vacation. We hope to include a report of that meeting in the next issue of "Delta-K".

Report on the NCTM Winnipeg Meeting

October 15-16-17, 1970

submitted by *Edwin R. Olsen*
CAMT Representative

Although all the sessions at the Winnipeg meeting were good and deserve mention, one session that many classroom mathematics teachers might be interested in dealt with the laboratory approach to teaching mathematics.

Dr. Jack Wilkinson from the University of Northern Iowa gave a brief description of "The Laboratory Method as a Teaching Strategy in Grades V to VIII." In his talk he stressed that there is no single teaching approach which can be successfully used in teaching mathematics and that the use of the laboratory approach is no exception.

In using this approach, Dr. Wilkinson set up a number of shoe boxes with instructions and materials necessary to carry out the instructions. In most cases the students had to record the results of their activity on a prepared record sheet or in their notes. For some labs the students worked individually, and for others they worked in pairs or in small groups.

Regarding the success of this approach, Dr. Wilkinson stressed that it should be used in conjunction with other teaching methods and techniques. From his own experience he found that students weak in mathematics tended to like the laboratory approach better than the lecture method, but that a few of the good students didn't seem to like it as well.

On the whole, the laboratory approach to mathematics as described by Dr. Wilkinson seemed to be an exciting experience well worth the effort used to produce the labs. Why not try to set up one or two shoe boxes (or other boxes) of materials and see if you like the approach?

Contemporary Mathematics and its Mathematicians

William J. Bruce (Editor)
Department of Mathematics
Universtiy of Alberta

Volume 39, 1970, of *Mathematical Reviews*, published by The American Mathematical Society, indicates that some 95 general subject classifications are being used for the many areas of mathematics. In this volume, there are approximately 900 different subject listings. This relatively small sample hardly does justice to the vastness of the field of contemporary mathematics.

Back in the year 1941, Richard Courant and Herbert Robbins first published their book *What is Mathematics?** After some attempt in their introduction to answer this profound question, they finally concluded that "For scholars and laymen alike it is not philosophy but active experience in mathematics itself that alone can answer the question: what is mathematics?"

For this article we selected a very small subset of the many areas of mathematics and asked various members of the Department of Mathematics, University of Alberta, to participate. Specifically, each professor was asked to identify an area of interest and specialization by briefly indicating what it is all about. He was then asked to name some of the outstanding contemporary mathematicians in his area, to tell where they are located today, and to state what aspect of their speciality they are investigating at this time. The following contributions have been received with thanks.

THEORY OF NUMBERS

Harvey L. Abbott

Number theory is the study of the properties of the positive integers. Because of their importance in counting, the positive integers were among the first objects of a mathematical nature to be considered by ancient civilizations. Since questions about numbers appeal to natural human curiosity, it is not surprising that number theory has a long history. For example, early Chinese mathematicians considered various questions concerning divisibility properties of integers, some of them of a decidedly non-trivial nature. One of the questions which they raised was whether the condition that n divides $2^n - 2$ implies that n is a prime. While they were not able to answer this question, it should be pointed out that it was not until the 18th century that the great German mathematician C.F. Gauss proved such an integer n is not necessarily a prime, and that the problem of finding all composite integers n which divide $2^n - 2$ is still unsolved. The ancient Greeks, noted mainly for their contributions to geometry, also investigated properties of numbers. Euclid was aware of the fact that there exist infinitely many primes, and in his "Elements" he also discusses the so-called perfect numbers; a number n, such as 6, 28, or 496, is said to be

*R. Courant and H. Robbins, *What is Mathematics?* Oxford University Press, Seventh Printing, 1956.

perfect if the sum of its divisors is 2n . It is still an unsolved problem as to whether there are infinitely many such numbers or, for that matter, whether there are any at all which are odd.

Number theory advanced very little during the dark ages. It can be said that modern number theory had its beginning during the 17th century with the work of the French mathematician Pierre Fermat. Fermat proved a large number of very interesting and often surprising results about the positive integers. For example, every prime, which leaves a remainder of 1 on divisions by 4, can be written in a unique manner as a sum of two squares. When one considers that there is no simple way of deciding whether a number is a prime or not (excepting the obvious but impractical method of finding all of its divisors) it becomes clear that this theorem of Fermat is remarkable. Practically all famous mathematicians of the 17th, 18th, and 19th centuries contributed to the theory of numbers.

During the past 100 years, number theory has developed in many directions. Perhaps the major advance has been the development of the analytic theory of numbers. This is not so much an "area" of number theory but a method of attack. Analytic number theory is characterized by the use of the calculus of functions of a complex variable. The first advances were made by Riemann in Germany, and Hadamard and de la vallée Poussin in France. Other pioneering work was done by Hardy and Littlewood in England and by Vinogradov in Russia. These and others have developed analytic tools very systematically, and now one can give answers to questions which would have been hopelessly difficult a century ago.

The additive theory of numbers is concerned with questions of the following type:

Given two sequences of positive integers $a_1 < a_2 < \dots$ and $b_1 < b_2 < \dots$, what can be said about the set of numbers which are of the form $a_i + b_j$?

While some questions in additive number theory have been in existence for a long time, most of the results have been obtained during the past 40 years. The most active and prolific mathematician in this area is P. Erdös of Hungary.

Finally, consider the geometry of numbers which is concerned with the following basic question: How many lattice points (points with integers for coordinates) are there in a given region in the plane or in a space of a higher number of dimensions? This is a surprisingly difficult question, and even for such simple regions as the interior of the circle with equation $x^2 + y^2 = n$ the complete answer is unknown.

Like all other branches of mathematics, number theory is growing at a very fast rate. This will continue, because while there is a great deal now known about positive integers, there are still many things which we do not know.

SET THEORY

Henry F. J. Lowig

Set theory is a mathematical discipline which is due to George Cantor, a German mathematician (1845 - 1918). Cantor's most important achievement is the discovery that the number of elements of an infinite set can be defined in a similar way as the number of elements of a finite set. Two finite sets have the same number of elements if, and only if, there is a one-to-one correspondence between them. Cantor extended this criterion of "having the same number of elements" to infinite sets. According to Cantor, any two sets have, by definition, the same number of elements if, and only if, there is a one-to-one correspondence between them. The number of elements of a set is called its "cardinal number" or "cardinality". A set which is in one-to-one correspondence with the set $\{1,2,3, \ldots\}$ of positive integers is called <u>countable.</u> This implies that any two countable sets are of the same cardinality. The set of odd positive integers and the set of all rational numbers are examples of countable sets.

On the other hand, Cantor proved that the set of all real numbers is uncountable. His argument runs essentially as follows. Assume that we have an enumeration of all real numbers x such that $0 < x \leq 1$, namely,

$$\begin{array}{l} 0.\ a_{11}\ a_{12}\ a_{13} \cdots \\ 0.\ a_{21}\ a_{22}\ a_{23} \cdots \\ 0.\ a_{31}\ a_{32}\ a_{33} \cdots \\ \qquad \cdots \end{array}$$

Here it is supposed that each of these numbers is written as a non-terminating decimal. For example, 1/4 would be written as 0.24999 ... , not as 0.25 or 0.25000 ... . Let, for $n = 1,2,3,\ldots$,

$$a_n = \begin{cases} 9 & \text{if } a_{nn} \neq 9 \\ 1 & \text{if } a_{nn} = 9 . \end{cases}$$

Then $0.\ a_1 a_2 a_3 \ldots$ is a real number > 0 and ≤ 1 which does not occur in the above enumeration, contrary to our assumption. Hence such an enumeration cannot exist, or the set of all real numbers x such that $0 < x \leq 1$ is uncountable. Now it is easily shown that the set of all real numbers (without restriction) is also uncountable.

Therefore, the cardinality of the set of all real numbers is different from the cardinality of the set of all positive integers; so there exist different infinite cardinal numbers just as there exist different finite cardinal numbers.

Another part of set theory which is very important for its applications is the theory of <u>well-ordered</u> sets. An ordering of a set is called a well-ordering if, under this ordering, every non-empty subset of the given set has a smallest (or first) element. For example, the set of positive integers is well-ordered by its natural ordering. The same set can be well-ordered as follows:

$$2,\ 4,\ 6,\ 8,\ \ldots;\ 1,\ 3,\ 5,\ 7,\ \ldots$$

(that is, all even positive integers precede all odd positive integers, while the even positive integers as well as the odd positive integers are left in their natural order). On the other hand, the usual ordering of the set of all non-negative real numbers is not a well-ordering because, for example, there is no smallest positive real number.

Nowadays, set theory is used in practically all branches of mathematics. A few mathematicians who are currently working in abstract (or pure) set theory are J. W. Addison, University of California at Berkeley; G. Fodor, Szeged, Hungary; K. Hrbáček, Charles University of Prague, Czechoslovakia; K. Kuratowski, Warsaw, Poland; E. C. Milner, University of Calgary, Alberta; J. Mycielski, University of Colorado, U.S.A.

GROUP THEORY

Ronald D. Bercov

The rotations about a point in the plane can be regarded as a mathematical system in which we obtain a third rotation, $R_1 o R_2$, from two given ones, R_1 and R_2, by following the first by the second. For instance, if R_1 and R_2 are 15° and 30° counterclockwise rotations, then $R_1 o R_2$ is a 45° counterclockwise rotation. In this mathematical system we have for any three rotations that $R_1 o(R_2 o R_3)$ is the same as $(R_1 o R_2) o R_3$ (associative law); the 0° rotation E plays a role analogous to that of the number zero for addition, namely R o E = R for all rotations R (identity element); and for each rotation R, the rotation -R, which has the same magnitude and is opposite in sense, satisfies R o (-R) = E (inverse). Such an associative system with an identity and inverses is called a group.

Groups first became important as a means of describing geometric systems. For instance, any rotation about its center takes a circle onto itself, but only 0°, 90°, 180° and 270° rotations take a square onto itself, and only 0°, 120°, and 240° rotations take an equilateral triangle onto itself. Thus we can identify these geometric objects by knowing which rotations take them onto themselves. We say that a geometric object is described by its "symmetry group".

Similarly, physical systems can be described by symmetry groups. Those transformations which leave the system unchanged give a description of the system. In this way group theory has become important for modern theoretical physics. Indeed, the group concept was used recently by the Nobel prize-winning American physicist Murray Gell-Mann, whose work has contributed to the classification of the subatomic particles, of which there are a large number. The group-theoretic approach even indicated the existence of an at that time unknown particle which was then found experimentally.

The mathematician studies groups by breaking them up into "simple" factors much as large whole numbers are broken up into their prime factors. New such simple groups have been found in the last few years by, among others, a Czech, Janko, then in Australia, now in the United States; a Canadian, Higman, also in the United States; an American, Hall; and an Englishman, Conway. Major discoveries had been made in the five to ten previous years by a Frenchman, Chevalley; a Japanese, Suzuki, and a Canadian, Steinberg, both in the United States; and a Korean, Rhee, in Canada. There is now hope that almost all such finite simple groups have been found. This hope has been bolstered by the outstanding work of two young Americans, Feit, now of Yale, and Thompson, now at Cambridge in England, who have proved that there are no new such groups with an odd number of elements. The international character of modern mathematics is well illustrated by the above partial list of distinguished group theorists. Another name prominently associated with these discoveries is that of Richard Brauer, who came to the University of Toronto as a refugee from Nazi Germany and who is now at Harvard University. Much recent progress in the theory of groups has been based on methods developed by Brauer.

ORDINARY DIFFERENTIAL EQUATIONS

Jack W. Macki

Ordinary differential equations were of great interest in the 18th and 19th centuries because the many laws of physics are easily written in terms of these equations.

For example, Newton's law, $f = ma$, becomes $f(y(t),t) = m\frac{d^2y}{dt}$ if $y(t)$ describes the position of a particle in a force field, f .

Interest in such equations declined in the early part of this century, but since about 1940 there has been a tremendous increase in interest in ordinary differential equations. At the University of Alberta, work in ordinary differential equations centers on several different areas as follows:

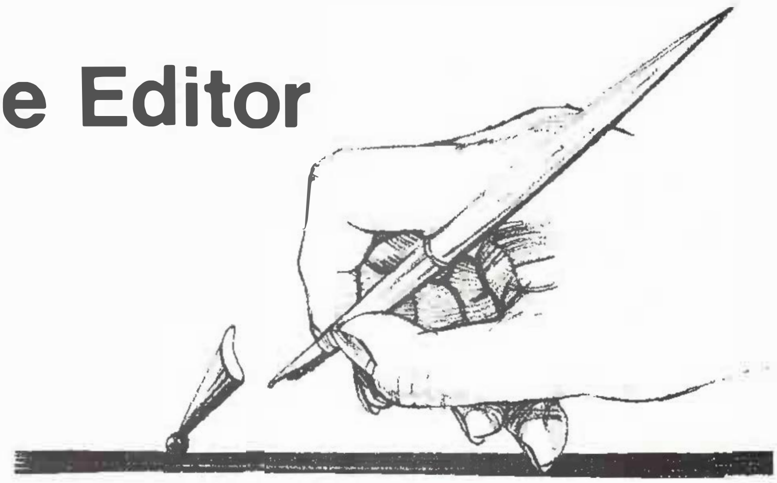
(1) Suppose that a particle is subjected to a periodic (repeating) force. Will its motion be periodic? In more mathematical terms, if $q(t + 2\pi) = q(t)$ for all t, when are solutions of $y''(t) + q(t)y(t) = 0$ periodic?

(2) How can one determine the behavior for large time of solutions of differential equations? A typical result is the following: If $q(t)$ is an increasing function of t, then solutions $y(t)$ of the equation $y''(t) + q(t)y(t) = 0$ oscillate (that is, have infinitely many zeros) and are decreasing in amplitude. Questions like this are important because most of the interesting differential equations that occur in applied areas cannot be solved exactly. Our task is to get information about the solutions, even though we cannot determine the solutions exactly.

Many talented people are working on ordinary differential equations today. Probably one of the best in the world is F. V. Atkinson, University of Toronto. In the United States, there are very strong groups located in the universities of Maryland, Minnesota, and Wisconsin. Among the most eminent men in the field in the United States are Philip Hartman at Johns Hopkins University, Einar Hille at the University of New Mexico, W. T. Reid at the University of Oklahoma, and Lamberto Cesari at the University of Michigan. European schools are generally weak in ordinary differential equations. The notable exception is the Italian school centered in Florence. This school is led by Robert Conti. There are many strong groups researching differential equations in the Soviet Union. Notable among the workers there are the famous blind mathematician L. S. Pontryagin, in Moscow, and Bogolinboff, Krylov, and Mitropolsky of the outstanding Ukrainian school.

Pages 5 to 9 (inclus.) of this publication contain the first part of Dr. William J. Bruce's edited article "Contemporary Mathematics and its Mathematicians". WATCH FOR THE SECOND HALF OF THIS ARTICLE IN THE NEXT ISSUE.

Letters to the Editor



Editor's Note: The February, 1971, issue of the *Newsletter* contained a letter from "Curious Reader", Calgary, inquiring about the policies of the Universities of Calgary, Lethbridge and Alberta with respect to admitting students who present the Mathematics 13, 23, 33 sequence. We referred the inquiry to the registrars of the three universities and received the following replies:

Dear Mr. Falk:

This will acknowledge receipt of your letter of January 25, 1971, regarding our policy on admitting students who present Mathematics 13, 23 and 33.

The University of Alberta will not consider accepting Math 33 as a matriculation subject until a departmental examination is given for this course. The Faculties of Physical Education and Education will accept Math 33 as the non-examination subject option provided Math 30 is not also presented.

I trust this is the information you were seeking, and if there is any further assistance I can provide, I should be happy to do so.

Sincerely yours,

W. A. D. Burns,
Assistant Registrar and
Admissions Officer
The University of Alberta

Dear Mr. Falk:

Re: Policy Statement of the University of Lethbridge re Mathematics 13, 23 & 33

Mathematics 33 may fulfil partial admission requirements to the University of Lethbridge as indicated under "c" below. Mathematics 33 does not serve as an official pre-requisite for any particular mathematics course offered by the University of Lethbridge, though as in all course offerings at this institution, if the student, after seeking advice from the department concerned, feels that he has sufficient background to handle the university course or courses, he may be permitted to enrol in them notwithstanding the fact that he does not have the recommended pre-requisite.

Admission Requirements:

Students seeking admission from Alberta high schools must present

a. a High School Diploma,
b. four 30 level Department of Education Examination subjects including English 30 with an overall average of 60% and a minimum mark of 50% in any subject, AND
c. a fifth subject which may be either a 30-level Department of Education Examination subject or a five-credit Grade XII non-examination subject with a minimum mark of 50%.

We hope this policy statement will clarify the situation for you.

Yours sincerely,

J. D. OVIATT
Registrar
University of Lethbridge

Dear Mr. Falk:

RE: Mathematics '33'

Dr. Stewart, Registrar and Academic Secretary, has asked that I send you a detailed explanation concerning The University of Calgary's position with regard to Mathematics '33'.

In April 1970, one of our assistant deans in the Faculty of Arts and Science wrote a letter which was easily misinterpreted to indicate that The University of Calgary would accept Mathematics '33' in lieu of Mathematics '30'. Unfortunately many of us were not aware such a letter had been sent. In discussion with this gentleman after that date, I can recall distinctly that he had felt acceptance would be given provided the course met with our standards and provided any student registered in Mathematics 13 or 23 could easily transfer to Mathematics 20 or 30 without any foreseeable difficulties. Nevertheless, the letter was misinterpreted and numerous students were encouraged to take Mathematics '33'.

On October 5, 1970, Dr. R. W. Wright, Dean of Arts and Science, wrote a letter to the Calgary School Board in which he indicated that ONLY the Faculty of Arts and Science would be willing to accept Mathematics '33' in lieu of Mathematics '30' for entrance provided, in addition, the applicant presented four Departmental Examination subjects. The average on all five would have to be 60% with no mark below 50% in any one subject. It was pointed out clearly in this letter that acceptance was for the 1971-72 Winter Session only.

All other faculties on this campus have agreed to accept Mathematics '33' as a Non-Departmental Examination subject only and not in lieu of Mathematics '30', where required.

On January 15, 1971, a letter was sent, by the Vice-Dean of Arts and Science, to the Department of Education indicating that unless certain conditions were met, The University of Calgary was not prepared to accept Mathematics '33' after the 1971-72 Session. I am not at liberty to divulge, at this time, these conditions. However, to this date no changes have been forthcoming.

I, therefore, reiterate that The University of Calgary can only indicate a policy of acceptance for 1971-72. The Registrar and myself apologize for the errors and misconceptions our Faculty of Arts and Science has caused. We can only hope that in the near future decisions are made which will be to the satisfaction of all parties concerned.

If I can be of any further assistance, please do not hesitate to contact me.

Yours sincerely

G. J. Krivy
Admissions Officer
The University of Calgary

Mathematics teachers in high schools might be well advised to refer these statements of policy to their department heads and to their guidance counsellors. (The Editor)