THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

delta-k

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"New Math" Sparks Lively Debate

Four eminent Canadian scholars fought to a draw in a lively battle over the merits of "New Math" that highlighted the annual convention of the Saskatchewan Mathematics Teachers' Society (SMTS) in Regina on May 14-15.

The distinguished combatants were *Professor Ralph A. Staal* of the Department of Pure Mathematics, University of Waterloo; *Professor A. D. Booth*, Dean of Engineering, University of Saskatchewan, Saskatoon; *Professor Roger Servranckx*, Department of Mathematics, University of Saskatchewan, Saskatoon and *Professor James Beamer*, Department of Curriculum Studies, College of Education, Regina.

Dr. Staal, in his keynote address to the conference, conceded that "New Math" had produced valuable fall-out in the '60s, notably the development of the inquiry approach at the elementary level and some first-class writing on teaching techniques. But he claimed these gains were outweighed by disadvantages in the "New Math" approach, including a decline in rigor and discipline, an over-emphasis on notation and the neglect of performance in "New Math's" overthrow of rote learning in favor of understanding.

In This Issue

Dr. Staal conceded that "New Math", 10 years after its conception, "is still alive and well but a little wiser," but the "ultimate verdict is in favor of its opponents". He concluded that "New Math" had suffered "transplant rejection" from its proper environment at the university level to lower educational altitudes.

The most unequivocal opponent of "New Math" on the SMTS panel proved to be Dr. Booth, who asserted that "new math will never be useful to society". As an engineer who had "to apply my math", Dr. Booth championed classical mathematics. Claiming that the present secondary school curriculum does not produce engineering students who can either "read, write or do math", he declared himself as a "missionary who wants to remove Christianity in the form of new math from the curriculum". Dr. Booth said the amount of time spent on math studies in Canadian high schools was "hopelessly inadequate". Whereas Swiss and British students spend 1,400 hours on math, Americans devote only about 1,000 hours and Canadians 840 hours, just above the "banana republic level". He agreed with Dr. Staal that "New Math" had produced too much icing and not enough cake and a "nibbling" at math that resulted in "an absence of any necessity of real thinking". He criticized current math texts for posing math problems in such a way as to make "the answers immediately obvious". "The whole of mathematics is being taught at much too superficial a level", Dr. Booth concluded. "There is not enough discipline to achieve any respectable mathematics. New Math involves too many words, exactly what mathematics is not."

Dr. Booth's stubborn defense of Euclid later produced a brilliant blackboard battle with Dr. Roger Servranckx who defended "new math" procedures for bringing enhanced "clarity and simplicity to classical problems". However, he joined Dr. Booth's assault on the "textbook racket", claiming that new texts dealt with slogans but left content unchanged. The Belgian-trained mathematician claimed that "new math" was actually a myth, in that most math dates from 1850. "There is really no new math to remove from the curriculum." He agreed with Dr. Staal's contention that modern math procedures had involved themselves too much with notation, but that this was not the fault of the procedures, but the manner of using them.

The fourth panelist, Professor James Beamer, also recognized that "New Math" had not achieved its objectives but he disagreed with contentions that "it has failed to improve the school curriculum". He quoted recent studies that showed "significant gains in math ability" because of "New Math".

Professor Beamer said that the secondary school math curriculum must strive to serve more than university-bound students. He suggested a multi-level math curriculum which would include options that would serve the abilities of top students as well as a lab-oriented program for low achievers.

The stalemate produced by the panel discussion was summed up later by Harold Leibel, vice-principal of Regina Central Collegiate, who observed that "the experts can't agree if we have New Math, and if we have it, what it is". Mr. Leibel's remarks were made in a progress report on the Division IV math curriculum revision committee, as an example of the difficulties facing that committee.

Reprinted from the *Saskatchewan Bulletin*, June 1, 1971, a publication of the STF.

What is CAMT?

H. J. Promhouse
President, CAMT

CAMT is an *association of associations*. Any individual who is a member of a provincial mathematics society is automatically a member of CAMT (Canadian Association of Mathematics Teachers), providing that the provincial society is itself a member of the national association.

HISTORY

The Canadian Teachers' Federation (CTF - all provincial associations are members) at its General Meeting (GM) in 1966 passed the following resolution:

> Be it resolved that the Canadian Teachers' Federation sponsor a Canadian council of teachers of mathematics.

As a result of this motion, a CTF mathematics teachers conference was held in Ottawa March 16-18, 1967. At this conference, the following resolution was passed:

> Be it resolved that this mathematics teachers' conference establish a Canadian Association of Mathematics Teachers as provided by the Canadian Teachers' Federation GM resolution of 1966 and in accordance with the guidelines established by the CTF.

A planning committee of four was elected and given the following duties:

1. to prepare a constitution
2. to carry out all organization affairs
3. to arrange or approve a national conference in 1968
4. to issue at least one newsletter before June 30, 1968.

By December, 1967, the planning committee had achieved all its objectives: the proposed constitution was completed in June, 1967; the first newsletter was distributed in December, 1967, and a national conference was organized and held in Ottawa on December 8-9, 1967.

FUNCTIONS

The functions of the CAMT are:

1. to provide a means of communication among provincial mathematics teachers' organizations such as the Saskatchewan Mathematics Teachers' Society (SMTS) and the Mathematics Council, ATA.

2. to coordinate common activities
3. to provide effective liaison with the Canadian Mathematics Congress (CMC - this organization is concerned chiefly with international conferences on education)
4. to provide effective liaison and contact with international organizations.

To facilitate these functions, a national newsletter is issued, provincial journals are exchanged, correspondence is carried on with the councillors from each province, annual meetings and conventions are planned, and representatives from CTF, CMC and NCTM are invited to executive and annual meetings.

CAMT has developed a close relationship with several other organizations, such as CTF, NCTM and CMC. At present CTF provides CAMT with secretarial aid and makes provision for a member of its executive staff to be an ex-officio member of the CAMT executive. (Dr. George Richert is presently the ex-officio member.) CTF works through our executive to get mathematics teachers to represent Canada at international conferences. In August 1969, *two Canadian representatives attended the first International Congress on Mathematical Education* in Lyons, France. In 1972, a second international conference will be held in England.

NCTM is kind enough to pay the expenses of a representative (Dr. Joan Kirkpatrick) as a liaison officer at all CAMT general meetings and conferences.

The CMC has maintained a very close liaison with CAMT. Dr. Dulmage, chairman of the CMC educational committee has taken a continued interest in our association from its initial organization, to the present. He has been present at all CAMT meetings, including executive meetings, and has proven to be a tower of strength to the executive.

The CMC through Dr. Dulmage asked CAMT to nominate a member for each of the three committees set up by CMC. The names of the committees and CAMT's representatives on them are:

National Mathematics Contest Committee Summer Institutes for Teachers Committee	- Rev. J. Egsgard, Toronto
Pre-Calculus Mathematics Committee	- Doug Polvin, Montreal

It is also CAMT's purpose to provide a means of communication among mathematics teachers' organizations in the provinces and to coordinate any common activities. Heretofore, most communication has been north-south through the NCTM. Now, a means of east-west communication exists through our newsletter which goes out to every member of a provincial mathematics organization. Our first conference, held at the time of our founding meeting in December, 1967, brought together for the first time representatives from the provincial Departments of Education and from provincial teacher organizations. The New Mathematics Curricula for the provinces were described and common problems were discussed. Our second conference was held in Toronto April 17-18, 1970 and representatives of the provinces described their use of Educational Television (ETV) in the mathematics classroom. This conference gave those committed to the use of ETV the opportunity to learn of new developments. Those undecided about ETV were able to gain knowledge which will be useful in making decisions about the future use of ETV as a method of instruction. The conference, attended by 150 mathematics educators from across Canada, was a decided success.

CAMT's only financial support now comes from its individual provincial mathematics organizations. We are presently exploring the possibilities of other means of obtaining financial assistance.

CAMT, though still in its infancy, has proved itself a necessary and worthwhile national organization. The support of the provincial organizations is required for it to continue to grow and provide much greater service and leadership in the field of mathematics to all Canadian teachers of mathematics.

From the Editor's Desk

* Here is one answer to the oft-heard question, "*What is Math Council doing?*":

At the regular meeting of the MCATA Executive on February 27, 1971, a decision was made to submit the following recommendations to the Executive Council of the ATA:

(a) That the High School and University Matriculation Examinations Board be requested to provide for Mathematics 31 an "open book" examination of the kind provided for the 30 level science examinations.

(b) That the committee on Articulation of High School and University Programs be requested to seek acceptance by the University of Calgary of Mathematics 33 in lieu of Math 30 as an entrance requirement to the Faculty of Arts and Science for students entering a non-mathematics science area of study.

Each recommendation was accompanied by a supporting statement which elaborated on the recommendation, and the reasons for which that recommendation was being made.

* Thanks to L. R. Adler of Fort MacLeod and G. Hanson of Edmonton for their letters. Mr. Hanson answered the questions raised in the last issue of *Delta-K* about the "numbers" i^i and i^{-i}: both are real, and $i^{-i} > i^i$. (The Editor will mail a copy of Mr. Hanson's proofs to any reader requesting them.)

* If you have not already done so, you should request a copy of *An Active Learning Unit on Real Numbers* by Dale N. Fisher: FREE to members, $1 to non-members. Write to

T. F. Reiger
The Alberta Teachers' Association
11010 - 142 Street
Edmonton 50.

Contemporary Mathematics and its Mathematicians

Dr. William J. Bruce, Editor
Department of Mathematics
University of Alberta

Here follows the conclusion of the article started in the last issue of *Delta-K* (Vol.X, No.3, May, 1971).

PARTIAL DIFFERENTIAL EQUATIONS

Walter Allegretto

The branch of mathematics called Partial Differential Equations deals, as its title indicates, with the study of equations involving a function of more than one variable and several of its derivatives. The main problem usually considered is to find solutions of these equations which satisfy specific conditions which, in turn, depend on the special problem under consideration. Often, one also investigates the properties of the solutions of such equations, without actually finding them.

Partial differential equations arise in almost every branch of modern science. Fluid flow, heat flow, wave motion, atomic physics, relativity theory, electromagnetic waves, quantum mechanics, and reaction rates are but a few of the fields which extensively use partial differential equations.

Together with the above-mentioned applications, in recent years, elegant new theories have been developed. Now the subject also forms an abstract branch of mathematics somewhat removed, at the present time, from physical problems.

Since the subject is so vast, the names of only very few of the mathematicians working in this area can be mentioned. Many omissions are clearly unavoidable. From the list of notable names emerge Browder, Bers, Protter, Courant and John, all of whom work in North America; Sobolev and Miklin in the USSR; Lions, Stampacchia and Hörmander in Western Europe; and Agmon in Israel.

GENERAL TOPOLOGY

Richard L. McKinney

The word "topology" is derived from the Greek word "τοπος" meaning "place" or "space". Thus, it is a rudimentary form of geometry which analyzes the fundamental properties of very general spaces. Some isolated problems of a topological nature were considered long ago by such famous mathematicians as Descartes (1640), Euler (1736) and Gauss (1794). The proper birth of the subject grew out of the work of Weierstrass in the 1860s in which he investigated the foundations of the calculus. Cantor's development of the theory of point sets a decade later was instrumental in determining the direction of general topology during its rapid growth in the present century. The adjective "general" is used to distinguish this point set topology from combinatorial or algebraic topology which started in the 1890s with some remarkable work of Poincaré.

As a consequence of its general nature, topology is a unifying branch of mathematics which has applications in many seemingly strange quarters. A brief perusal of the current crop of mathematical research journals will dispel any doubt that topology is now one of the most active fields of mathematical activity A few of the important twentieth century contributors to topology are Brouwer of Holland, Banach of Poland, Alexandroff of Russia and Moore of the United States. An excellent introductory survey of topology is available in the paperback ***First Concepts of Topology*** by W. G. Chinn and N. E. Steenrod, published in the New Mathematical Library series of Random House.

APPROXIMATION THEORY

Amram Meir

The best-known classical result of approximation theory is Weierstrass' Theorem which states that any continuous function on a bounded closed interval can be uniformly approximated by polynomials.

Since this theorem was proved, approximation theory has developed in several directions, had interactions with different other fields of mathematics and become a very fruitful and wide area of research. Important new research is being carried out at present in a number of mathematical centers.

The general problems usually can be formulated as follows:

1. Given a normed linear vector space and a certain subspace of it, how well, by what method and under what conditions can an element in the space be approximated by an element from the subspace?

2. Given a functional on a function space, how well can it be approximated by other functionals of a certain type?

To the first type of results belong all those statements which claim the approximability (in some sense) of a function of polynomials, spline functions, trigonometric functions, solutions of differential equations, and so forth. To the second type belong the quadrature formulae (integral approximation).

Some of the outstanding men who are doing important and active research in approximation theory today are A. F. Timan of the USSR; G. Freud and P. Turán at the Academy of Science in Hungary; G. Birkhoff at Harvard University; W. Cheney and G. G. Lorentz in Texas; A. Sard and D. J. Newman in New York; T. J. Schoenberg in Wisconsin, and J. L. Walsh in Maryland.

FUNCTIONAL ANALYSIS

John L. B. Gamlen and Sherman D. Riemenschneider

Functional analysis is an abstract branch of analysis which began earlier in this century when a new and very powerful approach to analysis succeeded in solving some difficult long-standing problems. Present day functional analysis is, broadly speaking, an extension of the same methods into many different mathematical situations. The methods have obtained many striking new insights, and have also simplified much of the classical analysis.

We illustrate the method with two general examples. In electric field theory, the action of certain charges is described by a function (the potential function). It is known that this function satisfies a differential equation (Poisson's equation). Very frequently, the main concrete information one has about the potential function is that it solves such an equation (its form being unknown). Solution of the equation produces the function, but for a long time mathematicians were unable to show that the equation must always have a solution. The classical methods dealt with detailed properties of individual functions, whereas the new method considers a collection of functions, and attempts to reason from the properties of the collection as a whole. This is the so-called global approach. The use of the new methods enabled mathematicians to solve the problem.

To illustrate further the global point of view, we consider the property of continuity for a function. Many functions have this property and many do not. The collection of all functions that have the continuity property is a linear space (the sum of two continuous functions is a continuous function, and a number times a continuous function yields a continuous function). The global point of view studies the linear space as a whole, and asks: What properties do all the continuous functions have in common, and can these properties be expressed in useful terms? It turns out that two continuous functions have a "distance" between them (with the same properties as the distance between real numbers), and many of the common properties of continuous functions can be expressed in terms of this distance.

This global approach to continuous functions leads to the study of other linear spaces of functions supplied with a distance or norm (a special distance). Among the linear spaces of functions are those composed of functions (called operators) that map one linear space into another. The study of these more general linear spaces and the operators defined on them yields practical and useful results including the problem that we posed earlier.

Thus, the functional analyst begins with "concrete" examples, performs an abstraction process with a global and linear point of view, to obtain a general picture of things that aid in the study of particular examples. Functional analysis has progressed rapidly and the abstraction process has been reiterated so that portions of the field no longer resemble concrete problems at all, but the usefulness is still there.

As one may suspect, the repeated abstractions lead to an immensely diverse field with many specializations and, consequently, to many outstanding individuals. As a sampling of these people and where they are located, we present the following list (which is by no means complete): Granirer, University of British Columbia; Gelfand, Naimark and M. Krein, USSR; L. Schwartz, J. Dieudonné and G. Choquel, France; L. Hörmander, Sweden; M. H. Stone, University of Massachusetts; J. T. Schwartz, New York University; S. Kahutani, Yale; and F. Browder, University of Chicago.

GENERAL RELATIVITY

Werner Israel

Einstein's theory of gravitation, after the initial burst of excitement around 1920, lay dormant for a long time because of a paucity of contacts with observation and with other branches of science. All this has changed in the last decade, with the discovery of quasars, radio galaxies, X-ray sources and pulsars, Weber's report of gravitational waves coming from the center of our galaxy, and direct observational evidence that our universe began as a highly compressed "primeval fireball". We now realize that regions of very intense gravity play a fundamental role in the universe, and that only by applying Einstein's theory in consort with nearly every other branch of theoretical physics can we hope to unravel their mysteries.

Probably the foremost relativist in the world today is Roger Penrose in London. He has made profound and highly original contributions to the mathematics of the theory. In the United States, the three leading groups are at Maryland under Charles Misner, studying the early history of the universe; the California Institute of Technology under Kip Thorne, studying gravitational fields of highly compressed bodies; and Princeton under John Wheeler, studying "black holes" and the formulation of a quantum theory of gravity. Mention must also be made of Zeldovich in Moscow, whose deep physical intuition has illuminated virtually every aspect of relativistic astrophysics.

PROBABILITY AND STATISTICS

E. S. Keeping

The mathematical theory of probability builds on intuitive notions of "chance" and "odds", derived from experience with horseracing, coin-tossing, playing with dice or cards, and so on, and formalizes these notions into a logical abstract theory. This theory in turn has many useful applications in such fields as insurance, engineering and quality control.

Mathematical statistics is based on probability and deals with the kinds of numerical inference that can be drawn from observations of the real world. Such inferences, or estimates, are never absolutely certain, but often decisions have to be made on this basis. Statistics, apart from the purely routine manipulation of data, is largely concerned with estimating the risks involved in making various types of decisions.

Among the great names in probability are A. Kolmogorov in Russia and W. Feller in the United States. The leading statistician, until his death in 1962, was Sir Ronald Fisher in England. Prominent still are H. Cramer in Sweden and J. Neyman in the USA.

THEORETICAL MECHANICS

Robert J. Tait

Mechanics seeks to describe and account for the motion of material bodies. In this discipline, one of the oldest branches of applied mathematics, the mathematician is interested in the formulation of its general principles, the development of its logical structure, and the efficiency and accuracy of its particular predictions. Along with philosophers, physicists and engineers, he is interested in deepening its foundations and widening its boundaries.

Classical mechanics, based on the "laws" of Newton, forms the foundation of much of physics, and has been the starting point of many subjects in pure mathematics. Its uses today range from designing bridges to arranging return trips to the moon.

If one applies the principles of classical mechanics to particles moving with very high speed or of very small size, one encounters conceptual and experimental difficulties. From these have stemmed the modern theories of relativity and quantum mechanics which are mathematically and physically more formal. In principle, these subjects, to which classical mechanics can be thought of as an approximation, are of universal application, and have raised questions of both a logical and computational nature.

On the other hand, if we deal with "ordinary" solids or fluids, Newtonian mechanics has been extended to formulate equations describing the behavior of such substances. We are then led to the study of continuum mechanics which includes elasticity, plasticity, fluid flow, aerodynamics, and related topics. A topic of present interest in this regard is rheology, the study of flowing materials such as lava or "Silly Putty". Among the large body of research workers in this area should be mentioned Truesdell and coworkers in North America and A. E. Green in Europe.

To indicate the mathematical subjects involved in discussions of these topics, and which have to some extent been influenced by them, one might list ordinary and partial differential equations, integral equations, calculus of variations, differential geometry, tensor calculus, probability and statistics, group theory, and topology.

New areas connected with mechanics are still opening up. To illustrate this, one can refer to S. Smale, working in differential topology, who has been examining the restricted three-body problem. As an example, consider the earth, moon, and a small satellite, and assume that all three travel in the same plane, and that the center of mass remains fixed. It is desired to find a mathematical model for this situation, one which is stable in the sense that small changes in the variables do not affect the essential character of the motion. Not all stability questions of this nature have been answered for the collection of models proposed.

Success Stories

NORTHEASTERN ALBERTA

For mathematics teachers in northeastern Alberta, this has been a great year. Within a period of one year, they have been able to organize and form an active regional council of MCATA.

Beginning last spring, a small group of enthusiastic teachers was inspired into considering the formation of a regional math council. Realizing the magnitude of the job, they proceeded. This nucleus of St. Paul math teachers developed an interesting program which evolved around the *BLAST OFF* theme, and sent out the information calling an inaugural meeting on January 16, 1971. Despite the inclement weather, the turnout was relatively good. The success of the meeting can be attributed partially to the group and greatly to George Cathcart from the U of A. The elected executive got to work immediately, and within two months, they had distributed a film to their membership, followed by a workshop held at the Regional School in St. Paul.

The workshop, which had as its theme *Get Set for the '70s*, was conducted by W. Bober, the math coordinator for the Edmonton Separate Schools. He pointed out and demonstrated some ways of effectively organizing a math class. The afternoon workshop was divided into the three major teaching levels.

At the end of the afternoon, the success of the institute was demonstrated by the enthusiasm expressed by the teachers before they returned to their schools. If discussions with math teachers from the regional membership is any indication, then future response should be extremely good.

Next year's activities are already in the planning stage, and the executive is showing much enthusiasm in mapping their future course.

Perhaps the one main drawback in the regional council is the lack of operating capital. This may be overcome if somehow there were more means of raising money at the local and provincial levels for this type of professional activity.

Eugene Katerenchuk

NORTH CENTRAL ALBERTA

Forty enthusiastic mathematics teachers met in the Jubilee Junior High School in Edson on March 27 for a day of mathematics. The teachers were from various schools in the Yellowhead and Lac Ste. Anne locals. They covered all grades from I to XII.

In his address - *Trends in Mathematics in the '70s* - Dr. Tom Kieren of the U of A gave particular attention to individualized instruction, process objectives, activity experiences in mathematics, and the use of the environment.

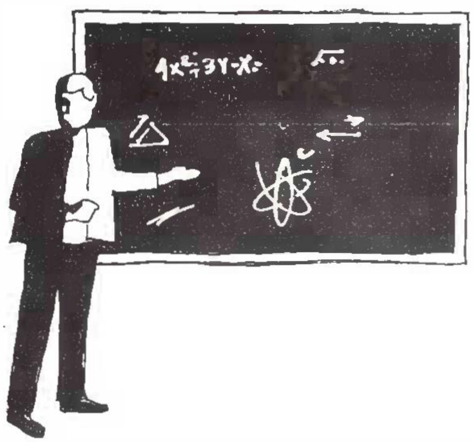
The afternoon was devoted to workshop sessions, one conducted by Dr. Kieren for the junior high and high schools and the other by Dr. Joan Kirkpatrick, also of the University of Alberta, at the elementary level.

The teachers at the meeting wished to continue such mathematical activity and formed a regional council of the Mathematics Council, ATA. They named it the North Central Math Council and elected the following officers:

President	*Denis Baudin*, Edson
Vice-President	*Sonya MacMillan*, Hinton
Past President	*A. O. Jorgenson*, Chairman of the Yellowhead Local PD Committee
Secretary-Treasurer	*Glen Kauffman*, Edson
Director for the Yellowhead Local	*Kay Griffith*
Director for Lac Ste. Anne Local	*Hillary Taylor*

Mr. Jorgenson and his PD Committee were responsible for the organization of this well-received program.

T. F. Rieger



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