THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

# delta-k

(Formerly Mathematics Council Newsletter)

VOLUME XI, NUMBER 1, NOVEMBER, 1971

# Curb your tongue, math teacher!

*David R. Johnson*, President
Wisconsin Mathematics Council

Many educational leaders claim that mathematics teachers talk too much in the classroom, that students will learn more if we, the teachers of mathematics, talk less. This may be true in part, but I hardly believe that talking less will in itself improve the learning of mathematics to any great extent in most classrooms. Should not the concern be directed toward the "what and how" of our speaking as well as the quantity of our speaking in the classroom? I believe so! To illustrate my point, let's look at a few examples from a typical high school algebra course.

How does the algebra student read the symbol $-x$? Common responses are "negative x", "minus x", "the opposite of x" or "the additive inverse of x". But are all these responses meaningful? No! In fact, the first two responses are misleading, if not incorrect. We are quite careful in the mathematics classroom to name a real number less than 0 (or to the left of 0 on the real number line) a *negative number*. Students quite easily grasp the meaning of the phrase "negative number". But all of a sudden we bring out the expression $-x$ and read it negative x! Trouble begins. Students immediately assume that this symbol stands for a number less than zero simply because its verbal name contains the word "negative".

## In This Issue

If you don't believe the verbal name (negative x) is a misleading name, I suggest that you write the symbol $-x$ on the board and ask your students how it relates to zero on the number line. You will hear a resounding cry, "To the left of zero!" Some students never fully understand that, if x is a real number, $-x$ could be positive, negative or zero and that more information is needed before a decision can be made. Students are so overpowered by the use of the word "negative" and the definition of a negative number that they cannot appreciate the meaning of the expression $-x$. We could have avoided the entire misunderstanding by reading the phrase $-x$ as the "additive inverse or opposite of x", for most students in a modern math program today appreciate the fact that additive inverses do not have to be negative. But the trouble with this phrase $-x$ is not over!

Before long, in the same classroom, we introduce the definition of absolute value. Though there are many homespun ways of defining the concept of absolute value, some are confusing and often incorrect. For example: The absolute value can be found by dropping off the sign. That idea is deadly. If b is less than zero, then $|b|$ in this case equals $-b$. No sign was chopped off here; in fact, one was added. When it comes to the definition of words such as absolute value, it is necessary to use a mathematically sound definition:

$a \in R$

If $a = o$, $|a| = o$

If $a > o$, $|a| = a$

If $a < o$, $|a| = -a$

This definition, however, demands real understanding. First, the student must know the size of the real number in relationship to zero. Secondly, he must understand that $-x$ is simply a symbol for the "inverse of x" and obeys the property of trichotomy. That is, $(-x)$ could be positive, negative, or zero. If, for example, you ask a student to define the absolute value of $-b$ where b is less than zero, it follows that the $|-b|$ equals $-b$. But for the student who believes that you must drop a negative sign to take the absolute value of a number, or for the student who does not appreciate that $-x$ is really a positive number in this case, he will not be able to apply the definition of absolute value to this expression correctly.

Talking less will not add understanding, but speaking correctly with meaningful phrases will aid in the understanding of the concept. Try a few of the examples with your students. I predict they will do well if they have a good understanding of the definition of absolute value and if they understand that the symbol $-x$ represents the inverse of x.

Simplify:

a. $|-b^3|$, if $b < o$

b. $|-3 - x|$, if $x > o$

c. $|-b| + |-b|$, if $b < o$

d. $|-3b|$, if $b < o$

(By the way, I believe we should introduce expressions with variables when teaching the concepts of absolute value in the first year algebra course. Using only constants may lead to poor techniques in simplifying absolute value expressions. That is, students may be able to get the correct answer and yet never realize that they do not understand the definition.)

Let's examine another area in the classroom in which we as mathematics teachers might curb our tongues, some classic statements used to motivate students.

*"HERE'S AN EASY QUESTION"*

How often have you used that statement! At first glance, it appears to be a very innocent comment. In fact, it might be considered a confidence builder for some students. But wait a minute! For what students does this comment build confidence? If the student answers the question, you have already told him it was an *easy* question and therefore answering the question is not too much of an accomplishment. Even worse, however, is when the student cannot answer the question. You have pretty much told him then that he is dumb. After all, he missed not only a question, but an *easy* question. "Curb your tongue, math teacher!"

Just one more example, a painful one with which to close any article but, I believe, important enough to include.

"Class, you must learn this technique because it will be on the test on Friday."

Is that why we want students to learn a new concept - for our tests? Does that phrase offer meaningful motivation for students to learn? If this is the only reason for learning a skill or concept in our mathematics class, we should consider one of two things:

- Do not teach the concept.
- Give a sound reason for learning the concept. The reason may be none other than it is a concept needed in solving more complex problems at some future time in the course.

Students are continually challenging the testing in many courses. Making statements like the above will only kindle their attack!

It is a new year - a great year to begin to "curb our tongues" - a great year to speak less - to speak with more meaning and to speak emphatically. The result - a classroom atmosphere that promotes real learning.

Reprinted from *Wisconsin Teacher of Mathematics*, Vol. XXII, No. 1, Winter, 1971.