THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

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(Formerly Mathematics Council Newsletter)

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Curb your tongue, math teacher!

David R. Johnson, President
Wisconsin Mathematics Council

Many educational leaders claim that mathematics teachers talk too much in the classroom, that students will learn more if we, the teachers of mathematics, talk less. This may be true in part, but I hardly believe that talking less will in itself improve the learning of mathematics to any great extent in most classrooms. Should not the concern be directed toward the "what and how" of our speaking as well as the quantity of our speaking in the classroom? I believe so! To illustrate my point, let's look at a few examples from a typical high school algebra course.

How does the algebra student read the symbol $-x$? Common responses are "negative x", "minus x", "the opposite of x" or "the additive inverse of x". But are all these responses meaningful? No! In fact, the first two responses are misleading, if not incorrect. We are quite careful in the mathematics classroom to name a real number less than 0 (or to the left of 0 on the real number line) a *negative number*. Students quite easily grasp the meaning of the phrase "negative number". But all of a sudden we bring out the expression $-x$ and read it negative x! Trouble begins. Students immediately assume that this symbol stands for a number less than zero simply because its verbal name contains the word "negative".

In This Issue

If you don't believe the verbal name (negative x) is a misleading name, I suggest that you write the symbol $-x$ on the board and ask your students how it relates to zero on the number line. You will hear a resounding cry, "To the left of zero!" Some students never fully understand that, if x is a real number, $-x$ could be positive, negative or zero and that more information is needed before a decision can be made. Students are so overpowered by the use of the word "negative" and the definition of a negative number that they cannot appreciate the meaning of the expression $-x$. We could have avoided the entire misunderstanding by reading the phrase $-x$ as the "additive inverse or opposite of x", for most students in a modern math program today appreciate the fact that additive inverses do not have to be negative. But the trouble with this phrase $-x$ is not over!

Before long, in the same classroom, we introduce the definition of absolute value. Though there are many homespun ways of defining the concept of absolute value, some are confusing and often incorrect. For example: The absolute value can be found by dropping off the sign. That idea is deadly. If b is less than zero, then $|b|$ in this case equals $-b$. No sign was chopped off here; in fact, one was added. When it comes to the definition of words such as absolute value, it is necessary to use a mathematically sound definition:

$a \varepsilon R$

If $a = o$, $|a| = o$

If $a > o$, $|a| = a$

If $a < o$, $|a| = -a$

This definition, however, demands real understanding. First, the student must know the size of the real number in relationship to zero. Secondly, he must understand that $-x$ is simply a symbol for the "inverse of x" and obeys the property of trichotomy. That is, $(-x)$ could be positive, negative, or zero. If, for example, you ask a student to define the absolute value of $-b$ where b is less than zero, it follows that the $|-b|$ equals $-b$. But for the student who believes that you must drop a negative sign to take the absolute value of a number, or for the student who does not appreciate that $-x$ is really a positive number in this case, he will not be able to apply the definition of absolute value to this expression correctly.

Talking less will not add understanding, but speaking correctly with meaningful phrases will aid in the understanding of the concept. Try a few of the examples with your students. I predict they will do well if they have a good understanding of the definition of absolute value and if they understand that the symbol $-x$ represents the inverse of x.

Simplify:

a. $|-b^3|$, if $b < o$

b. $|-3 - x|$, if $x > o$

c. $|-b| + |-b|$, if $b < o$

d. $|-3b|$, if $b < o$

(By the way, I believe we should introduce expressions with variables when teaching the concepts of absolute value in the first year algebra course. Using only constants may lead to poor techniques in simplifying absolute value expressions. That is, students may be able to get the correct answer and yet never realize that they do not understand the definition.)

Let's examine another area in the classroom in which we as mathematics teachers might curb our tongues, some classic statements used to motivate students.

"HERE'S AN EASY QUESTION"

How often have you used that statement! At first glance, it appears to be a very innocent comment. In fact, it might be considered a confidence builder for some students. But wait a minute! For what students does this comment build confidence? If the student answers the question, you have already told him it was an *easy* question and therefore answering the question is not too much of an accomplishment. Even worse, however, is when the student cannot answer the question. You have pretty much told him then that he is dumb. After all, he missed not only a question, but an *easy* question. "Curb your tongue, math teacher!"

Just one more example, a painful one with which to close any article but, I believe, important enough to include.

"Class, you must learn this technique because it will be on the test on Friday."

Is that why we want students to learn a new concept - for our tests? Does that phrase offer meaningful motivation for students to learn? If this is the only reason for learning a skill or concept in our mathematics class, we should consider one of two things:

- Do not teach the concept.
- Give a sound reason for learning the concept. The reason may be none other than it is a concept needed in solving more complex problems at some future time in the course.

Students are continually challenging the testing in many courses. Making statements like the above will only kindle their attack!

It is a new year - a great year to begin to "curb our tongues" - a great year to speak less - to speak with more meaning and to speak emphatically. The result - a classroom atmosphere that promotes real learning.

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From the Editor's Desk

Murray R. Falk

Most of us complain that the summer was too short, and we drag out all the old clichés which we pass on to each other this time of the year. However, I suspect that most of us are glad to get back into the classroom. We have hopes and aspirations for the coming school year which will eliminate those mistakes and failures of last year.

It seems to us that this is one of the important reasons for the existence of MCATA: it provides us with a vehicle whereby we can keep fresh in our thinking; it helps us to come together and get those ideas that others have, which we can use to the advantage of our own students.

Naturally, everything we encounter is not for us. Many of these things are very idealistic and without much practical application at the moment. But we need to keep ideals ahead of us so that we are always reaching and striving to improve mathematics education. We need to be thankful for those few who are able to take the idealists' concepts and put them to practical use. That is what helps us advance. That is how we grow professionally.

We take this opportunity to wish our colleagues the best year ever.

A little problem . . .

Ask students who know how to use variables to evaluate this expression in less than three minutes:

$$\frac{987654321}{987654322^2 - 987654321 \times 987654323}$$

Answer: 987654321

Hint: Let X = 987654321.

MCATA President 1971-72

Dick Daly

Richard Daly (B.Ed. 1966, M.Ed. 1971, University of Alberta), the MCATA President for 1971-72, is a very experienced and involved teacher - he has taught math and science to Grades VII, VIII and IX in Lamont and in the following Edmonton schools: D. S. McKenzie (2 years), Allendale (1 year) and McDougall (2 years). He is now teaching at Capilano Elementary.

Dick has always been active in mathematics organizations - he was a mathematics coordinator for four years, a member of the Junior High Curriculum Committee for four years, a member of the Secondary School Mathematics Curriculum Committee for three years, and Editor of the Math Council *Annual* in 1967.

He is presently an Elementary Curricular Associate for Math and Science with the Edmonton Public School Board.

He says, "I would have Math Council strive for a more unified math program throughout Alberta, provide more service for rural members, form more regional councils, press for another NCTM meeting in Alberta, promote the teaching of mathematics at all levels, and become a strong voice in curriculum decisions at the provincial level."

We are very fortunate to have a man of Dick's experience and dedication at the helm of the Math Council for the up-coming year.

What time is it?

To the second, at what times are the second hand, minute hand and hour hand of a watch over top of each other? We all know that this happens at 12 noon and 12 midnight. When else? A guess might be five minutes and five seconds after one o'clock. But this would be incorrect since at five seconds after the minute, the minute hand would have moved another 1/720 of the way around and the hour hand would have moved another 1/43200 of the way around. At what times are the hands above each other? Send the editor your reasoning and your answers (or the answers of your students).

Mathematics

The curriculum in secondary mathematics was still in the process of change. Former courses did not always serve the needs of students, and some of these were revised while others were phased out and replaced by new courses. All students in junior high school were registered in the courses for that level, but the senior high school program was divided into three levels of difficulty. Of these, matriculation courses were most popular and received the best quality instruction, because they were usually assigned to highly qualified and experienced teachers. There were some weaknesses in instruction:

1. There was often a lack of correlation with junior high school mathematics.
2. The geometry section of Mathematics 10 was sometimes neglected.
3. Inadequate use was made of teaching aids.
4. Some teachers gave little opportunity for individual students to use their initiative.

However, such conditions were not general, and instruction in Mathematics 10, 20, 30 and 31 was usually effective. Unfortunately, the same was not always true of lower level mathematics courses often assigned to inexperienced teachers whose major fields of preparation were in areas other than mathematics. They often used a stereotyped textbook approach which proved ineffective. Mathematics instruction was strong where teachers offered individualized instruction, particularly when teaching at the point of error, but this was not always true in the lower level courses, where a lack of individualized instruction was noted.

Students were generally assigned to high school mathematics classes according to achievement in junior high school. In some schools, this assignment followed a rigid formula based on stanine gradings in Grade IX mathematics, but more often students were permitted to select their courses after counselling. In very small schools where only the matriculation program was offered, students had no choice but to attempt these courses.

There were few in-service education programs conducted during the year. The Mathematics Council of The Alberta Teachers' Association conducted some meetings, and some city teachers attended orientation workshops in the new courses. In larger schools, department heads met with their teachers to coordinate instruction, but the mathematics teachers in small rural centers had little opportunity to participate in in-service progams.

Construction of conic sections

A. Horovitch
Vauxhall High School

Certain difficulties are encountered in teaching the unit on Conic Sections in the Math 30 course. Students find it hard to understand the graphical implications of the constant b, the length of the shorter semi-axis of the hyperbola or ellipse. Rather than presenting this constant theoretically as part of the proof of a theorem, I have found it useful to give students the opportunity to construct the graph using the appropriate definition. This can be followed by a derivation of the equation in standard form.

Each of the four problems is presented as a lab exercise at the beginning of the study of each of the four conic sections. Poster paper is unrolled by the students and stapled to the bulletin boards. This serves as the material to draw diagrams. Other materials required are yardsticks, string, pins and thumbtacks. For construction of the parabola, a trapezoid made of stiff cardboard or plywood is required. This should be made by the students in advance. A ring is required to construct the hyperbola. It is preferable to use one without a stone, as this tends to tear the paper. Students seem to work best in groups of two or three. If there is not enough bulletin board space in your room, several pieces of "Ten-Test" can be cut from a 4' x 8' sheet and used as a drawing surface.

- PROBLEM 1: *To graph the locus of all points which are equidistant from a fixed point, and to find an equation for the locus.*

PART 1

1. Draw an x-y coordinate system with each axis about 24" long (including positive and negative portions).
2. Tie a cord around the pin of a thumbtack and push the thumbtack into the board at the origin. Tie the pencil to the other end of the cord, so that the distance between the tack and the pencil is about 5". (Leave an extra 1" to loop the string around the pencil and tie a knot.)
3. Keeping the cord tight, draw a continuous curve with the pencil.
4. Choose any point on the locus and call it (x_2, y_2). Then, using the distance formula, write an equation for the locus you have just drawn: The distance between any point and the fixed point - in this case the origin (0, 0) - is equal to a constant of 5".

   distance formula: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   Simplify the equation by squaring to remove radicals.

PART 2

1. Using the same coordinate system, locate the point (2, 3) on the graph.
2. Using the same radius as in Part 1, and fixed point (2, 3), draw the locus using the same method.
3. Using the distance formula, write an equation for the locus with fixed point or center (2, 3) and constant distance or radius of 5".

   After squaring to remove radicals, leave the equation in the form

   $(x_2 - x_1)^2 + (y_2 - y_1)^2 = r^2$, where "r" is the constant distance.

   This results in an equation called "the equation of the circle in standard form".
4. By writing the equation of the circle in standard form, what information can be obtained directly from the constants in the equation?

PART 3 - no construction required, but a sketch would be useful.

1. Let (h, k) be the center of a circle anywhere on the plane and let its radius be "r".
2. By methods outlined previously, derive a formula for this circle. Simplify by squaring to remove radicals. Leave the equation in standard form.
3. What are quantities required in order to write the equation of a circle in standard form?

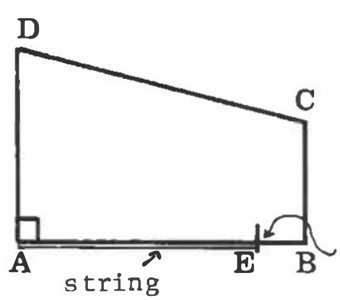
● *PROBLEM 2: To graph the locus of all points which are equidistant from a fixed point and a fixed line, and to find an equation for the locus.*

PART 1

1. Draw an x-y coordinate system with each axis about 20" long.
2. Draw the line x = -2 (the fixed line) and mark the point F (2, 0) - the fixed point.
3. Cut a trapezoid out of thick cardboard to the dimensions shown and letter it as follows:

Note: AB = 15"
$\angle$ DAB = 90°
AE = length of string, approximately 12"

other dimensions not critical

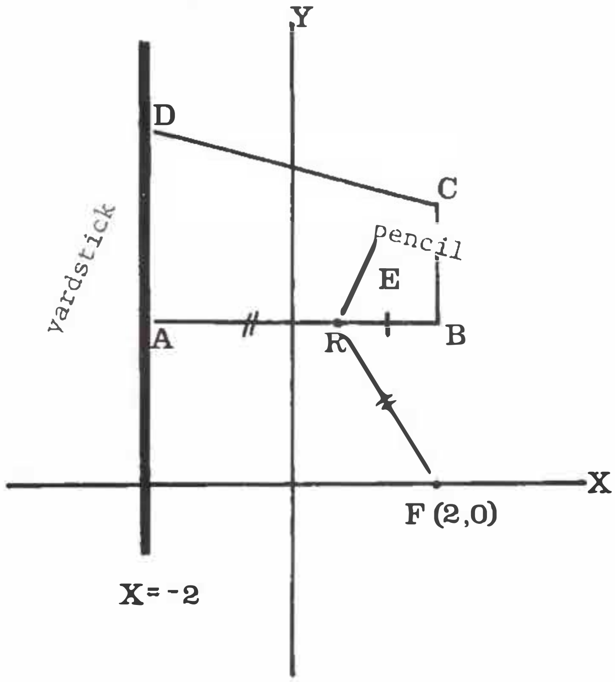


punch hole near bottom edge of trapezoid

4. Place the trapezoid on the paper so that AD lies along the fixed line $x = -2$ and the end of the cord is fastened at point (2, 0) with a thumbtack. The cord should be left slack.

5. Using the point of your pencil, pull the cord tight along AB (do not press down on the paper during this process).

6. Keeping the cord tight, move the trapezoid downward, with AD moving along the fixed line $x = -2$. This is accomplished by pushing the pencil slightly upward and to the left and holding the straight edge along the fixed line.



Note: AE = length of cord
AE = AR + RE

Also: AE = RF + RE
AR = RF

∴ the pencil is equidistant from point F and fixed line x = -2.

7. Move the trapezoid down until AE is on the x-axis. Then start below the x-axis and move upward. *Do not remove the thumbtack.*

8. Repeat the whole procedure twice more, making the following changes:
   (a) let the fixed line be $x = -1$, and fixed point (1, 0)
   (b) let the fixed line be $x = 1$, and fixed point (-1, 0). (Flip the trapezoid over for this one.)

PART 2

You now will have drawn three different loci which obey the same kind of condition: The distances between a general point on the graph and a *fixed point* is the same as the distance between the general point and a *fixed line*– e.g., in the diagram for Part 1, 6 - RF = RA.

Note: Distance from a point to a line is always considered to be distance measured *perpendicular* to the line.

1. Choose any point other than the origin which is on the graph having fixed point (2, 0) and fixed line x = -2. Let this point be P (x, y).

2. Using the chosen point, write the equation of the locus: The point is equidistant from the fixed point and the fixed line.

3. Simplify the equation by removing the radicals. This results in the "Standard form of the equation".

4. Using a different diagram, let A be any point on the locus with coordinates (x, y). Let (p, o) be the fixed point and x = -p be the fixed line. Draw a sketch and using the method outlined above, write the standard form of the equation of the locus.

● PROBLEM 3: *To graph the locus of all points such that the sum of their distances from two fixed points is a constant, and to find an equation for the locus.*

PART 1

1. Draw a coordinate system making each axis about 24" long, including the positive and negative portions. Let 2" equal one unit.

2. Mark the points F_1 (3, 0) and F_2 (-3, 0).

3. Place a thumbtack at each point F_1 and F_2 and fix the string at these two points. The total length of the string should be 21" (20" plus 1" for the loop to hold the pencil).

4. Make a notch around the pencil about 1/2" from the point. This will be used to hold the cord in place without slipping.

5. Loop (do not tie a knot) the cord around the notch and, keeping the string tight, trace out a curve with the pencil. Start with the pencil on the positive x-axis, and move around toward the negative x-axis, finishing the graph by moving below the x-axis up to the starting point.

6. List the x and y intercepts of the graph you have drawn.

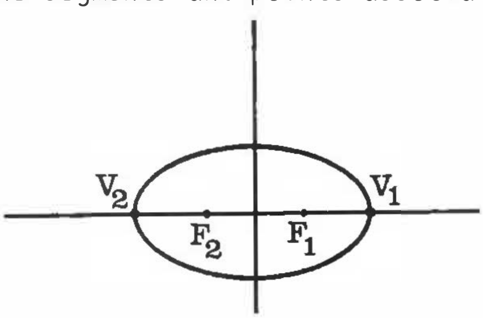
7. Repeat the above procedure, using the same length of string and

   (a) F_1 (2, 0), F_2 (-2, 0)

   (b) F_1 (0, 3), F_2 (0, -3)

PART 2

Several important line segments and points associated with the ellipse are illustrated below.



V_1 and V_2 are *vertices* of the ellipse.

F_1 and F_2 are *foci*.

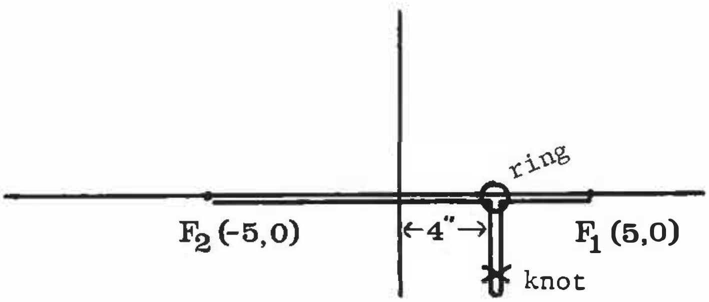
a = distance from origin to either vertex (x-intercepts in this case)
b = distance from origin to either of the remaining intercepts
c = distance from origin to either focus

1. Take any point on the ellipse $P(x, y)$. Measure PF_1 and PF_2.
2. Measure V_2V_1. How are PF_1, PF_2 and V_2V_1 related?
3. Using the idea of locus, write a definition for the ellipse.
4. Find by measurement the relation between a, b and c.

● PROBLEM 4: *To graph the locus of all points such that the difference of their distances from two fixed points is a constant, and to find the equation of the locus.*

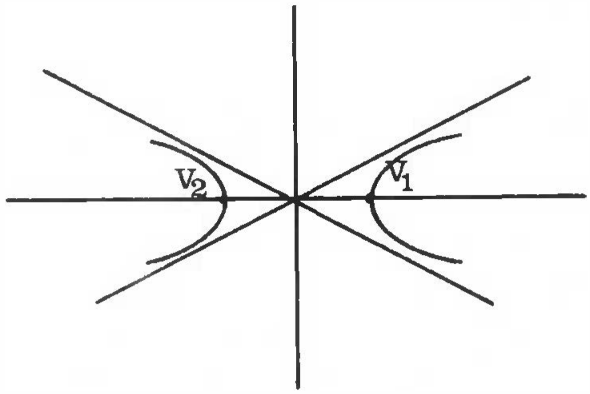
PART 1

1. Draw a coordinate system with axes about 18" long, letting 1" equal one unit. Mark the points F_1 (5, 0) and F_2 (-5, 0).
2. Using a string exactly 23" long, fasten its ends at F_1 and F_2.
3. Slip a ring over the cord and move it along the doubled portion until the string is tight along the x-axis. Tie a knot near the bottom of the doubled end.



4. Put the end of a pencil in the ring and, pressing the pencil against the upper portion of the ring, trace out a curve by moving the pencil upward while keeping the cord tight, but allowing it to slip through the ring.
5. To obtain the portion of the graph below the x-axis, swing the doubled part of the cord above the x-axis, and repeat the procedure, moving the ring downward.
6. Remove the tacks, *but keep the length of the string between the tacks unchanged.* The tack which was at F_2 is now moved to F_1, and vice versa.
7. Repeat parts 4 and 5 to obtain the second branch of the graph.

PART 2



1. Using a straight edge, draw two lines which pass through origin and approach each part of the graph *but do not touch it*. These lines are called *ASYMPTOTES* of the hyperbola.
2. At V_1 and V_2, construct perpendicular segments to cut the asymptotes in four places.
3. Join these four points to form a rectangle.
4. Find the points at which the horizontal lines of the rectangle cross the y-axis. The distance between these points of intersection and the origin is the quantity b for the hyperbola.
5. Having obtained b, find the relation between a, b, and c for the hyperbola.

   a is the distance from the origin to either vertex of the hyperbola, and c is the distance from the origin to either focus (F_1 and F_2).
6. Take any point P (x, y) on either branch of the hyperbola. Measure PF_1 and PF_2. Measure V_1V_2. How are these three quantities related?

Census in the year 2000

If we continue to live our lives as we are at present, facts show that human existence may soon come to an end. The world's peoples are faced with extinction from pollution, shortage of food, the hydrogen bomb, germ warfare and depletion of natural resources. Let us put these problems aside and concentrate on another problem - population increase: in the year 2000 AD, what will be the world's population? We can derive this answer by the formula:

$$A = Pe^{rn}$$

where

A = population in the future - 2000 AD

P = present population - 3.8 billion*

e = natural logarithm base - 2.7183

r = human growth per thousand
= births - deaths
= 34/1000 - 15/1000*
= 19/1000
= 0.019

n = 30 years

$$A = Pe^{rn}$$

$$A = 3.8 \times 10^9 \, (2.7183)^{(.019) \times 30}$$

$$\text{Log } A = \text{Log } 3.8 \times 10^9 + .57 \text{ Log } 2.7183$$

$$= 9.57978 + .57\,(.43428)$$

$$= 9.57978 + .24754$$

$$= 9.82732$$

$$A = \text{Antilog } 9.82732$$

$$A = 6.7192 \times 10^9$$

$$A = 7,000,000,000$$

This result, showing a world population of seven billion by the year 2000 AD, indicates that the world's population will almost double in only 30 years. Do you think the world can support such a large population? Assuming Canada's population of 21.5 million* is growing at a rate of 0.02* per year, can you show that Canada's population in the year 2000 should be about 39,000,000? If the world's increase over 30 years is

$$\frac{6.7 \times 10^9}{3.8 \times 10^6} \times 100 = 180\%$$

what is the comparable figure for Canada?

**Encyclopedia Americana*, 1970 edition.

Noah Weizner, student
Garden City Collegiate

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Book Review

TECHNICAL MATH by H. F. R. Adams. Publisher: McGraw-Hill Co. of Canada Ltd., 330 Progress Avenue, Scarborough (Toronto), Ontario. First edition. $7.75. 345 pages, paperbound.

The text does not appear to be part of a series. I would judge it to be useful for the Grade XI level, although it could be used for supplementary resource problems and as a guide for Grades VII to XI. It could serve as a text for non-matriculation Grade XI, or for vocational school mathematics classes.

The text consists of six sections: Arithmetic, Measurement and Scale Reading, Algebra, Indices and Logarithms, and Geometry and Trigonometry, with the usual tables in the appendix. There is an interesting approach to each chapter: each is introduced by an examination for the student. The results indicate whether or not the student needs further practice in that type of problem or skill. This idea makes the book adaptable to individual student progress, but because the answers are not given, the student cannot really work entirely on his own. I personally feel that answers should be accessible to high school students when working in mathematics.

The price is rather high, especially since the book has a soft cover. Otherwise, it compares favorably with other mathematics texts at this level, and is especially valuable as a resource text.

Mrs. Louise L. Hathaway
Marwayne

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