The Wisconsin Mathematics Council, through action by the executive board and lots of hard work by members William Miller, Pat Lautenschlager, John Knutson, and Donna Kringen, has begun a war on pollution. The first skirmish was a cooperative effort with the state Department of Public Instruction to produce a teacher's handbook, *Pollution: Problems, Projects, and Mathematical Exercises, Grades VI-IX.* This handbook contains problems and exercises involving actual pollution data that are keyed to mathematical topics studied in Grades VI-IX as well as descriptions of individual and group projects that will make students aware of environmental pollution situations. It can be obtained from DPI Publication Order Service, 162 Langdon Street, Madison, Wisconsin 53702, for 50¢ per copy.

The rationale behind the efforts to produce this publication revolves around the conviction that today's youth must become aware of earth's ecological state if they are to influence adults and their peers to do something to stop man's pell mell acceleration towards total disaster long before 2437 A.D.

What's it all about? What's really important? If we are willing to stop the busy, irrelevant activities that use up our precious time and think, we all know that the population explosion and the environmental pollution of earth's resources must be slowed down, and soon!

What can you do? I suggest two immediate actions: obtain and study a copy of "Guidelines for Citizen Action on Environmental Problems", free from The Conservation Education Association, P. O. Box 450, Madison, Wisconsin 53701, and bring into your teaching as much environmental pollution information as you can.

Good Luck!

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Two Preference Paradoxes

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Given A is preferred to B
and B is preferred to C

under some reasonable scheme for decision-making, one normally deduces that, under the same scheme and at the same time,

A is preferred to C.

This note is written to convince the reader that such a deduction is invalid.

EXAMPLE 1

Suppose that three candidates A, B, and C run for office. To get the most accurate feeling from each voter, it is decided that each voter should list the three candidates in the order of his own preference. Suppose the voters list their preferences in this way:

30 percent	A	B	C
5 percent	A	C	B
5 percent	B	A	C
25 percent	B	C	A
30 percent	C	A	B
5 percent	C	B	A

Then 65 percent of the voters have listed A before B, and 60 percent of the voters have listed B before C. So A is preferred to B and B is preferred to C. One would think that A is preferred to C. Not so! A full 60 percent of the voters prefer C to A. Hence we have A preferred to B, B preferred to C, and C preferred to A.

Notes on Example 1: This example could be repeated with taste preferences of people, TV watching, public opinion polls, and so forth. The reader is encouraged to attempt to try corresponding situations where four objects are being compared. Is it possible to prefer A to B, B to C, C to D, and also D to A?

Perhaps the reader feels that Example 1 was "cooked up", in that the preferences of the voters were arbitrarily ascribed. He should then examine the following situation: suppose there were 600 voters and they liked the candidates rather evenly. That is, the six possible orders of preference had the following number of voters:

Order:	A-B-C	A-C-B	B-A-C	B-C-A	C-A-B	C-B-A
No. of Voters:	100	100	100	100	100	100

Then, as is backed by intuition, 50 percent (300) of the voters prefer A to B, 50 percent prefer B to C, and 50 percent prefer C to A. If only two voters switch, we can obtain the following distribution of preferences:

Order:	A-B-C	A-C-B	B-A-C	B-C-A	C-A-B	C-B-A
No. of Voters:	101	99	100	100	101	99

Now 301 voters prefer A to B, 301 voters prefer B to C, and 302 voters at the same time prefer C to Z. This gives the same paradoxical result as before: A is preferred to B who is preferred to C who is preferred to A.

EXAMPLE 2

This example is different from Example 1. Instead of asking individuals to rate objects, let us rate the objects by some performance standard. For instance, suppose three runners X, Y, and Z are entered in the 100-yard dash. Their previous times for this distance give some indication (perhaps the best indication) of how well they will do in the race. Here is a table of the number of races in which each runner might have run the distance in a certain time.

Runner/Time	9.8	9.9	10.0	10.1	10.2
X		2			1
Y			4		
Z	1			2	

Who is the best runner? Notice that two-thirds of X's times beat the times of Y (the consistent one). One would expect X to beat Y in a race. X is preferred to Y. Y, in turn, beats Z two-thirds of the time. So Y is preferred to Z. But Z, when he runs 9.8 (one-third of the time), always beats X. When he runs 10.1 (two-thirds of the time), he has a 1 in 3 chance of beating X. The probability that Z beats X is thus $1/3 + 2/3 \cdot 1/3 = 5/9$. So Z ought to be preferred to X.

Again we have X preferred to Y who is preferred to Z who is preferred to X.

Notes on Example 2: In sports, favorites for a game are often chosen on the basis of past performance against a "common opponent". The above example shows possible pitfalls of such a strategy of choice. It is entirely possible for A to have beaten B over half of the time, and for B to have beaten C over half of the time, yet have C better than A. What appear to be "upsets" may not be upsets at all.

It should be noted that this example could also be applied to the climates of cities. Given distributions of temperatures and preferring the warmer climate, it is possible that one might prefer city A to city B, city B to city C, and city C to city A.

Here are two problems which indicate further seemingly paradoxical situations.

Given triangle ABC, find a set of points, in the plane, so that 2/3 of the points are closer to A than to B, 2/3 of the points are closer to B than to C, and (simultaneously) 2/3 of the points are closer to C than to A.

Draw graphs of three continuous functions f, g, and h with domain $[x: 0 < x < 1]$ in which for over half of the values of the domain, $f(x) > g(x)$, for over half of the values, $g(x) > h(x)$, and for over half of the values $h(x) > f(x)$. (Answers to these problems are available from the author at the University of Chicago.)

These results and problems, for most of us, violate all "common sense". One is forced to ask why the situations given above behave so contrarily to our intuition. One reason is that it is more common to compare individual events than distribution of events. We assign one number as an evaluation of an event (or candidate). Since numbers obey the transitive property: (a > b and b > c) implies a > c, we expect distributions to do likewise. The above has shown that preferences are not necessarily transitive.

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Medicine Hat

Math Seminar

The Senior High Mathematics Seminar was held at C.H.H.S. on Saturday, October 16. Dr. Blummel and Wilf Lencucha presented many ideas which were well received by the teachers.

Dr. Blummel stated that the three levels of mathematics were to accommodate the different ability levels of students in the high school. He did not foresee any changes in the mathematics curriculum in the near future, but there was a question mark as to the future of math 31. He felt that the 13, 23, 33 series would be better received if it were accepted for entrance into university and technical institutions.

Dr. Blummel also pointed out that teachers should make subject matter more relevant by:

1. teaching for application
2. teaching appropriate content
3. showing the role of mathematics in occupations
4. relating mathematics to other subject areas
5. bringing out the history of mathematics on occasion
6. use of visual materials.

On guiding students into the correct stream, Mr. Lencucha stressed the use of the following criteria:

1. intellectual maturity and ability
2. previous achievement in mathematics
3. interest in the subject
4. work habits
5. cultural background
6. learning style.

He also stressed that it was entirely up to the school as to how students moved from one stream to the other. He also pointed out that teachers tend to concentrate more on content, rather than on material. He felt that teachers should take a textual, laboratory and environmental approach to mathematics teaching.