THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

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Educational Performance Contracting

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In past months we have all heard of "Performance Contracting" or "Contracted Learning" as something "new" in education. On October 13-15, 1970, the Michigan Association of State and Federal Programs Specialists, in cooperation with the Michigan Department of Education, sponsored a conference on Educational Performance Contracting.

"Performance Contracting" is not a simple concept nor someone's overnight brainstorm. It is an outgrowth of the public's demand for "accountability" in its schools. Very simply stated, it is the process of contracting with an independent, profit-oriented company to teach students subjects such as mathematics and reading. The contract is paid on the basis of student achievement as measured by pre- and post-testing.

"Accountability" has been defined as "responsibility for satisfactory performance". Dr. Norman Weinhermer, former Superintendent of the Grand Rapids Public Schools, has stated, "Accountability needs responsibility and authority - authority to operate." He went on to say, "We educators truly must have individualized instruction - now." Until about the 1950s, not much

In This Issue

money was really involved in public education. Since then, larger sums of money have been spent on education and as a result, we, the educators, are being required to be accountable.

In the American educational system, change from within is slow, but pressure from the outside, for some yet unexplained reason, produces change. One needs only to look at the revolution in mathematics to see how outside pressures cause change. Performance contracting is a method of allowing outside forces to make controlled changes. *Almost all performance contracts have been in the areas of mathematics and reading*, generally at the elementary level. There are two basic reasons for this fact. First, these two areas are of prime importance, and second, their results are the easiest to measure. In addition, these two areas are conducive to the development of individualized programs of instruction.

Many contractors involved in these programs first change the learning environment by use of carpeting and air conditioning. How many school districts do you think could convince their taxpayers to spend the money necessary to carpet and climate-control their existing classrooms? The rationale is that after the contractor has demonstrated that these changes (and many others) affect learning positively, the public would accept the additional costs. Air conditioning and carpeting are, of course, not the prime changes made by the contractor, but they are a starting point and perhaps the only changes that the man on the street sees.

The contractor places his confidence for results in four areas: environment, both physical and non-physical; motivation on the part of the entire staff; staff and resources. The emphasis placed on these four areas varies with the contractor. Let's take a closer look at each of them.

ENVIRONMENT

The physical aspects of this have been mentioned. Things such as furniture and decorating come into play as well. The non-physical aspects are a little more difficult to pinpoint, but they come about through the use of enthusiastic staff members and new philosophies.

MOTIVATION

Both staff and student motivation is a necessary ingredient. Student motivation might be achieved through the use of payment by tokens or stamps for completion of certain objectives. These tokens can then be converted to money or used to buy time in the game room where the student can relax and listen to records, drink pop, play pinball machines, and so on. Other forms of motivation might be free time when the student can do whatever activity he likes. Yet another aspect is to have some learning experiences which are special and must be earned.

RESOURCES

Most contractors make extensive use of equipment. Equipment is provided for both the staff to use as well as the students. It includes, but is not limited to, reading machines, typewriters, diversified text materials, manipulative games and toys, filmstrip and -loop viewers with films, cassette tape recorders and players with appropriate tapes, and a game room.

STAFF

Most contractors either provide their own staff or more often use the best staff the school has to offer. Contractors pick the staff from those persons indicating a desire to be involved. This staff, both professional and para-professional, then undergoes an intensive training program prior to the opening of school.

An expected reader reaction at this point is, "Give me all these things, the para-professional help, and I, too, will produce." Why don't we do this and eliminate the outside force? The answer to this question has been alluded to earlier. Schools are generally conservative and not willing, or able, to spend money necessary to meet these ends. Contractors, on the other hand, know that they can produce through these means and can borrow money as private businesses. They will be paid, not by vote of the taxpayers, but through a fulfilled legal document. Almost all existing performance contracts in force, both in Michigan and throughout the country, are financed by federal dollars. Consequently, no one can lose.

Let's illustrate how performance contracting is financially sound. One school system traditionally spends, on the average, $110 per child to teach reading per year. Suppose this child requires three years to achieve one full year of achievement in reading. A contractor says that he will bring this same child up one full level in one school year for, say, $230. If this goal is not accomplished, the contractor will be paid either nothing or a portion of the contracted sum corresponding to the measured amount of learning which was accomplished, depending on the terms of the contract. Granted, assuming full production, this is a large outlay of money for the student for one year. However, when equated with the traditional program, the contracted program is $100 less expensive for the same product. Of more importance, the student has accomplished what he is in school to accomplish. Assuming this level of production can continue, the student will theoretically be three times better educated when he leaves school than he would have been under the traditional program - at a cost of $100 per year more. It should be noted that at present, most performance contracts involve the "educationally deprived" student.

How involved and complicated is this business of contracting the school's responsibilities to some one outside the school? The performance incentive contract between the Dallas Independent School District, Dallas, Texas, and New Century, Educational Division, Meredith Corporation, is a 26-page legal document. In addition to this basic contract, others must be negotiated for a management support system, for evaluation and still another for the audit. As payments are made on the basis of production, there must be a means for testing this production and a system to relate these test results to payments. Thus the need for a firm to perform an audit. It is considered best to have both the testing contractor and auditor independent of the firm holding the performance contract. This interplay necessary between the performance contractor and testing contractor requires a management support system. It soon becomes obvious that the total cost does not involve just that of the performance contractor but the costs of the firms providing these additional needed functions as well.

The testing part of the complete program is probably the biggest philosophical bug-a-boo in the entire system. This is the area which has drawn the most attention in the reports on the Texarkana program. This is unfortunate as it is reported that a great deal of good and productivity resulted from that project.

A performance contract usually has built into it a "turnkey" section which spells out how the school should be able to take over the program. No one expects the contractor to be around for more than three years on a particular project.

The efforts and frustrations involved in establishing the first performance contract for a school system are more monumental than those encountered in the negotiation of teacher contracts throughout the State. Why, then, has the Office of Economic Opportunity been pumping Federal dollars into performance contracts? The basic reason is that no independent school system alone could afford to, either financially or philosophically. Yet, some educators have felt that the methods to be used by contractors were the best ones at this time and for the desired goals. Dr. John W. Porter has said, "In my opinion, performance contracting is going to be the salvation in helping the classroom teacher serve more students and give more individual help. I think the educational community will eventually come around to this..." What Dr. Porter refers to are the concepts and methods of performance contracts, not the turning of public schools over to private enterprise.

If Dr. Porter, the State Board of Education, the Michigan Legislature, and Federal Government are willing to reorder their priorities and put more money in all areas of education, not just the select areas as is presently being done, then all teachers will be as those working for performance contractors and more accountable.

The desire for accountability and productivity exists in almost all educators, but the funds to carry out change does not. If there is one thing that is being demonstrated through the present OEO program, it is this very fact.

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From the Editor's Desk

* Several letters have been received from readers concerning the properties of i^1 and i^{-1} raised in an issue last year. We are pleased to print the most interesting article in this issue on page 15.

* Here's a puzzle for that student in your class who is always the first to finish his work:

> Using the digits 1, 9, 7, 2 and the signs of standard mathematical operations, represent the integers from 1 to 25 (or some higher limit) without gaps. Here's a start:

$$1 = 1 + 9 - 7 - 2$$

$$2 = 1 + [9 \div (7 + 2)]$$

$$3 = -1 + 9 - 7 + 2$$

* A reader sent this little poem (source unknown). If it reflects your feelings, may we hear from you?

CHANGES,
CHANGES,
CHANGES

I took a course in curriculum
And thought it rather queer
That I am now a 'resource'
To the pupils sitting near.

I must not lecture nor talk too much
But let the pupils do
And then I shall be fostering
Creativity through and through.

The school was built with kids in mind
The course around them planned
And I exist to jump about
At their whim and command.

Into groups I'll break them
With ideas for their mind,
And should they find it boring
I must try some other kind.

And should the classroom noise level
Disturb the Richter Scale
I know that they are learning
For in noise learning prevails.

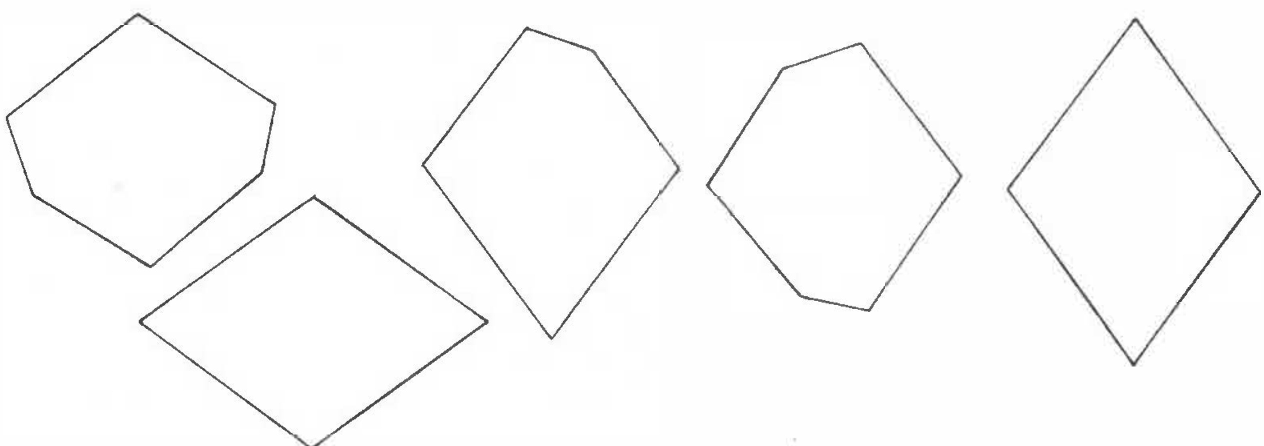
No rows of desks will I permit
No lining at the door
For this is passé don't you know?
Thank God they've left the floor.

But what am I to teach, you ask.
Don't look at me confused
I have a teachers' plan book
And I'm told it must be used.

Now to reconcile these concepts
With a curriculum in print
Is like making papier mâché
Out of solid rock cement.

But I'll try in my endeavors
Though my ways must all be changed
And then in my retirement
I might see just what was gained.

* For the stars in your class - assemble these five pieces to form a five-pointed star.



What's It All About?

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As we struggle through life busy with so much of what we, in our more serious moments, call trivia, sometimes we stop and ask ourselves, "What's it all about? What's really important?"

Mathematics teachers really are as responsible for considering the ultimate worthwhileness of their energy-devouring professional activities as ministers, social workers, government workers, politicians and physicians. In fact, by our very nature, we may be better trained and better equipped to examine our activities logically in perspective.

So, what's really important? Consider the following:

$$2^{\frac{x}{35}} N = Y$$

This equation can be used to compute the number of years, (x) it will take the mass of human flesh and blood to equal Y if the world's population mass doubles every 35 years (the present rate).

Substituting 180,000,000 for N (accepting 3,650,000,000 as the number of people on earth averaging 100 pounds each yields 180,000,000 tons of humanity) and solving for x gives

$$x = 115 \text{ (Log } Y - 8.25).$$

Since Y represents a limit beyond which the mass of humanity cannot go, let's arrive at a reasonable value for this limit via the following argument presented by scientist Isaac Asimov:

"The total mass of living tissue on earth today is estimated to be something like 20 million million tons, and this cannot really increase as long as the basic energy source for life is sunlight. Only so much sunlight reaches earth; only so much sunlight can be used in photosynthesis; and therefore only so much living plant tissue can be built up each year. The amount built up is balanced by the amount destroyed each year, either through spontaneous death or consumption by animal life.

"Animal life may be roughly estimated as one-tenth the mass of plant life or about two million million tons the world over. This cannot increase either, for if, for any reason, the total mass of animal life were to increase significantly, the mass of plants would be consumed faster than it could be replaced, as long as sunlight is only what it is. The food supply would decrease drastically and animals would die of starvation in sufficient numbers to reduce them to their proper level.

"To be sure, the total mass of human life has been increased throughout history, but only at the expense of other forms of animal life. Every additional ton of humanity has meant, as a matter of absolute necessity, one less ton of non-human animal life.

"Not only that, but the greater the number of human beings, the greater the mass of plants that must be grown for human consumption as food (either directly, or indirectly by feeding animals destined for the butcher) or for other reasons. The greater the mass of grains, fruits, vegetables and fibers grown, the smaller the mass of other plants on the face of the earth.

"Suppose we ask, then, how many years it will take for mankind to increase in numbers to the point where the mass of humanity is equal to the present mass of all animal life? Remember that when that happens there will be no other animals left - no elephants or lions, no cattle or horses, no cats or dogs, no rats or mice, no trout or crabs, no flies or fleas.

"Furthermore to feed that mass of humanity, all the present mass of plant life must be in a form edible to man; which means no shade trees, no grass, no roses. We couldn't afford fruits or nuts because the rest of the trees would be inedible. Even grain would be uneconomic, for what would we do with the stalks? We would most likely be forced to feed on the only plants that are totally nutritious and require only sunlight and inorganic matter for rapid growth - the one-celled plants called "algae."

If the total mass of animal life is 2,000,000,000,000 tons, then x = 115 (1230 - 8.25). Simple calculation puts x at 466 years. This means that by the year 2437 A.D. the last animal other than man will have died, and the last plant other than algae will also have died.

It should be noted also that the human population by 2437 A.D. will be over 8,000 times the present number, that there will be 200,000 people per square mile of total earth surface - desert, ocean, mountain, river, plain. The present density of crowded Manhattan (New York City) at noon is estimated at only 100,000 persons per square mile!

So what's important? Something has got to give! The human population of earth cannot continue to grow at the present rate. And, along with the population explosion, we have pollution of our environment. The National Wildlife Federation states it this way:

> America is in trouble because our greed has put us on a collision course with disaster. Our Second Annual EQ Index - measuring national Environmental Quality - reveals the sobering fact that we are still losing ground on almost every front. Our air is dirtier. Our water is more polluted. Land for food, wildlife and living space is deteriorating. Certain minerals may soon be exhausted. Apathy is our biggest problem.

The Wisconsin Mathematics Council, through action by the executive board and lots of hard work by members William Miller, Pat Lautenschlager, John Knutson, and Donna Kringen, has begun a war on pollution. The first skirmish was a co-operative effort with the state Department of Public Instruction to produce a teacher's handbook, *Pollution: Problems, Projects, and Mathematical Exercises, Grades VI-IX.* This handbook contains problems and exercises involving actual pollution data that are keyed to mathematical topics studied in Grades VI-IX as well as descriptions of individual and group projects that will make students aware of environmental pollution situations. It can be obtained from DPI Publication Order Service, 162 Langdon Street, Madison, Wisconsin 53702, for 50¢ per copy.

The rationale behind the efforts to produce this publication revolves around the conviction that today's youth must become aware of earth's ecological state if they are to influence adults and their peers to do something to stop man's pell mell acceleration towards total disaster long before 2437 A.D.

What's it all about? What's really important? If we are willing to stop the busy, irrelevant activities that use up our precious time and think, we all know that the population explosion and the environmental pollution of earth's resources must be slowed down, and soon!

What can you do? I suggest two immediate actions: obtain and study a copy of "Guidelines for Citizen Action on Environmental Problems", free from The Conservation Education Association, P. O. Box 450, Madison, Wisconsin 53701, and bring into your teaching as much environmental pollution information as you can.

Good Luck!

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Two Preference Paradoxes

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Given A is preferred to B
and B is preferred to C

under some reasonable scheme for decision-making, one normally deduces that, under the same scheme and at the same time,

A is preferred to C.

This note is written to convince the reader that such a deduction is invalid.

EXAMPLE 1

Suppose that three candidates A, B, and C run for office. To get the most accurate feeling from each voter, it is decided that each voter should list the three candidates in the order of his own preference. Suppose the voters list their preferences in this way:

30 percent	A	B	C
5 percent	A	C	B
5 percent	B	A	C
25 percent	B	C	A
30 percent	C	A	B
5 percent	C	B	A

Then 65 percent of the voters have listed A before B, and 60 percent of the voters have listed B before C. So A is preferred to B and B is preferred to C. One would think that A is preferred to C. Not so! A full 60 percent of the voters prefer C to A. Hence we have A preferred to B, B preferred to C, and C preferred to A.

Notes on Example 1: This example could be repeated with taste preferences of people, TV watching, public opinion polls, and so forth. The reader is encouraged to attempt to try corresponding situations where four objects are being compared. Is it possible to prefer A to B, B to C, C to D, and also D to A?

Perhaps the reader feels that Example 1 was "cooked up", in that the preferences of the voters were arbitrarily ascribed. He should then examine the following situation: suppose there were 600 voters and they liked the candidates rather evenly. That is, the six possible orders of preference had the following number of voters:

Order:	A-B-C	A-C-B	B-A-C	B-C-A	C-A-B	C-B-A
No. of Voters:	100	100	100	100	100	100

Then, as is backed by intuition, 50 percent (300) of the voters prefer A to B, 50 percent prefer B to C, and 50 percent prefer C to A. If only two voters switch, we can obtain the following distribution of preferences:

Order:	A-B-C	A-C-B	B-A-C	B-C-A	C-A-B	C-B-A
No. of Voters:	101	99	100	100	101	99

Now 301 voters prefer A to B, 301 voters prefer B to C, and 302 voters at the same time prefer C to Z. This gives the same paradoxical result as before: A is preferred to B who is preferred to C who is preferred to A.

EXAMPLE 2

This example is different from Example 1. Instead of asking individuals to rate objects, let us rate the objects by some performance standard. For instance, suppose three runners X, Y, and Z are entered in the 100-yard dash. Their previous times for this distance give some indication (perhaps the best indication) of how well they will do in the race. Here is a table of the number of races in which each runner might have run the distance in a certain time.

Runner/Time	9.8	9.9	10.0	10.1	10.2
X		2			1
Y			4		
Z	1			2	

Who is the best runner? Notice that two-thirds of X's times beat the times of Y (the consistent one). One would expect X to beat Y in a race. X is preferred to Y. Y, in turn, beats Z two-thirds of the time. So Y is preferred to Z. But Z, when he runs 9.8 (one-third of the time), always beats X. When he runs 10.1 (two-thirds of the time), he has a 1 in 3 chance of beating X. The probability that Z beats X is thus $1/3 + 2/3 \cdot 1/3 = 5/9$. So Z ought to be preferred to X.

Again we have X preferred to Y who is preferred to Z who is preferred to X.

Notes on Example 2: In sports, favorites for a game are often chosen on the basis of past performance against a "common opponent". The above example shows possible pitfalls of such a strategy of choice. It is entirely possible for A to have beaten B over half of the time, and for B to have beaten C over half of the time, yet have C better than A. What appear to be "upsets" may not be upsets at all.

It should be noted that this example could also be applied to the climates of cities. Given distributions of temperatures and preferring the warmer climate, it is possible that one might prefer city A to city B, city B to city C, and city C to city A.

Here are two problems which indicate further seemingly paradoxical situations.

Given triangle ABC, find a set of points, in the plane, so that 2/3 of the points are closer to A than to B, 2/3 of the points are closer to B than to C, and (simultaneously) 2/3 of the points are closer to C than to A.

Draw graphs of three continuous functions f, g, and h with domain $[x: 0 < x < 1]$ in which for over half of the values of the domain, $f(x) > g(x)$, for over half of the values, $g(x) > h(x)$, and for over half of the values $h(x) > f(x)$. (Answers to these problems are available from the author at the University of Chicago.)

These results and problems, for most of us, violate all "common sense". One is forced to ask why the situations given above behave so contrarily to our intuition. One reason is that it is more common to compare individual events than distribution of events. We assign one number as an evaluation of an event (or candidate). Since numbers obey the transitive property: ($a > b$ and $b > c$) implies $a > c$, we expect distributions to do likewise. The above has shown that preferences are not necessarily transitive.

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Medicine Hat

Math Seminar

The Senior High Mathematics Seminar was held at C.H.H.S. on Saturday, October 16. Dr. Blummel and Wilf Lencucha presented many ideas which were well received by the teachers.

Dr. Blummel stated that the three levels of mathematics were to accommodate the different ability levels of students in the high school. He did not foresee any changes in the mathematics curriculum in the near future, but there was a question mark as to the future of math 31. He felt that the 13, 23, 33 series would be better received if it were accepted for entrance into university and technical institutions.

Dr. Blummel also pointed out that teachers should make subject matter more relevant by:

1. teaching for application
2. teaching appropriate content
3. showing the role of mathematics in occupations
4. relating mathematics to other subject areas
5. bringing out the history of mathematics on occasion
6. use of visual materials.

On guiding students into the correct stream, Mr. Lencucha stressed the use of the following criteria:

1. intellectual maturity and ability
2. previous achievement in mathematics
3. interest in the subject
4. work habits
5. cultural background
6. learning style.

He also stressed that it was entirely up to the school as to how students moved from one stream to the other. He also pointed out that teachers tend to concentrate more on content, rather than on material. He felt that teachers should take a textual, laboratory and environmental approach to mathematics teaching.

Grades I and II

A Working Bulletin Board

Mrs. Bonnie Trudeau
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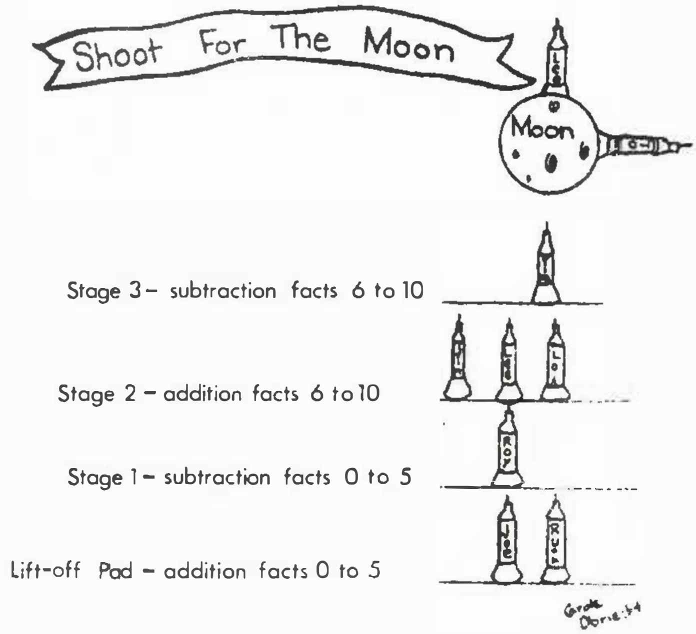
SHOOT FOR THE MOON

This game is intended to motivate the children to learn their addition and subtraction facts. Each child receives a sheet of construction paper, cuts out his own rocket ship and writes his name on it. As soon as he knows his addition facts from 0 to 5, he is permitted to put his rocket on the lift-off pad.

Each stage represents a timed test. In order to advance from one stage to the next, a child must score 100 percent on that test. He reaches his goal when he "lands on the moon".

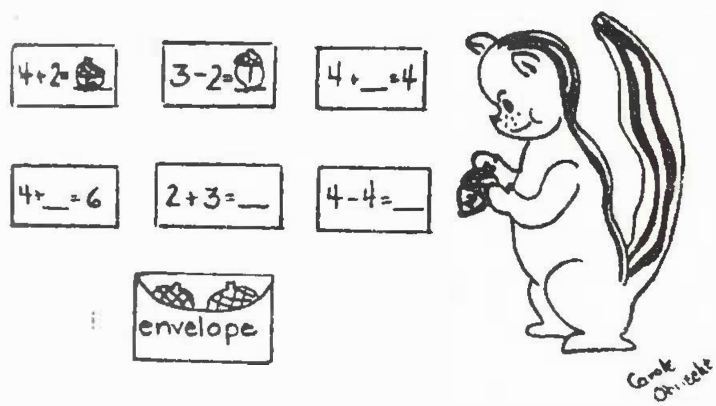
It's great fun to see how eagerly the children learn these number facts in order to reach their visible goal.



CHIPPER'S PROBLEM

Another game I use to motivate the children to learn their number facts is a game we call "Chipper's Problem". A cut-out of a large chipmunk (any friendly animal can be used, of course) is put on the bulletin board along with the question, "Can you help me?" Next to Chipper are several open sentences that Chipper can't answer, as well as an envelope containing acorn-shaped pieces of construction paper (approximately 1"x2") with a numeral on each. The children work independently or in groups of two or three to find the "acorn" with the correct answer to an open sentence. As soon as all the blanks are filled, the open sentences are changed so that there is a variety of problems.

CAN YOU HELP ME?



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And Then the Imaginary Becomes Real

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The number "i" has the interesting property to become real every time its exponent is of the form $2k$, $k \in \mathbb{N}^*$.

$$x = 2k,\ k \in \mathbb{N}^* \Rightarrow i^x \in \mathbb{R}.$$

What will happen when the exponent is i itself?

The demonstration given here is based upon the famous Euler's Formula:

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

This formula is proved in any first course of Complex Variable or Complex Analysis. It is a key to successful work in the complex field. One of the first uses of this formula leads to a result that is always a surprise for the beginner

What happens when $\theta = \pi$?

$$e^{i\pi} = \cos\pi + i\sin\pi$$

$$e^{i\pi} = -1.$$

We have a relation between irrational numbers, imaginary and integers.

What happens when $\theta = \pi/2$?

$$e^{i\pi/2} = \cos\pi/2 + i\sin\pi/2$$

$$e^{i\pi/2} = i.$$

If we want to get the value of i^i the temptation will then be to write

$$i^i = \left(e^{i\frac{\pi}{2}}\right)^i$$

$$= e^{i^2\frac{\pi}{2}} = e^{-\pi/2}$$

$$\text{so } i^i \in \mathbb{R}$$

Let us face the problem in another way.

We know that

a) $e^{i\theta} = \cos\theta + i\sin\theta$

b) $\ln e^{i\theta} = i\theta \ln e = i\theta$

Thus if we write down

$$\omega = i^{i}$$

$$\ln\omega = \ln i^{i} = i \ln i.$$

$$= i \ln e^{i(\pi/2 - 2k\pi)}, \quad k \in \mathbb{Z}$$

$$= i \cdot i(\pi/2 - 2k\pi)$$

$$= -\pi/2 + 2k\pi$$

then $\omega = e^{-\pi/2 + 2k\pi} = i^{i}$

$$\therefore i^{i} \in \mathbb{R}.$$

Thus i^{i} belongs to the real field, but there are many values for this expression, all of them being real. We can have a restriction and take the *principal value* only, in which case $i^{i} = e^{-\pi/2}$.

The same procedure for i^{-i} will give $i^{-i} = e^{\pi/2 + 2\lambda\pi}$, $\lambda \in \mathbb{Z}$

Here again we are dealing with a multi-valued expression whose principal value may be taken for technical purposes.

Is i^{i} greater or smaller than i^{-i}? Are these expressions sometimes equal? The comparison is easily handled with the preceding results.

Let us first answer the second question:

$$i^{i} \stackrel{?}{=} i^{-i}$$

$$e^{-\pi/2 + 2k\pi} \stackrel{?}{=} e^{\pi/2 + 2\lambda\pi}, \quad k, \lambda \in \mathbb{Z}$$

$$-\pi/2 + 2k\pi \stackrel{?}{=} \pi/2 + 2\lambda\pi$$

$$k - \lambda \stackrel{?}{=} 1/2$$

$k, \lambda \in \mathbb{Z}$ we cannot have $k - \lambda = 1/2$ so i^{i} is never equal to i^{-i}.

Which one is the greater ? A simple look at the principal values will give

$$e^{-\pi/2} < e^{\pi/2}$$

$$\Rightarrow i^{i} < i^{-i}$$

but this relation is not the same if we are looking at the different values of i^i and i^{-i}. The values will vary according to the variation of k and λ.

If $k \leqslant \lambda$ then $i^i < i^{-i}$ and if $k > \lambda$ then $i^i > i^{-i}$.

Further studies will bring the following relations:

a) $i^i = (-i)^{-i}$

b) $i^{-i} = -i^i$.

All these results are astonishing at first sight but they remind us that we are in a land of interesting discoveries when moving in the complex field.

MATH LUNCHEON

Math Luncheon of GREATER EDMONTON TEACHERS' CONVENTION will be held in Rupert's Land Room of the Macdonald Hotel, 12:30 pm, Thursday February 24.

Speaker: Dr. Doyal Nelson, University of Alberta.

Topic: "Mathematics Education in Japan"

Price: $3.50

Contact William Gee at Afton School, 16604 - 91 Avenue, Edmonton.

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