THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

T. W. Rieger
file

delta-k

(Formerly Mathematics Council Newsletter)

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Cards, Anyone?

The use of games instead of drill increases in popularity. Games can be invented to correspond to the concept of your concern. The following is adapted from an article by Stephen Krulik in the *New York State Mathematics Teachers' Journal*, June, 1971.

Card games are essentially mathematical in nature, but games of bridge or poker may not yet be acceptable math class activities, especially if there is money on the table. Simple games, such as fish, casino or rummy, are easily adaptable to the lesson you are teaching, and the "deck" is easily made.

FRACTION WAR

To practice comparing values of fractions. Sixty-six cards about two and one-half inches by three inches (half a three inch by five inch index card will do) are marked with a fraction on each as follows: 1/2, 1/3, 2/3, 1/4, 2/4, 3/4, 1/5 ... Do not reduce to lowest terms. Shuffle and deal entire deck face down to any number of players up to six. To play, each player turns up top card. High card wins and winner places all exposed cards face down at the bottom of his deck. To break a tie, tied players turn up another card each. Winner is the player who has the most cards at the end of the game. Game ends if a player runs out of cards.

FACTOR CASINO

Casino is essentially a matching game. Prepare a 45-card deck by marking in black on each of 20 cards an algebraic expression to be factored; mark in red on another card the factored form of each of these. On each

in this issue

of the remaining five cards, mark in red some incorrect factored form, preferably common student errors.

Shuffle and deal five cards to each player and five cards face up on the table; the remainder of the deck is placed face down on the table. Play proceeds around the table in the usual fashion. A player takes a trick when he matches an expression from his hand with its alternate form from the table. Tricks are held in front of each player on the table, face down, in a manner that permits them to be counted easily. If a player takes a trick, he replaces the card from his hand with the top card from the deck. If he cannot take a trick, he must place one card from his hand face up on the table and replace it with the top card from the deck. If a card placed on the table matches one already there and the player discarding it does not notice the matching, the trick belongs to the first player who notices it. A renege occurs if an incorrect matching is made, and the trick belongs to the player who calls the renege. Game ends when deck is finished and there are no more tricks to be taken. Winner is the player who has the most tricks.

FACTORUMMY

A little more complex in play, this game follows the usual rummy pattern. Fifty-two cards are prepared in four suits, using differently colored felt-tipped markers. Each of five cards in a suit will be marked with polynomial expressions to be factored (for best results, some should require three factors, even if one factor be a constant). The remaining eight cards in each suit are marked with a factor each. All suits are marked the same, but each in a different color. Cards are dealt seven to a player, face down; the remainder of the deck is placed on the table, face down, with the top card turned up, as in regular rummy.

Each player attempts to arrange his hand so that it consists of groups (three or four cards having the same expression) or runs (a polynomial expression together with its factors).

Play proceeds thus: each player in turn chooses either the top card of the facedown deck or the top card of the faceup deck on the table. He adds this card to his hand. He then discards from his hand one of the eight cards he now holds so that between plays his hand consists of seven cards only. Play ends when a player has no card that does not belong to a run or a group, but no card in that hand may belong to a run and a group at the same time. The winner, of course, must declare the win by the use of the word "rummy".

Each of these games will require a deck for each five players or thereabouts. The level of difficulty can be tailored to suit your needs, all the way from practice in addition to identities in trigonometry. Cost is practically nothing.

If your students enjoy this form of activity, they will invent their own variations. Listen well; you are about to meet the *real* mathematicians.

Letters to the Editor

Dear Sir:

After reading the article "Flow Charting and Mathematics" in the April issue of *Delta-K*, I finally decided to try this technique with a Grade VIII option class. The students had been experiencing some difficulty in the solution of problems, and analysis showed invariably the problems stemmed from the inability to solve an equation.

The topic of flow charting was introduced and discussed and, as is usual, students made flow charts for the completion of everyday tasks with very little difficulty and a great deal of individuality. Finally, we tackled the problem of flow charting for the solution of equations, and the final product is enclosed.

This chart is by no means complete nor necessarily accurate in every respect, but all the decisions which had to be made were selected by the students, as were the operations involved, and the color code, except in some cases for the use of the symbolism.

I have found that this type of procedure definitely assists students in doing this type of work. It is not very long before they begin simplifying the chart and combining various steps, so the chart eventually becomes a useful tool for "troubleshooting" rather than a crutch on which they have to lean.

Yours truly,

T. Cooke, Math Instructor
Eaglesham School
Eaglesham, Alberta.

Editor's note: Teachers who would like a copy of Mr. Cooke's flow chart should contact him c/o Eaglesham School.

Dear Sir:

In the April 1972 issue of *Delta-K*, there was a review of *Mathematics For A Modern World*, Book 1. The reviewer mentioned that Book 2 was in preparation. This book is now available and I am forwarding a copy under separate cover. I trust that you will pass this copy on to the anonymous reviewer with my compliments.

We have a number of new mathematics publications which I am sure would be of interest to the members of the Mathematics Council, ATA. I am wondering what procedure should be followed in order to get materials reviewed. At the present time, we do forward a copy of every new Canadian publication to the Edmonton office of The Alberta Teachers' Association for review purposes. However, I am wondering whether it might not be more appropriate to send titles of specific interest to the editors of the subject councils for review in the individual journals.

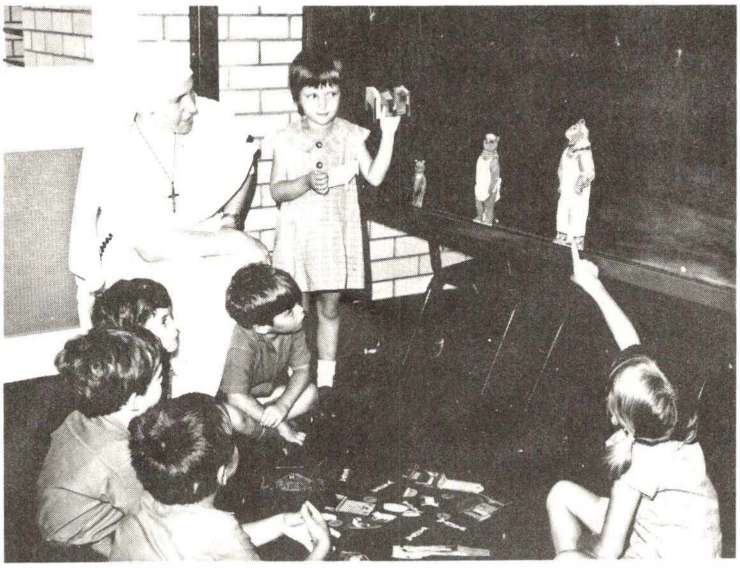
I thank you for your cooperation and look forward to hearing further from you.

Sincerely,

E.D. Lucas, Western Sales Manager
Gage Educational Publishing Limited

Editor's note: Teachers interested in reviewing *Mathematics For A Modern World*, Book 2, or any of the other materials alluded to by Mr. Lucas, should contact the Editor.

TWO AND TWO EQUALS FUN



Mathematics without tears for Grade I pupils at Our Lady of Lourdes School in Brisbane (March, 1972). Australian Information Service. Photograph by Bob Nicol.

A novel method of teaching mathematics to primary school children has been developed in Australia. Known as TRIAD, it aims at encouraging a child to "experience" the concept of mathematics, rather than learn from a book, and uses such things as puppets, brightly colored blocks, dice, picture cards, music and rhyme. For further information, contact: Jacaranda Press Pty. Ltd., 46 Douglas Street, Milton, Brisbane 4064, Australia.

Finding the Non-real Roots of a Quadratic Equation by Graphing

Wayne Harvey
New Trier West High School
Illinois

NOTE: Most teachers and students of mathematics would agree that the quadratic function is most easily graphed when the equation is written in the form $y = a(x-h)^2 + k$. But to find the roots of a quadratic equation, the same teachers and students use the form $ax^2 + bx + c = 0$ and the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The following theorem shows that it is not necessary to change from the first form to the second to find the roots. In fact, this theorem provides a much easier way to find the roots of a quadratic equation than the more traditional use of the quadratic formula.

Theorem: $a(x-h)^2 + k = 0 \leftrightarrow x = h \pm \sqrt{-\frac{k}{a}}$

$$a(x-h)^2 + k = 0$$

$$(x-h)^2 + \frac{k}{a} = 0$$

$$(x-h)^2 - \left(-\frac{k}{a}\right) = 0$$

$$(x-h)^2 - \left(\sqrt{-\frac{k}{a}}\right)^2 = 0$$

$$\left(x-h - \sqrt{-\frac{k}{a}}\right)\left(x-h + \sqrt{-\frac{k}{a}}\right) = 0$$

$$x-h - \sqrt{-\frac{k}{a}} = 0 \quad \text{or} \quad x-h + \sqrt{-\frac{k}{a}} = 0$$

$$x = h + \sqrt{-\frac{k}{a}} \quad \text{or} \quad x = h - \sqrt{-\frac{k}{a}}$$

$$x = h \pm \sqrt{-\frac{k}{a}}$$

Since these steps are reversible:

$$a(x-h)^2 + k = 0 \leftrightarrow x = h \pm \sqrt{-\frac{k}{a}}$$

I presented this proof to my advanced junior math class and challenged each of them to come up with a clever, simple way to find the solution of a quadratic equation *by graphing*, when the roots were *non-real, complex numbers*, that is, when the parabola did not intersect the x-axis. (I didn't know the answer.) The accompanying article is a response to this challenge.

George Kelso

The graph of a quadratic equation is a parabola. If the equation is written in the form $y = a(x-h)^2 + k$, then:

1. The coordinates of the vertex are (h,k).
2. The minimum or maximum value of y is k.
3. i. If $a>0$, the parabola opens up.
   ii. If $a<0$, the parabola opens down.
4. i. If $|a| >1$, the parabola is "acute".
   ii. If $|a| <1$, the parabola is "obtuse".
5. i. The constants "a" and "k" are both positive or both negative, if and only if the parabola does not intersect the x-axis. In this case, the "discriminant" $-\frac{k}{a} <0$ and the roots are not real and can be written as

$$x = h \pm \sqrt{-\frac{k}{a}} \quad \text{or} \quad x = h \pm \sqrt{\frac{k}{a}}\,i$$

   ii. One of the constants "a" and "k" is positive and the other is negative, if and only if the parabola intersects the x-axis $(y=0)$. In this case, the "discriminant" $-\frac{k}{a} > 0$ and the roots are real and can be written as

$$x = h \pm \sqrt{-\frac{k}{a}}$$

6. The equation of the axis of symmetry is $x = h$.
7. i. If the parabola intersects the x-axis, then the real roots are the x-coordinates and both y-coordinates are 0. That is, the coordinates of the points of intersection are:

$$(x_1, y_1) = \left(h + \sqrt{-\frac{k}{a}}, 0\right) \qquad (x_2, y_2) = \left(h - \sqrt{-\frac{k}{a}}, 0\right)$$

   ii. One point is $\sqrt{-\frac{k}{a}}$ units to the left of the line of symmetry $(x=h)$, and the other is $\sqrt{-\frac{k}{a}}$ units to the right of this line.

The above information enables us to approximate the roots of a quadratic equation when it is written in the form $0 = a(x-h)^2 + k$, by graphing the corresponding equation $y = a(x-h)^2 + k$, *even if the roots are non-real, complex numbers!*

Suppose we are given a parabola which does not intersect the x-axis. Then, according to 5.i. above, the roots are non-real numbers. Our problem is to find the approximate values of these roots by graphing. If the given parabola appears as it does in Figure 1, let us invert it to obtain the parabola in Figure 2.

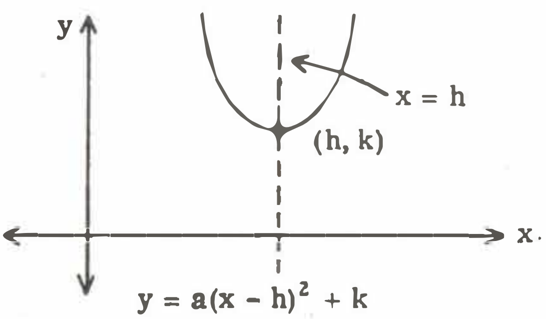


Figure 1

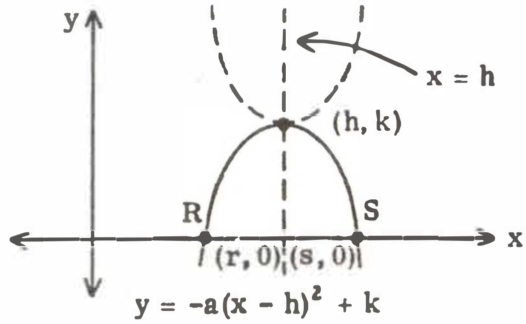


Figure 2

From the figures, it is obvious that both parabolas have the same vertex (h,k). Therefore, the values for "h" and "k" in the equations of the given parabola and the inverted parabola will be the same. And inverting a parabola merely changes the sign of the constant "a" in the corresponding equation. Therefore, if the equation of the given parabola is $y = a(x-h)^2 + k$, then the equation of the inverted parabola is $y = -a(x-h)^2 + k$.

But the inverted parabola in Figure 2 intersects the x-axis. According to 7.i. above, the real roots are:

$$x = h \pm \sqrt{\frac{k}{-a}} \qquad \text{or} \qquad x = h \pm \sqrt{\frac{k}{a}} \quad \left(\frac{k}{a} > 0\right)$$

That is, the *real* roots of the inverted parabola in Figure 2 are

$$x = h \pm \sqrt{\frac{k}{a}}$$

and the *non-real* roots of the given parabola in Figure 1 are

$$x = h \pm \sqrt{\frac{k}{a}}\, i$$

In other words, the real roots of the equation, $0 = -a(x-h)^2 + k$, which corresponds to the inverted parabola in Figure 2, provide us with the real numbers "p" and "q" of the non-real numbers "p + qi" and "p - qi", where p = h and $q = \sqrt{\frac{k}{a}}$.

How can "h" and "$\sqrt{\frac{k}{a}}$" be determined from the graph? Suppose that the inverted parabola in Figure 2 intersects the x-axis at points R and S, where the coordinates of R are (r, 0) and the coordinates of S are (s, 0),

then $s + r = \left(h + \sqrt{\frac{k}{a}}\right) + \left(h - \sqrt{\frac{k}{a}}\right) = 2h$

or $h = \frac{s + r}{2}$

and $s - r = \left(h + \sqrt{\frac{k}{a}}\right) - \left(h - \sqrt{\frac{k}{a}}\right) = 2\sqrt{\frac{k}{a}}$

or $\sqrt{\frac{k}{a}} = \frac{s - r}{2}$.

The non-real, complex roots $p \pm qi$ can, therefore, be found from the graph of the inverted parabola and expressed in the form

$$\frac{s + r}{2} \pm \frac{s - r}{2}\, i, \text{ where } p = \frac{r + s}{2} \text{ and } q = \frac{r - s}{2}.$$

To illustrate this method, supposing a student is given the graph in Figure 3 and asked to approximate the roots of the corresponding quadratic equation (where y =0). To do so, he merely does three things:

1. invert the parabola
2. determine the x-coordinates of the points where the inverted parabola intersects the x-axis
3. substitute for r and s in the expression $\frac{s + r}{2} \pm \frac{s - r}{2}\, i$

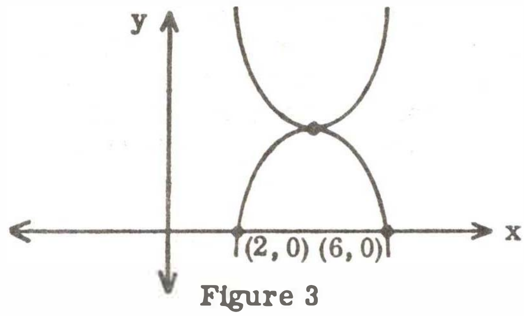


Figure 3

Solution: From the graph, $r = 2$ and $s = 6$.

Therefore, the non-real roots of the equation that correspond to the given parabola (where $y = 0$) are:

$$x = \frac{6+2}{2} \pm \frac{6-2}{2} i \qquad \text{or} \qquad x = 4 \pm 2i.$$

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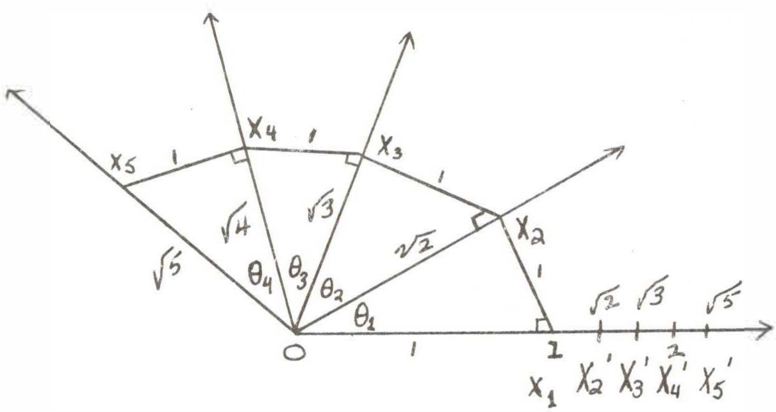
A Problem



Most teachers will be familiar with this diagram in which segments OX_i are constructed with lengths equal to $\sqrt{2}$, $\sqrt{3}$, and so on. The problem is to determine how many repetitions of the construction are necessary so that

$$\theta_1 + \theta_2 + \theta_3 + \ldots + \theta_i \geq 360°$$

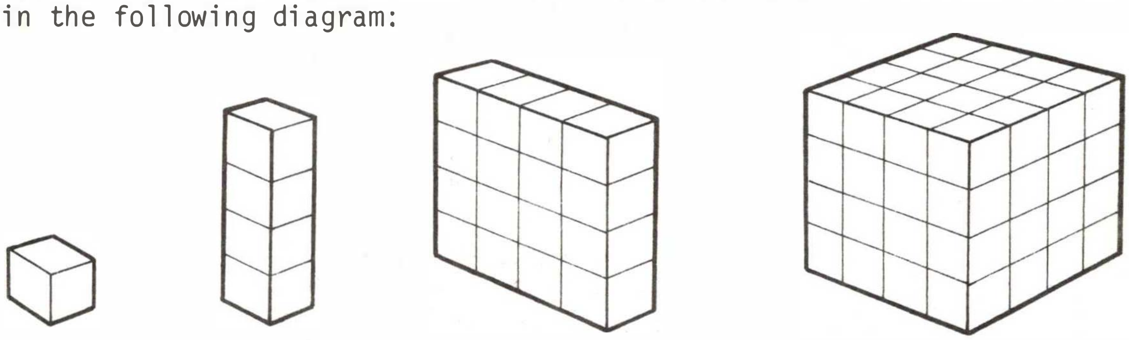
Motivating Mathematics Materials for the Elementary School

Dr. Mary Beaton
University of Calgary
Calgary, Alberta

Activities with real things in an experimental setting help children to develop mathematical concepts. Sylvia Farnham-Diggory has shown why mathematics activities are so important: "Neurologically correct instruction in mathematics involves the development of 'strong' connections between the visual and motor systems ... The symbol systems (notational systems) are processed visually and must then be connected to another kind of information (action in the case of mathematics) if comprehension is to happen ... Every school needs to be stocked with materials to provide visual and action-based mathematical experiences for all children."[1]

Certain types of equipment highlight the mathematical concepts taught in the elementary grades more vividly than do others. In choosing materials for the teaching of mathematics, we need to decide which materials help children most in discovering mathematical relationships. Some of the most useful types of materials in this respect are multi-base blocks, the abacus, the geoboard, a set of balances and logical blocks.

A set of materials which teaches the concept of *BASE* is the Multi-base Blocks, invented by Z.P. Dienes. In three wooden boxes, there are very carefully made wooden blocks which represent quantities useful in demonstrating Base Three, Base Four, Base Five, Base Six, and Base Ten. In Base Four, the materials are as shown in the following diagram:

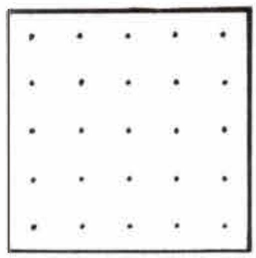


[1]Sylvia Farnham-Diggory, "On readiness and remedy in mathematics instruction", *The Arithmetic Teacher*. Washington, D.C.: NCTM, November, 1968. Pp. 616,621.

Children can learn the meaning of a base system with these blocks apart from the idea of place value. In this way, they can distinguish more readily between the idea of base and the idea of place value.

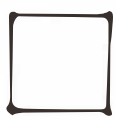
The abacus is useful in teaching place value. The type in which moveable beads can be shifted from the front over the wire to the back can help to clarify the idea that 10 ones can be replaced by 10. Ginn and Co. produces Arithme-Sticks of this kind. For the study of larger numbers to the millions, an abacus having 10 rows of beads is available from Moyer-Vico. A Japanese soroban or abacus could also be used for computing with large numbers. Experienced operators of the soroban in Japan can compute more rapidly than one can use a desk calculator.

An indispensable pupil aid is the geoboard. It can be purchased commercially or can be homemade. Twenty-five nails having round heads are placed in a five-by-five matrix on a six inch by six inch board. A class set of 40 pupil boards and a large demonstration board should be kept in the materials center. A good supply of colored elastics is needed.



The geoboard can be used in teaching the meaning of multiplication. Equal groups can be shown with one color and the total group can be shown with another color. The commutative property and the associative property of multiplication can be discovered by the students with a little guidance from the teacher or with the use of activity cards.

Perimeter and area can be taught using the nail board. Care must be taken here to emphasize that the distance between two nails is the unit of length, and the space enclosed by four nails is the unit of area. Geoboards can also be used extensively in the teaching of geometry in the elementary grades. Concepts such as line segment, angle, polygons can be illustrated by the students on their individual boards.



Geometric solid models may be obtained commercially in clear plastic or in wood. Individual kits provide children with the opportunity to handle and experiment with the models. Poly-O Labs are real time-savers. They contain triangles, squares, pentagons and hexagons made of heavy cardboard. They can be easily attached along the edges with elastic bands. Poly-O Lab C#3043 contains five instruction books plus 100 triangles, 60 squares, 25 pentagons, 15 hexagons, 100

rubber bands. This kit is large enough for 15 students. Students could prepare additional cardboard polygonal shapes as needed. The instructional booklet shows nets and photos or drawings for the five regular solids and for many other polyhedral shapes. Poly-O Labs may be obtained from Book-Lab, Inc., Dept. AT1
11218, 1449 - 37 Street
Brooklyn, New York.

An equalizer balance with 20 washers for weights can be used for discovery lessons in addition, subtraction and multiplication. True or false sentences can be pictured on the balance by the children.

Logical blocks can be used in a variety of ways to build mathematical concepts through experimentation. These blocks consist of sets of large and small circles, squares, rectangles and triangles, each of which has two thicknesses in the colors red, yellow and blue. Games can be played in which a player changes one, two or three attributes during each turn. For example, have the students take turns placing pieces in a line such that the piece being laid down is different from the previous piece by exactly one attribute. If the first piece is a small, thin red square, the next piece to be laid down could be a small, thin blue square. This is the "one-difference game".

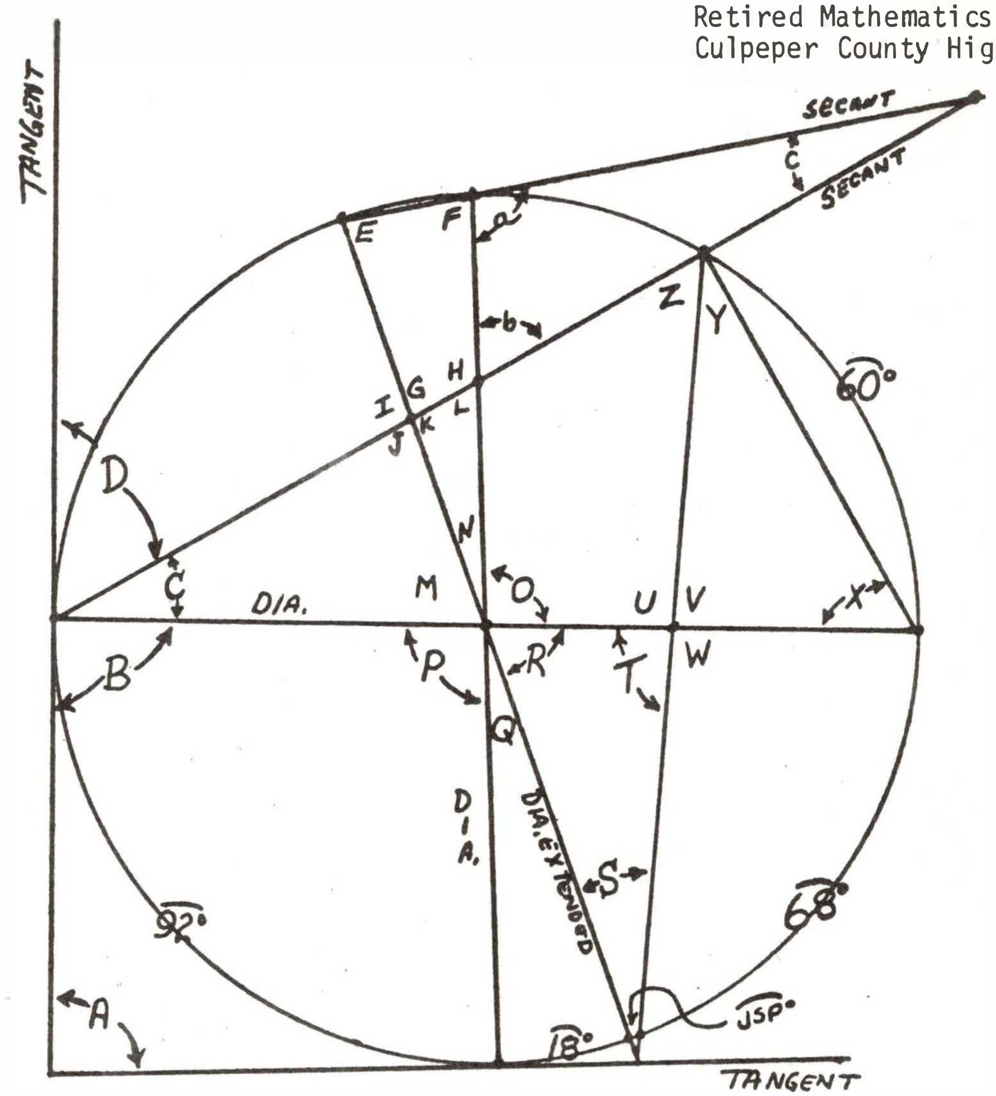
Another game which can be played with the logical blocks requires a rectangular grid. The pieces can be placed in such a way that there is one difference in one direction and two differences in the other. To score this game, allow as many points as there are differences between the piece on the board and the one laid down.

The logical blocks can be used with hoola hoops or ropes to show intersection and union of sets. The experimental approach with real blocks can lead to greater understanding than the visual approach through chalkboard diagrams and textbook illustrations.

Experimental work in elementary mathematics is motivating because it results in greater understanding of the major mathematical concepts.

Angular Measurement

John S. Peifer
Retired Mathematics Teacher
Culpeper County High School



Find the measure of each angle without using a protractor. All answers are integral.

/A = ____	/F = ____	/K = ____	/P = ____	/U = ____	/Z = ____
/B = ____	/G = ____	/L = ____	/Q = ____	/V = ____	/a = ____
/C = ____	/H = ____	/M = ____	/R = ____	/W = ____	/b = ____
/D = ____	/I = ____	/N = ____	/S = ____	/X = ____	/c = ____
/E = ____	/J = ____	/O = ____	/T = ____	/Y = ____	Arc JSP = ____

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PLAN NOW TO ATTEND !

Mathematics Council, ATA

* * ANNUAL CONFERENCE * and * BUSINESS MEETING *

Red Deer,

October 13 (evening) and 14, 1972

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