From the Editor

The Editor apologizes to members for the unusual lapse of time between your last receipt of *Delta-K* and the present mailing. We had planned to devote an entire number to "Mathematics and Reading", which would have been especially useful to our elementary members, but we were unable to obtain material as we had anticipated. Having thus missed our submission deadline, we put together this 'Double Issue': numbers 2 and 3 of volume XII. We sincerely hope that every reader will find in it something to pique his imagination, improve his teaching, or stimulate his professional growth.

The Mathematics Teacher Looks at Reading

Alfred Capoferi
Macomb Intermediate School District
Michigan

It has been said that every teacher is a teacher of reading. If this is true, then all math teachers are reading teachers as well as math teachers. But do all mathematics teachers teach their students to read mathematics? Is there a distinct difference between ordinary reading and reading of mathematics? The main purpose of this article is to focus attention on reading mathematics and how improved reading skills can facilitate learning and understanding of mathematics.

Reading is one of the most complex learning tasks that students encounter in their learning experiences. Frequently, the degree to which children master ordinary reading tasks is correlated to their success in learning mathematics. To complicate matters, the reading tasks of mathematics are even more complex than those normally encountered in the basic reading programs of the schools. Therefore, extra attention must be taken to emphasize special reading skills needed in the area of mathematics.

The following are some of the skills necessary for successful reading of mathematics content:

1. The students must be able to translate the words, phrases, sentences and sentence combinations. This translating process requires that students possess four vocabularies:
   - a verbal symbol vocabulary,
   - a numerical symbol vocabulary,
   - a literal symbol vocabulary,
   - an operational symbol vocabulary and the ability to attack new words which the student may have never encountered before in print.

2. The students must also be able to organize the materials into meaningful thought units. This process is generally associated with reading comprehension. The major skills necessary for adequate comprehension include recognition and utilization of thought relationships, forming generalizations, drawing conclusions, making inferences, interpreting symbols and graphic information such as graphs and formulas, and critical interpretation.

For example, some mathematical statements use only verbal symbols: the area of a rectangle is equal to the product of its length and width. Some math statements use only numerical and operational symbols: 10 x 5 = 50. Whereas some mathematical statements use only literal and operational symbols: A = lw. Further, some math statements mix all four vocabularies together: what is the area of a rectangle where l = 5 ft. and w = 10 ft.?

In a mathematical vocabulary, the referent for which each kind of symbol stands has to be clear before the student can use the symbol with meaning and understanding.

An examination of the three principal levels at which mathematics concepts are presented will establish a better perception of the complexities of reading as they apply to mathematics instruction:

1. Many math concepts are presented on a *concrete* level. This type of presentation involves students in multi-sensory experiences with equipment that can be manipulated by the students and supported with auditory stimulus from the teacher.
2. The presentation of mathematics ideas at the *semi-concrete* and/or *representational* level is more complex than at the concrete level, the student derives information and experiences from such learning aids as models, films, transparencies, illustrations, etc. Visual learning receives primary emphasis at this level.
3. The third and most complex learning is presented at the *abstract* level. Learning at this level requires the use of higher cognitive skills because the student is asked to perform most mathematical tasks with symbolic representations only.

Therefore, to read well in any discipline is to think well in the discipline. Each discipline has a conceptual vocabulary. Each has a unique manner of statement. Each has structures through which it develops, applies, and appraises its ideas. The following will illustrate this point further in mathematics.

How can a mathematics teacher improve reading skills? The following are a few examples where teachers can identify the basic difference between reading literature, newspapers, magazines, and mathematics texts and reference books. For example, when a student reads an ordinary book, he always reads left to right. This is not true when reading a mathematics book. While reading an ordinary book, each student deals predominately with the letters of the alphabet only, whereas in reading a math book he must interpret many more symbols. Furthermore, the additional symbolism involved in reading a math book requires specific interpretation such as order of the symbols in a given problem.

Each teacher of mathematics has the responsibility of teaching each student mathematics such that he/she can:

1. Read It - understand the vocabulary, symbols, charts and graphs.
2. Write It - use and understand the mathematics processes and symbolism.
3. Say It - be capable of verbalizing math ideas.

The following are some illustrations of reading skills that should contribute to a better instructional program with emphasis on the understanding of mathematics.

1. Reading Rate - The math reading rate is slower than the reading rate of ordinary English. This is influenced by the number of symbols, charts and graphs per page. The reading rate of mathematics requires special interpretation and understanding of symbols over and above the alphabet. The following is an example of this:

   Jim and Henry estimate 17 x 96. Here is how each arrived at his estimate:

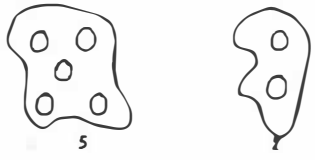
   Jim: 17 is about 20
   96 is about 100
   17 x 96 is about 2000

   Henry: 17 is less than 20
   96 is more than 90
   17 x 96 is about 1800

   Without finding the answer, explain what estimate you would use. Reading this example requires: inference, understanding symbols, different possible solutions.

   These reading skills become an integral part of teaching mathematics.

2. Ordering Symbols - Left-to-right orientation
   Order of symbols? Reading math requires more than left-to-right order. It requires many kinds of order and eye movements to determine order of symbol understanding.



Math problems do not use left-to-right as often as plain reading. Eye movement in math requires more training.

Order of symbols:

$$3/7 + 2/7 = \frac{3+2}{7} = \frac{\Delta}{7} \text{ or } \frac{3}{7} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{array}{r} \\ +\ \frac{2}{7} \\ \hline \\ \frac{\Delta}{7} \end{array}$$

3. Reading With Paper and Pencil
Reading math for understanding requires use of paper to interpret symbols — for example,

$$\begin{array}{r} 4000 + 600 + 20 + 7 \\ \hline \square + \square + \square + \triangle \end{array} = \begin{array}{r} 4627 \\ \times 5 \\ \hline \end{array}$$

Do this problem using both methods: 5263 x 7 = ___?___

4. Reading Graphic Materials - tables, charts, graphs
Special reading skills are necessary to read and understand graphic materials in a mathematics book. Each component part of a table must be identified, such as: specific elements (factors and products), main categories (products), elements within each category (number facts, commutative property)

		(Factor)				
	x	1	2	3	4	5
(Factors)	1	1	2	3	4	5
	2	2	4	6	8	10
	3	3	6	9	12	15
	4	(Products)				

Reading graphic materials in math books requires high-level cognitive skills such as: literal translations of words; interpretation of symbols (order of operations and translation of symbols); identification of the functional relationship of the symbols and words; identification and interpretation of the different kinds of graphic representations of mathematics ideas such as bar graph, line graph, pictograms, histograms, frequency distributions, circle graphs, function graph.

5. Organization of Texts
Teach children to use all aspects of the text: chapter headings, topics - subtopics, index, glossary, appendices.

Teachers should make maximum use of the organization of the books to teach greater understanding of the content. More effort should be made to teach children to use the index, glossary, chapter headings and appendices. Sensitize students to the use of text organization.

6. Main Ideas and Important Details
Main idea of a lesson or chapter may be written in different colored type. Teach children to discover how the main ideas are put together: What is the main idea?

A detail is important in context only. What is the lesson all about? Why read a lesson? What question helped you to get the main idea?

7. Words with Many Meanings

How many words can you identify from the following quotation with a mathematics meaning? "Mary Beth was to *set* the *table*. She heard the roar of a *plane and* ran to the *yard carrying* the dishes. She saw Harry *point* to the sky *and* say, "Wow, *what power*!"

The vocabulary of mathematics is fundamental to the basic understanding of the language of mathematics. Special effort must be made by the teacher to help students discover the special meanings that many words have in a math context. The above quotation illustrates this point.

Mathematics language (vocabulary, especially) development is sometimes overlooked by some teachers unintentionally. This oversight is due sometimes to a greater eagerness to cover more course content. In the early grades, teachers should teach vocabulary development in both mathematics and language simultaneously. Mathematics understanding is greatly facilitated by proper development and use of an adequate mathematics vocabulary.

What is an adequate math vocabulary? Dr. Robert Kane, Professor of Mathematics Education at Purdue University, conducted research in this area and suggested that approximately 1200-1400 technical words and 160 symbols constituted an adequate math vocabulary between Grades IV-XII.

The following is a language skill that should be employed to increase math word meaning:

Using Math Words - Derivations
divide - division - divisor - dividend - divisible - divisibility
commute - commutative - commutativity
associate - associative - associativity
point - mid-point
circle - circular - circumcircle

Teachers must teach students how to derive new math words from the root words.

8. How to Use Math Words

Math vocabulary is used to communicate mathematics ideas in such a way that math words have special meanings in a mathematics context. For example, the word "set" has a basic meaning to mathematics which is an extension of the meaning of the word "set" in an ordinary language translation.

A further example of this is to literally translate the following statement "Take away 2 from 21." A possible literal translation could be "1". Naturally, this answer is a literal translation with total disregard for math meaning that is assumed as part of this statement. Therefore, it is important that teachers teach the mathematics language for meaning and understanding from a mathematics context. The following suggestions for improving language skills should be helpful in carrying out your instructional program with greater understanding and clarity of mathematics ideas and words:

- Separate math words from ordinary English.
- Semantics of math relates to special words and symbols.
- A student who can read ordinary English does not have a guarantee that he can read a math text.

9. Seeing, Using and Understanding Symbols

Reading of mathematics requires that each student recognize words and symbols and understand their meaning in a mathematics context. Therefore, a special effort must be made by teachers to help students to better recognize, understand and functionally translate math symbols as part of their regular instructional program. Symbol understanding provides for a greater student ability to communicate math ideas. Solving "story" problems would become a more simple task for many students if they could translate ordinary English into math symbols and vice versa. Reading the following math symbols requires a specific math skill level and understanding that will have a direct bearing on a student's ability to translate effectively:

Seeing and Understanding Symbols

$(68 + 79) + 83 = 68 + (79 + 83)$

$p \longrightarrow q \longleftrightarrow q \longrightarrow \sim p$

$V = 1/3 \ \pi \sim^{2} h$

10. Helps in Computing

A given math algorithm or process is often taught by referring students to a given example. Students are asked to "read" the example. Here is where some possible difficulty may arise because reading math examples is possible only if the students have had the necessary experiences and background to interpret the symbols in the process. The following example will illustrate the point:

(a)
$$\begin{array}{r} 63 \\ \times\ 8 \\ \hline \end{array} = \begin{array}{r} 60 + 3 \\ \times\ 8 \\ \hline 480 + 24 = 504 \end{array}$$
Expanded notation

(b)
$$\begin{array}{r} 63 \\ \times\ 8 \\ \hline 24 \\ 480 \\ \hline 504 \end{array}$$
Three Steps

(c)
$$\begin{array}{r} 63 \\ \times\ 8 \\ \hline 504 \end{array}$$
One Step

A student can understand the reading of each algorithm if he has had experience in doing algorithms. Reading Example "a" and Example "c" requires different experience levels.

An attempt was made to illustrate how a teacher of mathematics can have a more viable instructional program by including some emphasis on reading skills in the teaching of mathematics.

BIBLIOGRAPHY

Allen, Richard V. and Claryce, *Language Experiences in Reading*. Chicago: Encyclopaedia Britannica Educational Corporation, 1966.

Calfee, Robert C., and Richard L. Venezky, *Component Skills in Beginning Reading*. Madison, Wisconsin: University of Wisconsin, Research and Development Center for Cognitive Learning, 1968.

_____, R. Chapman and R.L. Venezky, "How a Child Needs to Learn to Think to Learn to Read", in *Cognition in Learning and Memory*, L. Greg (ed.). New York: Wiley, forthcoming.

Chall, Jeanne S., *Learning to Read: The Great Debate*. New York: McGraw - Hill Book Company, 1967, pp.83 and 137. Reprinted by permission of the author.

Gage, N.L., (ed.), *Handbook of Research on Teaching, a Project of the American Educational Research Association*. Chicago: Rand McNally and Company, 1963.

Groff, Patrick J., "Comparisons of Individualized and Ability Grouping Approach in Teaching Reading", *Elementary English*, Vol. 41, March 1964, pp.238-241.

King, Martha L., et al (eds.), *Critical Reading*. Philadelphia: J.B. Lippincott Company, 1967.

Lee, Dorris May, and Richard V. Allen, *Learning to Read through Experience*. New York: Appleton Century Crofts, Inc., 1943.

Reprinted from *Mathematics in Michigan*, Michigan Council of Teachers of Maths, Vol. 12, No. 2, November 1972.

EMD Booklets Available

Experiences in Mathematical Discovery, EMD, is a series of self-contained units designed for use by students of Grade IX general mathematics. The two titles just released are "Mathematical Thinking" (Unit 6), with 64 pages, and "Rational Numbers" (Unit 7), with 96 pages. Units 1-5 and 9 were released earlier, so eight booklets of the planned 10-booklet series are now available. An answer key for Units 1-5 is also out. Each pamphlet in the series now sells for $1. They are available from National Council of Teachers of Mathematics, 1201 - 16 Street, NW, Washington, D.C., 20036.