THE ALBERTA
TEACHERS'
ASSOCIATION
MATHEMATICS
COUNCIL

delta-k

VOLUME XII, NUMBERS 2&3, APRIL 1973

Mathematics Festival

The first annual Mathematics Festival will be held Saturday, May 12, at McNally Composite High School in Edmonton, 10 a.m. - 12 noon and 2 - 6 p.m. Judging of entries in the "Fair" section will take place Friday, May 11, at 7:00 p.m. at the same location.

The Math Festival has been planned to create interest and enjoyment in mathematics for *all* students, regardless of ability. For 1973, the festival of events will be restricted to those related to an understanding of Grade IV, V, and VI mathematical concepts.

Activities will include drop-in booths, sideshows, a midway of math games, and a display of fair entries, all relating to mathematical concepts.

Students, parents, and teachers are all welcome. No admission will be charged.

In this issue

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ATTENTION, SUBSCRIBERS: At the recent meeting of the Executive Committee of the Council, it was decided that the subscription rate be $5.00, the same as the membership fee. This would entitle a subscriber to all Math Council publications.

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From the Editor

The Editor apologizes to members for the unusual lapse of time between your last receipt of *Delta-K* and the present mailing. We had planned to devote an entire number to "Mathematics and Reading", which would have been especially useful to our elementary members, but we were unable to obtain material as we had anticipated. Having thus missed our submission deadline, we put together this 'Double Issue': numbers 2 and 3 of volume XII. We sincerely hope that every reader will find in it something to pique his imagination, improve his teaching, or stimulate his professional growth.

The Mathematics Teacher Looks at Reading

Alfred Capoferi
Macomb Intermediate School District
Michigan

It has been said that every teacher is a teacher of reading. If this is true, then all math teachers are reading teachers as well as math teachers. But do all mathematics teachers teach their students to read mathematics? Is there a distinct difference between ordinary reading and reading of mathematics? The main purpose of this article is to focus attention on reading mathematics and how improved reading skills can facilitate learning and understanding of mathematics.

Reading is one of the most complex learning tasks that students encounter in their learning experiences. Frequently, the degree to which children master ordinary reading tasks is correlated to their success in learning mathematics. To complicate matters, the reading tasks of mathematics are even more complex than those normally encountered in the basic reading programs of the schools. Therefore, extra attention must be taken to emphasize special reading skills needed in the area of mathematics.

The following are some of the skills necessary for successful reading of mathematics content:

1. The students must be able to translate the words, phrases, sentences and sentence combinations. This translating process requires that students possess four vocabularies:
   - a verbal symbol vocabulary,
   - a numerical symbol vocabulary,
   - a literal symbol vocabulary,
   - an operational symbol vocabulary and the ability to attack new words which the student may have never encountered before in print.

2. The students must also be able to organize the materials into meaningful thought units. This process is generally associated with reading comprehension. The major skills necessary for adequate comprehension include recognition and utilization of thought relationships, forming generalizations, drawing conclusions, making inferences, interpreting symbols and graphic information such as graphs and formulas, and critical interpretation.

For example, some mathematical statements use only verbal symbols: the area of a rectangle is equal to the product of its length and width. Some math statements use only numerical and operational symbols: 10 x 5 = 50. Whereas some mathematical statements use only literal and operational symbols: A = lw. Further, some math statements mix all four vocabularies together: what is the area of a rectangle where l = 5 ft. and w = 10 ft.?

In a mathematical vocabulary, the referent for which each kind of symbol stands has to be clear before the student can use the symbol with meaning and understanding.

An examination of the three principal levels at which mathematics concepts are presented will establish a better perception of the complexities of reading as they apply to mathematics instruction:

1. Many math concepts are presented on a *concrete* level. This type of presentation involves students in multi-sensory experiences with equipment that can be manipulated by the students and supported with auditory stimulus from the teacher.
2. The presentation of mathematics ideas at the *semi-concrete* and/or *representational* level is more complex than at the concrete level, the student derives information and experiences from such learning aids as models, films, transparencies, illustrations, etc. Visual learning receives primary emphasis at this level.
3. The third and most complex learning is presented at the *abstract* level. Learning at this level requires the use of higher cognitive skills because the student is asked to perform most mathematical tasks with symbolic representations only.

Therefore, to read well in any discipline is to think well in the discipline. Each discipline has a conceptual vocabulary. Each has a unique manner of statement. Each has structures through which it develops, applies, and appraises its ideas. The following will illustrate this point further in mathematics.

How can a mathematics teacher improve reading skills? The following are a few examples where teachers can identify the basic difference between reading literature, newspapers, magazines, and mathematics texts and reference books. For example, when a student reads an ordinary book, he always reads left to right. This is not true when reading a mathematics book. While reading an ordinary book, each student deals predominately with the letters of the alphabet only, whereas in reading a math book he must interpret many more symbols. Furthermore, the additional symbolism involved in reading a math book requires specific interpretation such as order of the symbols in a given problem.

Each teacher of mathematics has the responsibility of teaching each student mathematics such that he/she can:

1. Read It - understand the vocabulary, symbols, charts and graphs.
2. Write It - use and understand the mathematics processes and symbolism.
3. Say It - be capable of verbalizing math ideas.

The following are some illustrations of reading skills that should contribute to a better instructional program with emphasis on the understanding of mathematics.

1. Reading Rate - The math reading rate is slower than the reading rate of ordinary English. This is influenced by the number of symbols, charts and graphs per page. The reading rate of mathematics requires special interpretation and understanding of symbols over and above the alphabet. The following is an example of this:

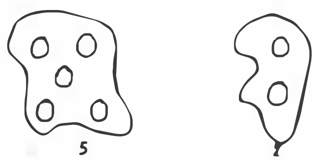
    Jim and Henry estimate 17 x 96. Here is how each arrived at his estimate:

Jim: 17 is about 20	Henry: 17 is less than 20
96 is about 100	96 is more than 90
17 x 96 is about 2000	17 x 96 is about 1800

    Without finding the answer, explain what estimate you would use. Reading this example requires: inference, understanding symbols, different possible solutions.

    These reading skills become an integral part of teaching mathematics.

2. Ordering Symbols - Left-to-right orientation
   Order of symbols? Reading math requires more than left-to-right order. It requires many kinds of order and eye movements to determine order of symbol understanding.



Math problems do not use left-to-right as often as plain reading. Eye movement in math requires more training.

Order of symbols:

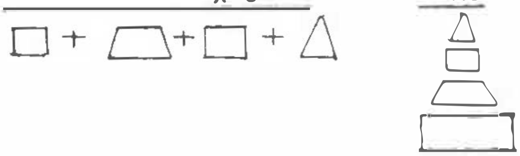
$$3/7 + 2/7 = \frac{3 + 2}{7} = \frac{\Delta}{7} \text{ or } \frac{3}{7}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{array}{r} \frac{3}{7} \\ + \frac{2}{7} \\ \hline \frac{\Delta}{7} \end{array}$$

3. Reading With Paper and Pencil
Reading math for understanding requires use of paper to interpret symbols — for example,

4000 + 600 + 20 + 7 = 4627
x 5 x5



Do this problem using both methods: 5263 x 7 = ____?____

4. Reading Graphic Materials - tables, charts, graphs
Special reading skills are necessary to read and understand graphic materials in a mathematics book. Each component part of a table must be identified, such as: specific elements (factors and products), main categories (products), elements within each category (number facts, commutative property)

		(Factor)				
	x	1	2	3	4	5
(Factors)	1	1	2	3	4	5
	2	2	4	6	8	10
	3	3	6	9	12	15
	4	(Products)				

Reading graphic materials in math books requires high-level cognitive skills such as: literal translations of words; interpretation of symbols (order of operations and translation of symbols); identification of the functional relationship of the symbols and words; identification and interpretation of the different kinds of graphic representations of mathematics ideas such as bar graph, line graph, pictograms, histograms, frequency distributions, circle graphs, function graph.

5. Organization of Texts
Teach children to use all aspects of the text: chapter headings, topics - subtopics, index, glossary, appendices.

Teachers should make maximum use of the organization of the books to teach greater understanding of the content. More effort should be made to teach children to use the index, glossary, chapter headings and appendices. Sensitize students to the use of text organization.

6. Main Ideas and Important Details
Main idea of a lesson or chapter may be written in different colored type. Teach children to discover how the main ideas are put together: What is the main idea?

A detail is important in context only. What is the lesson all about? Why read a lesson? What question helped you to get the main idea?

7. Words with Many Meanings
How many words can you identify from the following quotation with a mathematics meaning? "Mary Beth was to *set* the *table*. She heard the roar of a *plane and* ran to the *yard carrying* the dishes. She saw Harry *point* to the sky *and* say, "Wow, *what power*!"

The vocabulary of mathematics is fundamental to the basic understanding of the language of mathematics. Special effort must be made by the teacher to help students discover the special meanings that many words have in a math context. The above quotation illustrates this point.

Mathematics language (vocabulary, especially) development is sometimes overlooked by some teachers unintentionally. This oversight is due sometimes to a greater eagerness to cover more course content. In the early grades, teachers should teach vocabulary development in both mathematics and language simultaneously. Mathematics understanding is greatly facilitated by proper development and use of an adequate mathematics vocabulary.

What is an adequate math vocabulary? Dr. Robert Kane, Professor of Mathematics Education at Purdue University, conducted research in this area and suggested that approximately 1200-1400 technical words and 160 symbols constituted an adequate math vocabulary between Grades IV-XII.

The following is a language skill that should be employed to increase math word meaning:

Using Math Words - Derivations
divide - division - divisor - dividend - divisible - divisibility
commute - commutative - commutativity
associate - associative - associativity
point - mid-point
circle - circular - circumcircle

Teachers must teach students how to derive new math words from the root words.

8. How to Use Math Words
Math vocabulary is used to communicate mathematics ideas in such a way that math words have special meanings in a mathematics context. For example, the word "set" has a basic meaning to mathematics which is an extension of the meaning of the word "set" in an ordinary language translation.

A further example of this is to literally translate the following statement "Take away 2 from 21." A possible literal translation could be "1". Naturally, this answer is a literal translation with total disregard for math meaning that is assumed as part of this statement. Therefore, it is important that teachers teach the mathematics language for meaning and understanding from a mathematics context. The following suggestions for improving language skills should be helpful in carrying out your instructional program with greater understanding and clarity of mathematics ideas and words:

- Separate math words from ordinary English.
- Semantics of math relates to special words and symbols.
- A student who can read ordinary English does not have a guarantee that he can read a math text.

9. Seeing, Using and Understanding Symbols

Reading of mathematics requires that each student recognize words and symbols and understand their meaning in a mathematics context. Therefore, a special effort must be made by teachers to help students to better recognize, understand and functionally translate math symbols as part of their regular instructional program. Symbol understanding provides for a greater student ability to communicate math ideas. Solving "story" problems would become a more simple task for many students if they could translate ordinary English into math symbols and vice versa. Reading the following math symbols requires a specific math skill level and understanding that will have a direct bearing on a student's ability to translate effectively:

Seeing and Understanding Symbols

$$(68 + 79) + 83 = 68 + (79 + 83)$$

$$p \rightarrow q \longleftrightarrow q \rightarrow \tilde{p}$$

$$V = 1/3 \ \pi r^2 h$$

10. Helps in Computing

A given math algorithm or process is often taught by referring students to a given example. Students are asked to "read" the example. Here is where some possible difficulty may arise because reading math examples is possible only if the students have had the necessary experiences and background to interpret the symbols in the process. The following example will illustrate the point:

(a) $\begin{array}{r} 63 \\ \underline{\times 8} \end{array} = \begin{array}{r} 60 + 3 \\ \underline{\times 8} \\ 480 + 24 = 504 \end{array}$ Expanded notation

(b) $\begin{array}{r} 63 \\ \underline{\times 8} \\ 24 \\ \underline{480} \\ 504 \end{array}$ Three Steps

(c) $\begin{array}{r} 63 \\ \underline{\times 8} \\ 504 \end{array}$ One Step

A student can understand the reading of each algorithm if he has had experience in doing algorithms. Reading Example "a" and Example "c" requires different experience levels.

An attempt was made to illustrate how a teacher of mathematics can have a more viable instructional program by including some emphasis on reading skills in the teaching of mathematics.

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EMD Booklets Available

Experiences in Mathematical Discovery, EMD, is a series of self-contained units designed for use by students of Grade IX general mathematics. The two titles just released are "Mathematical Thinking" (Unit 6), with 64 pages, and "Rational Numbers" (Unit 7), with 96 pages. Units 1-5 and 9 were released earlier, so eight booklets of the planned 10-booklet series are now available. An answer key for Units 1-5 is also out. Each pamphlet in the series now sells for $1. They are available from National Council of Teachers of Mathematics, 1201 - 16 Street, NW, Washington, D.C., 20036.

101 Math Ideas

J.F. Woloshchuk

Editor's Note: The following article was submitted for publication under the title "101 Ideas for a Mathematics Department Head". While a few of the ideas might be applicable only to a person with the "Department Head" title, we felt that the number of ideas applicable to any mathematics teacher was sufficient to merit publication. While many of the ideas are perhaps strictly aimed at the large high school, we hope that every reader will find at least one idea that he may pursue. Mr. Woloschuk is Mathematics Department Head at Dr. E.P. Scarlett High School, Calgary.

1. Form a mathematics club - activities for such a club could include field trips to various local institutions and industries.
2. Interview students - priority would be to speak to failures.
3. Accept suggestions from teachers on improving instruction and improving the mathematics department in general.
4. Develop a list of free teaching materials and other resources (speakers) for mathematics education in your area.
5. Arrange student tutorials - extra help beyond class time.
6. Arrange mathematics scholarships and awards.
7. Examine and discuss experimental programs, innovation - for example, different forms of individualized instruction.
8. Conduct a study and write it up. For example, set out the objectives for mathematics education in your school and have the students assess the extent to which objectives have been met. Then do the appropriate analysis and written report.
9. Encourage zone meetings of teachers in order to exchange ideas, examine displays, student projects, new methods and materials. A zone might contain three senior high schools with their respective junior high feeder schools.
10. Discuss course coverage with other teachers.
11. Arrange visitation of other schools.
12. Teach students how to study mathematics. A special NCTM booklet entitled *How to Study Mathematics* can be obtained for this purpose.
13. Examine and discuss various methods of instruction. These methods could be: audio-visual presentations, computer-assisted instruction, communication lessons, creative learning lessons, discovery lessons, enrichment lessons, individualized instruction, laboratory lessons - active learning, mathematical games, small group instruction, student-directed class discussions, teacher lecture.
14. Spend time on teacher professional development at department meetings.

15. Visit E.D.C.I. department at a university, university library, public library, A-V center at school board; screen these centers for ideas and resources.
16. Keep a detailed inventory of equipment and supplies.
17. Encourage teacher involvement in departmental planning.
18. Assess teachers' feelings on new ideas and approaches.
19. Subscribe to mathematics magazines such as *The Mathematics Teacher*, *Delta-K*, *Mathematics Magazine*, and math teachers' journals from neighboring provinces and states.
20. Develop an "Academic Corps" - top students help others.
21. Encourage computer projects.
22. Bring in appropriate personnel to speak on mathematics and computer science programs at universities, SAIT, NAIT, and junior colleges.
23. Develop a testing file - shared by all mathematics teachers.
24. Develop a worksheet file - shared by all mathematics teachers.
25. Facilitate proper program placement of students.
26. If you are the department head, teach some of the less desirable courses.
27. Encourage common examinations in courses.
28. Promote departmental and interdepartmental communication in your school.
29. Give *positive* comments and suggestions to other teachers. Credit should be given when due.
30. Invite teachers into your classes.
31. Consult teachers about courses they would like to teach.
32. Develop a handout, "How to Study Mathematics" and distribute through the department.
33. Develop a mathematics skill test specifically designed to make students *read* directions on examinations.
34. Provide leadership on interpretation of test marks. Discuss departmental mark analysis.
35. Arrange for an A-V orientation for teachers of your staff.
36. Do public library research for games, projects and experiments.
37. Circulate book displays and charts through the department.
38. Design a portable mathematics showcase. This can be built at the school.
39. Encourage coordination with science and other departments in the school.
40. Develop a mathematics laboratory.
41. Develop a mathematics resource center for your school. Involve the library.
42. Why not develop a public relations program in cooperation with the science department?

43. Computer terminal at your school?
44. Discuss homework assignments. Are they functional if too lengthy?
45. Initiate a two-way communication - staff $\leftrightarrows$ administration.
46. Discuss the "core curriculum" problems, possible changes, and such.
47. Arrange meetings with junior feeder high schools.
48. Participate in MCATA and NCTM. Their conferences are often excellent.
49. Ponder on best possible staff placement and deployment.
50. Mathematics Christmas Tree, Easter Basket, Cornucopia, Valentine.
51. Ensure close communication with guidance personnel in your school.
52. Use calculators (Mathematics 15, 25 or elsewhere).
53. Arrange speakers from banks and industries for your mathematics classes.
54. Encourage student projects - creative constructions, mathematical models, experiments.
55. Have students do mathematics reports from a list of 30 or more mathematical topics. Reports encourage library research in mathematics.
56. Encourage student demonstration lessons on special projects.
57. Exchange information and ideas with the school library.
58. Ask staff members to report on classroom techniques which have proved effective.
59. Arrange a library orientation of teachers and students.
60. Business education department may be willing to do some typing of exams, worksheets for the mathematics department.
61. Discuss strengths and weaknesses of the mathematics programs in your school.
62. Arrange displays and demonstrations related specifically to the application of mathematics such as newspaper articles, concrete objects, charts.
63. Publish a mathematics student magazine.
64. Use local shop facilities to build wooden mathematical models - to be used in classroom instruction.
65. Collect a file of brain teasers to begin classes.
66. Encourage teachers to vary the beginning of a mathematics class period.
67. Conduct workshops using books such as *Freedom to Learn*.
68. Provide a list of varied activities for the Mathematics 15 and 25 programs.
69. Arrange mini-courses in areas not presently covered in our high school curriculum such as computers, topology, non-Euclidean geometry.
70. Stimulate departmental participation in local or provincial curriculum development.
71. Consider mental mathematics for beginning a class in mathematics.
72. Procure filing cabinets with distinct slots for "Hard to Teach" Mathematics classes. These would be used to store assignments.

73. Keep an inventory of course references for teachers.

74. Develop a Mathematics 15 unit entitled "Mathematics in Business and Industry". This can be done by writing businesses for sample problems which demonstrate the applicability of mathematics.

75. Develop evaluative criteria for your department.

76. Publish relevant techniques and activities in the NCTM journal or *Delta-K*.

77. Demonstrate the application of computers.

78. Develop a form letter which can be utilized by any member of the department when he has concern over a particular student. This form letter would be mailed home.

79. Communicate with parents. Have them phone the school when their son or daughter is to be absent.

80. Encourage teachers to phone parents when students are late for class, causing discipline problems, or underachieving.

81. Show students mathematical errors in graphs that are displayed by newspapers and other media.

82. Consider short mathematical quizzes, oral or written.

83. Establish a computer club.

84. Invite guidance, administration and library personnel to attend any of the department meetings.

85. Arrange an agenda a few days before the department meeting. This allows teachers to think about the items of discussion.

86. Expose teachers to the *Self-Evaluation Guide for High Schools - Part IV* - prepared by the ATA.

87. Provide members of the department with a list of important dates and happenings in the school. Do so on a regular basis.

88. Meet on a social basis with your staff.

89. The ATA has a series on the improvement of instruction. One could obtain these and discuss certain sections of them.

90. Provide all members of the department with teachers' editions and solution keys (if available) for the courses they teach.

91. Approach the local media (TV, radio or newspaper) to be at your school at a mathematics public relations night.

92. Conduct small surveys within the school and use the results in teaching statistics.

93. Allow accelerated students to do independent study.

94. Develop a teacher evaluation questionnaire which they may use if they wish.

95. Purchase class sets of slide rules, geometry sets, yardsticks, rulers, Knott's tables.

96. If possible, obtain outdated references and books from your local school board stores.

97. Invite central office personnel to public relations nights at your school.

98. Develop a set of slides that demonstrate the application of mathematics in the construction of your school or nearby building projects.

99. Maintain a positive attitude with students and teachers.

100. Develop summary sheets on certain aspects on the mathematics courses; for example, the various types of factoring can be summarized on one sheet.

101. Be flexible and be prepared to accept educational change.

Elementary Ideas

Elementary Teachers! Are you looking for a drill activity, or for something to give your fast students when they have finished their seatwork? Perhaps one of the following activities would fill the bill exactly. These items have appeared in several NCTM affiliate publications.

COMBINATIONS

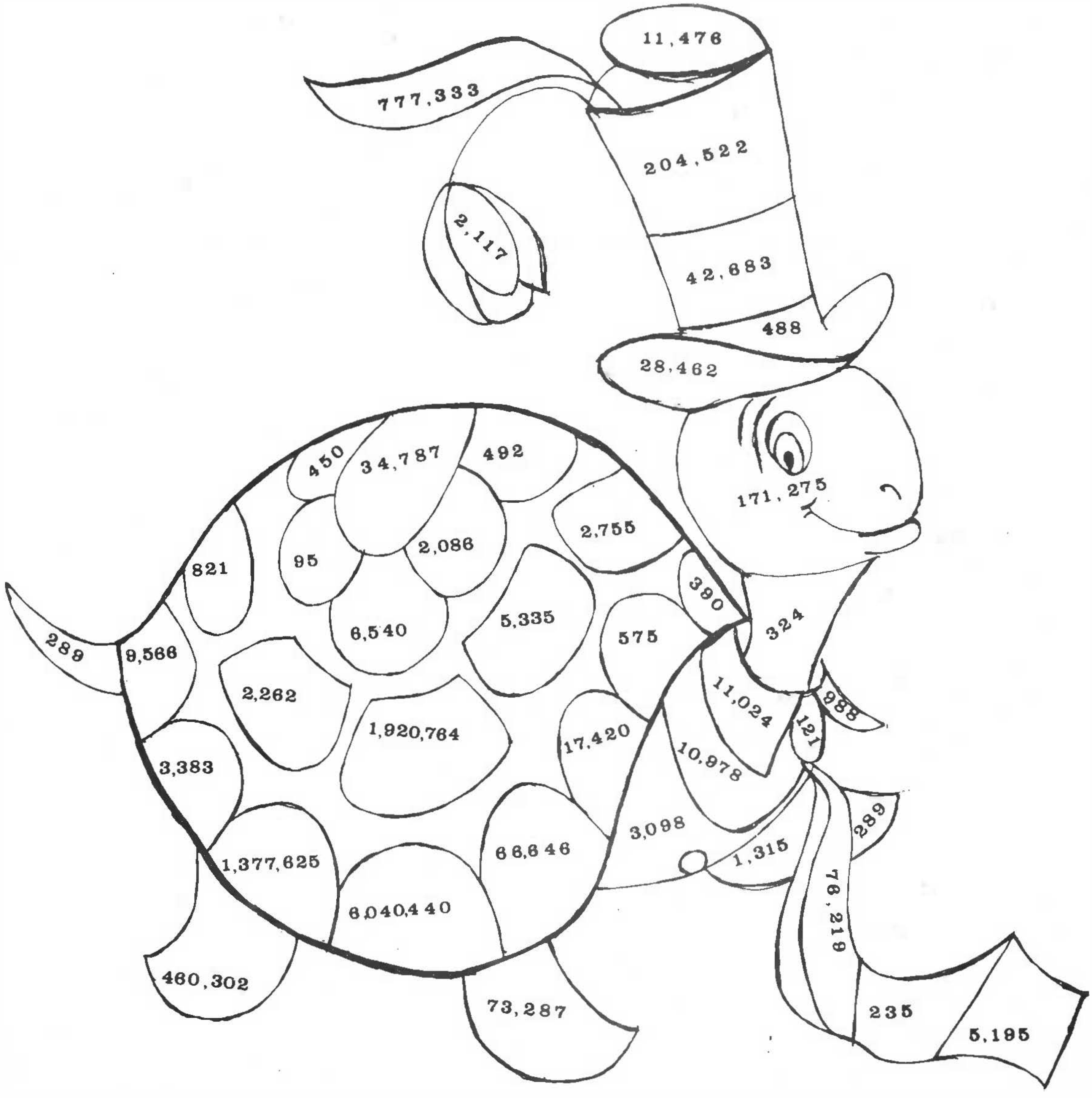
Number combinations appear in the grid vertically, horizontally, and diagonally. If you examine the grid closely, you will find many of the basic facts for addition, subtraction, multiplication, and division. See how many you can find! Insert the correct sign of operation and the equal sign .
Do not overlap!

23	9	3	6	81	60	7	12	19	57	76
45 −	2	3	12	48	6 +	8 =	14	25	33	58
3	35 =	18	4	72	42	30	56	3	44	66
15	5	10	9	12	5	28	11	17	35	16
72	38	8	36	6	6	4	27	31	5	1
49	8	80	4	20	11	7	4	9	36	2
64	4	24	8	3	5	15	45	40	3	10
7	13	12	32	35	19	8	3	6	18	3
32	33	28 ÷	4 =	7	6	9	4	19	21	5 ×
58	18	40	36	3	7	21	28	36	4	9 =
24	54	37	26	63	13	49	57	20	25	45
3	72	17	4	7	6	52	2	26	50	72
14										

ROUNDING TO NEAREST HUNDRED

If "hundred's" place is

1 - color the space red
2 - color blue
3 - color green
4 - color brown
5 - color yellow
6 - color orange
7 - color black
8 - color purple
0 - don't color

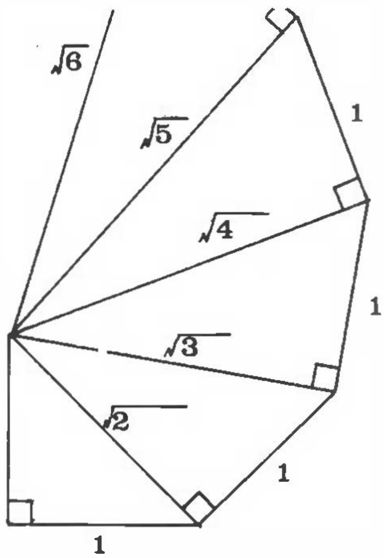


A Computer Application in Trigonometry

William J. Bruce

Editor's note: The following article was submitted in response to "A Problem", which was published in *Delta-K*, Volume XI, Number 4, August 1972, p.8. Dr. Bruce is a member of the Department of Mathematics at the University of Alberta.

We consider the compass and straight-edge construction of the square roots of the natural numbers, a result that leads to a spiral effect as shown.



Starting with a right triangle with unit legs, we obtain a measure of $\sqrt{2}$ units for the hypotenuse. With this hypotenuse as base, another right triangle with one unit leg can be constructed. Its hypotenuse measures $\sqrt{3}$ units. The process can be repeated to produce segments that measure $\sqrt{n}$, where n is any natural number. Incidentally, the area of each triangular region so formed is given by $\frac{1}{2}\sqrt{n}$.

It is intuitively obvious that eventually the triangular regions will overlap, but when does this first occur? Careful drawing makes it apparent that the segment representing $\sqrt{18}$ is the first one to intersect the original triangle. However, it is rather close, so we need to examine the problem more closely to be sure.

When the sum of the central angles of the triangles is first greater than 360° (or 2π) we shall know that the overlap has occurred. The measures of these angles are given by Arccot $\sqrt{n}$, where n is a natural number and principal values are indicated. Clearly, we need to compute

$$\sum_{n=1}^{k} \text{Arccot } \sqrt{n}$$

to find the necessary integral value of "k" for the first overlap.

Whether we use one of the new pocket computers, or otherwise, we find that Arccot $\sqrt{n}$ is not programmed. However, Arctan $\sqrt{n}$ is programmed. Now

$$\sum_{n=1}^{k} \text{Arccot } \sqrt{n} = k(\pi/2) - \sum_{n=1}^{k} \text{Arctan } \sqrt{n}.$$

The computer print-out gives us the following results in radian measure for n=1 to n=17:

n=1	0.7853981634	n=7	3.557622403	n=13	5.369914672
n=2	1.400877872	n=8	3.897459312	n=14	5.631072083
n=3	1.924476648	n=9	4.219209866	n=15	5.883752338
n=4	2.388124257	n=10	4.525487236	n=16	6.128731001
n=5	2.808658592	n=11	4.818330007	n=17	6.366672126
n=6	3.196255279	n=12	5.099364909		

The last entry is the first one to exceed 2π (approx. 6.2832). Hence the triangle with base $\sqrt{17}$ is the first one to overlap the first triangle. The line segment of measure $\sqrt{18}$ intersects the first triangle. In degree measure, the computer yields

n=16 351.15°
n=17 364.78°.

The pocket computer yields $\sum_{n=1}^{k}$ Arctan $\sqrt{n}$ in degree measure. This has to be subtracted from k(90°) to get the above results.

Since the central angles are continually getting smaller, it is obvious that considerably more terms of the series will need to be summed to find when the second overlap will occur. Try to make a conjecture and test it on a computer.

A diller, a dollar,
A witless trig scholar
On a ladder against a wall.
If length over height
Gives an angle too slight,
The cosecant may prove his downfall.

L.A. Graham

"Math teachers never die; they just reduce to lowest terms."

Positivizing Operations

Murray R. Falk
Mathematics Teacher
Ernest Manning High School
Calgary

During a recent lesson on absolute value equalities with my Mathematics 20 class, we were graphing the relation $|x| + |y| = 6$. The graph is a square, intersecting the axes at ± 6 as in Figure 1, below. One student asked how, when the intercepts were found, we knew it wasn't a circle. We had been graphing circles and other conics the few days previously, and she was surprised the graph was not a circle. So I sketched Figure 2 on the board, and asked her for the equation of the circle. She responded correctly with $x^2 + y^2 = 36$. I wrote the 36 as 6^2.

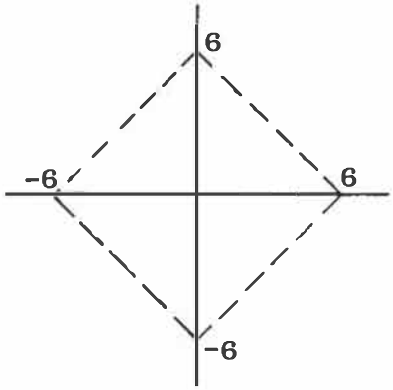


Figure 1: $|x| + |y| = 6$

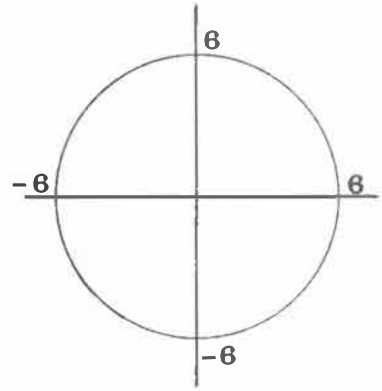


Figure 2: $x^2 + y^2 = 6^2$

Then the students were asked to look at the two relations carefully. Most of them were puzzled as to just what to look for, so I wrote $y = x^2$ and $y = |x|$ on the board. These were familiar to them, and they were asked to sketch the two side-by-side. Figures 3 and 4 were sketched by most of the students.

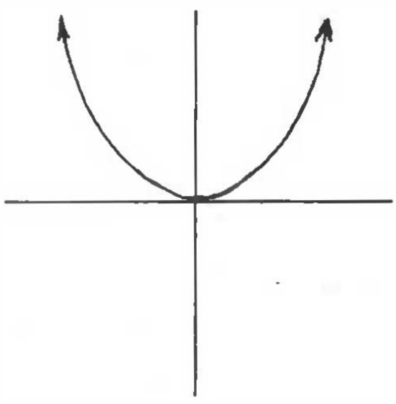


Figure 3: $y = x^2$

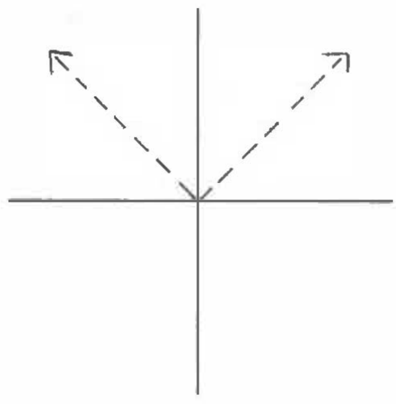


Figure 4: $y = |x|$

Someone suggested that "the curves are always the same, except one is straight". We tried the pair of equations $\frac{x^2}{4} + \frac{y^2}{1} = 1$, and $\left|\frac{x}{2}\right| + \left|\frac{y}{1}\right| = 1$, which are sketched below.

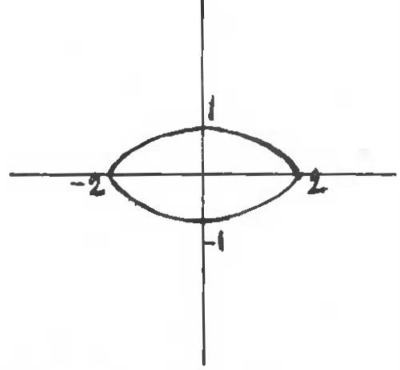


Figure 5: $\frac{x^2}{4} + \frac{y^2}{1} = 1$

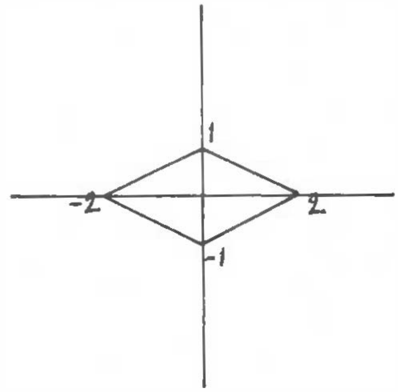


Figure 6: $\left|\frac{x}{2}\right| + \left|\frac{y}{1}\right| = 1$

They were familiar with hyperbolas too, so we tried $x^2 - y^2 = 1$ and $|x| - |y| = 1$, which yielded the graphs in Figures 7 and 8.

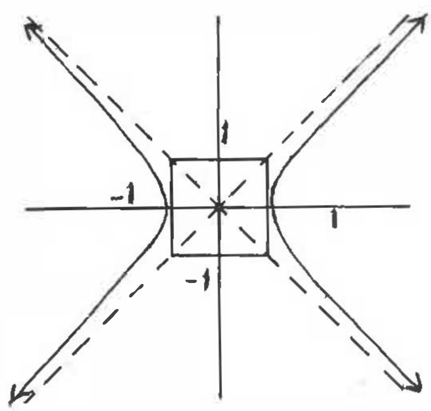


Figure 7: $x^2 - y^2 = 1$

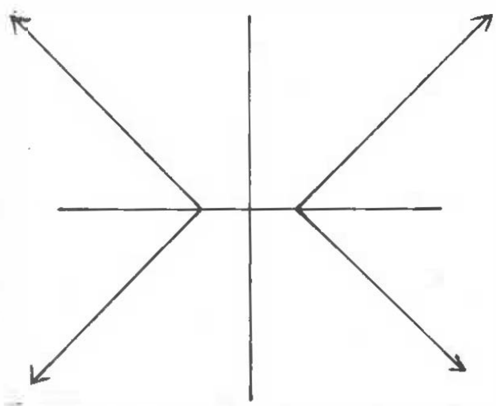


Figure 8: $|x| - |y| = 1$

By now, practically everyone had the idea, and many of the students were wondering "Why?". I heard, "Hey, that's neat!" from the back of the room.

There were only a very few minutes left in the period for an explanation, so it had to be brief. It centered around the fact that the operations of squaring and of taking the absolute value are "Positivizing" operations - that is, they always yield a positive number, whether the operand is positive or negative. Hence the relations behave similarly, but with the difference that one kind produces curves that are curved, the other produces curves which are straight.

For homework, I suggested that they test the hypothesis by comparing $y = (x - 1)^2$ with $y = |x - 1|$; $y = x^2 - 5$ with $y = |x| - 5$; $y = -2(x - 3)^2 + 4$ with $y = -2|x - 3| + 4$; and $x = y^2$ with $x = |y|$. Why don't you try these examples, too?

Also, can you come up with a better explanation than mine?

Models for Teaching Fractions and Percent

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Reprinted from *Math-O-Gram*, Virginia Education Association, Spring 1972.

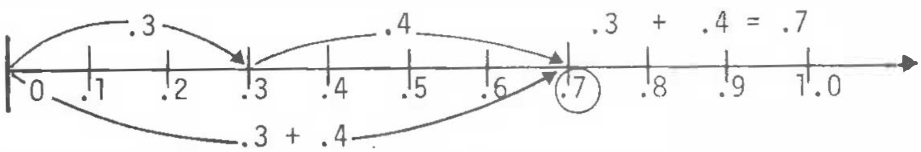
Today the study of fractional numbers in the form of common fractions begins in the primary grades of the elementary school. In the intermediate grades, pupils learn that decimal fractions and percents are other ways to express fractional numbers. Although most textbooks and teachers delay the introduction of decimal fractions and percent to Grades V and VI, recent research has indicated that some of these concepts can be taught earlier. Furthermore, the anticipated future adoption of the metric system may in time expedite this matter. In fact, in 1969 the Cambridge Conference on the Correlation of Science and Mathematics in the Schools suggested that children in the primary grades should learn the metric system and decimal notation.

To help children studying decimal fractions and percent for the first time in any grade, it is useful to relate these new concepts to some ideas with which the children are familiar, such as common fractions and operations on sets of numbers. Instructional aids can demonstrate the close relationship between these "old" and "new" concepts. This article attempts to present a variety of physical models which elementary school teachers can use when developing concepts of decimal fractions and percent.

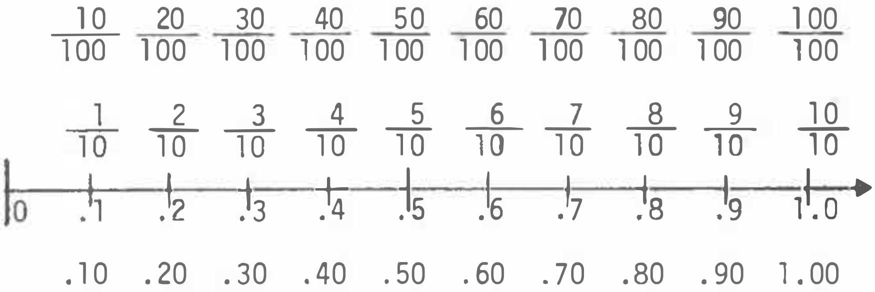
The models described here include some patterns which have resulted from the author's classroom experience and research and also some ideas collected by preservice teachers enrolled in "Teaching Mathematics in the Elementary School" at Madison College. Providing for the creative involvement of pupils in discovery-type situations, these models emphasize observation and exploration.

NUMBER LINE. The decimal number line can be used as early as Grade I, provided the whole number line is introduced first.

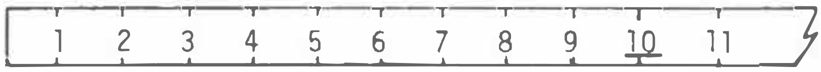
1. With the help of the decimal number line, the teacher can demonstrate the basic number operations.



2. The composite number line which includes both common and decimal fractions is useful for teaching the reading of decimals; it shows clearly the relationship between the number of zeros in the denominator of a common fraction and its respective decimal fraction.



METER STICK. If a meter stick is used to represent a unit, then the decimeter and centimeter will demonstrate tenths and hundredths, respectively.

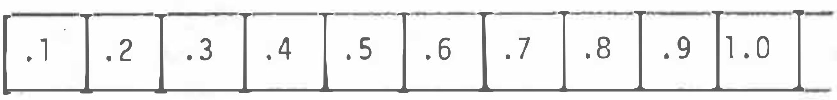


MONEY. Dollars can easily be used to represent whole numbers; dimes, tenths; and cents, hundredths. A chart can be prepared summarizing the use of money to demonstrate the equivalence of fractions, decimals, and percents.

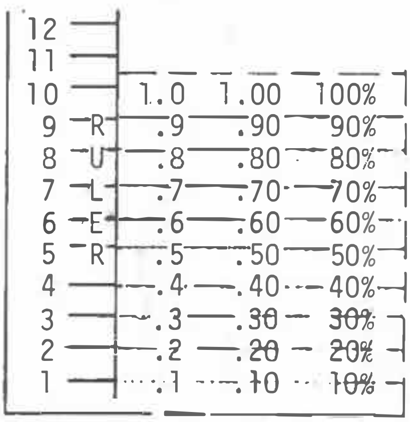
	decimal	percent	fraction
1¢	.01	1% of $	1/100 of $
10¢	.10	10% of $	10/100 of $
25¢	.25	25% of $	25/100 of $
50¢	.50	50% of $	50/100 of $

SNAP OR POP BEADS. Assemble nine beads of one color and a tenth bead of a different color. This bead ruler can be as long as 100 beads, and can be used to measure things in the classroom.

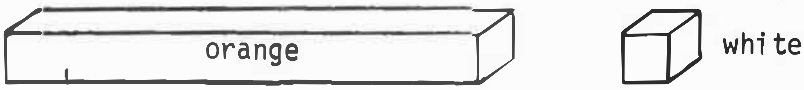
METRIC RULERS. Such rulers can be prepared from paper grided with centimeter squares. The students will then use the rulers to measure many things in the classroom.



TABS. Equivalent fractions, decimals, and percents can be displayed by preparing and taping tabs to a ruler.



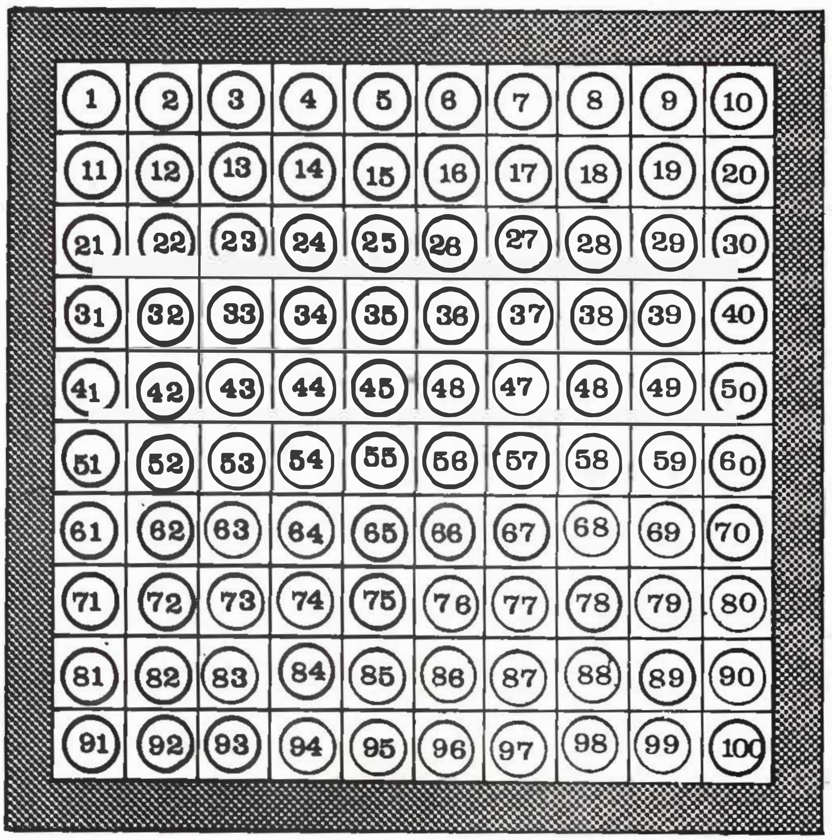
CUISENAIRE RODS. The white and orange rods can be used to represent units and tenths, respectively.



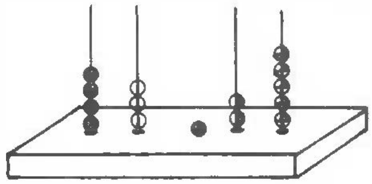
PAPER FOLDING AND SHADING. Strips of paper can be folded to show equivalent fractions, decimals, and percents.

1/4 = .25 = 25%

HUNDRED BOARD. This board can be used to show relationships between fractions, decimals, and percents. Let the entire board represent one or 100% so that each of the 100 squares or disks represents 1/100, .01, or 1%.

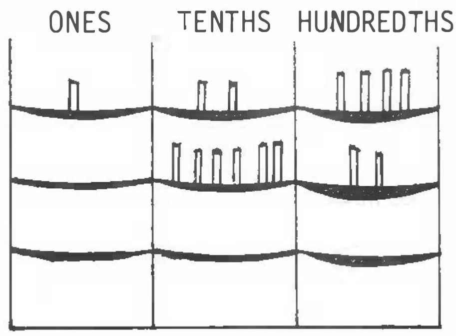


ABACUS. This device is effectively used to demonstrate decimal place value and the four basic operations involving decimal fractions.

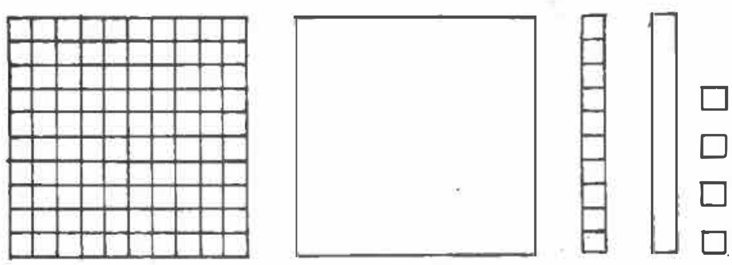


43.25

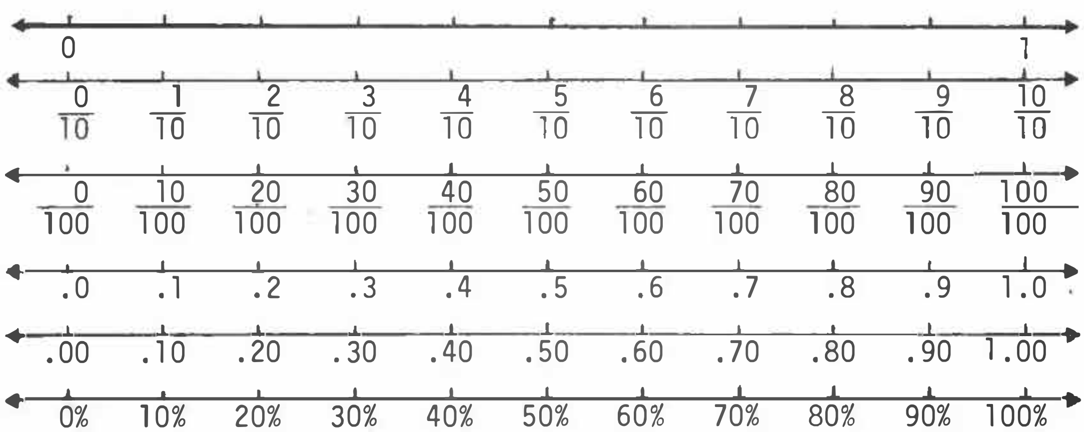
DECIMAL PLACE VALUE CHART. Place value and the basic operations involving decimal fractions can be demonstrated using this chart.



SQUARES AND RECTANGULAR STRIPS. The larger square with one side ruled and the other plain represents 1, 1.00, or 100%. A single strip represents 1/10, .1, and 10% whereas the small squares represent 1/100, .01, and 1%.



FRACTION-DECIMAL-PERCENT EQUIVALENCE CHART

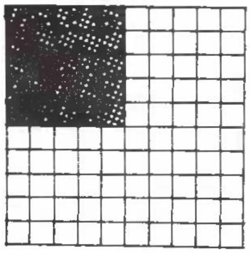


HUNDRED GRID. To display equivalents of fractions, decimals, and percents, shade parts of the hundred grid.

$\frac{1}{4}$

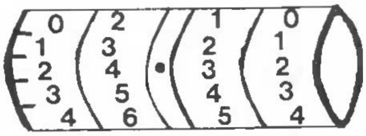
.25

25%

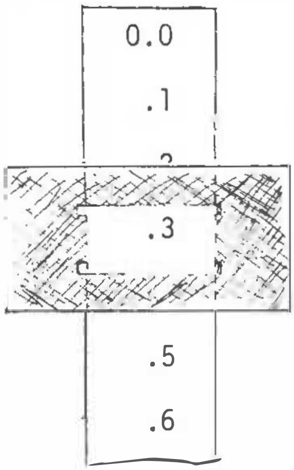


ODOMETER. This device can be used to illustrate decimal place value. Prepare (1) by encircling a can using individual 1"-wide strips of paper marked with a decimal point or the numerals 0-9

Reads
24.32



or (2) by cutting a window in an index card and inserting a strip bearing the decimal numbers.



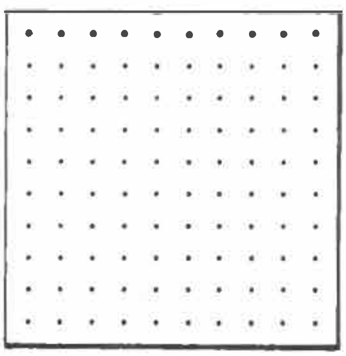
Odometer (1) also can be used to show addition, subtraction, and multiplication.

PEG BOARD. This board, in addition to showing equivalent fractions, decimals, and percents, can be used to demonstrate the four basic operations involving common and decimal fractions.

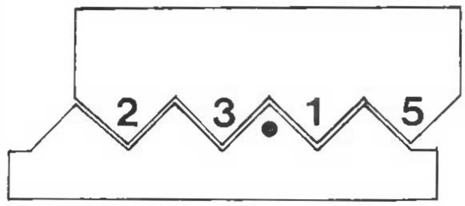
$\frac{1}{10}$

.10

10%



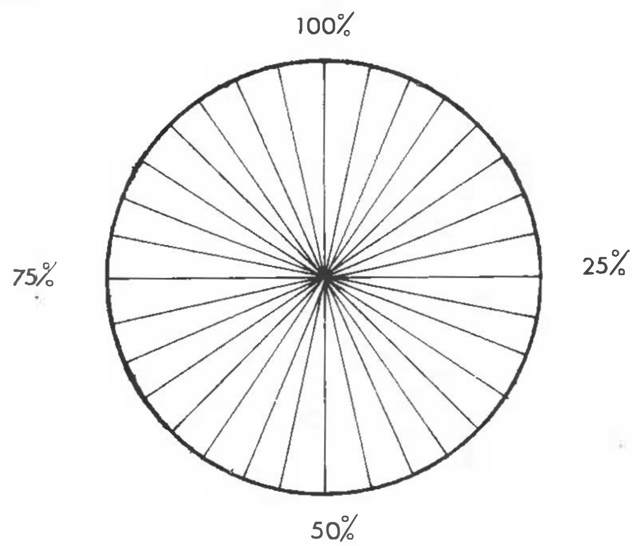
NOTCH CARDS. Prepare notched cardboard; the upper card should be covered with plastic on which different numerals can be written with a grease pencil. The lower card should be marked with a single decimal point. By moving the numerals to the right or left of the decimal point, the value of the number displayed will change in multiples of 10. The upper card can also be used in front of a class if the decimal point is placed on the chalkboard.



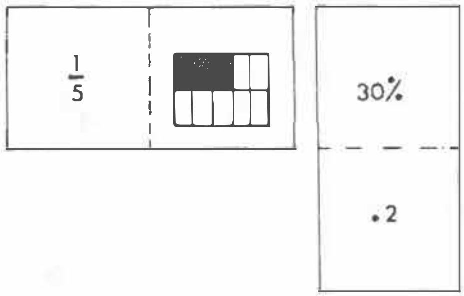
MIXED, DECIMAL, AND FRACTIONAL NUMBER COMPARISON CHART. This chart should assist students in extending their comparison skills to numbers in excess of one.

Mixed Number	Hundreds	Tens	Ones		Tenths	Hundredths	Fraction
6 + 7/10			6	.	7		67/100
25+83/100		2	5	.	8	3	2583/100

CELLOGRAPH PIE CHART. Such a chart is used to illustrate equivalent fractions, decimals, and percents. It is marked from 0-100 with numerals appearing at every fifth interval.



DOMINOES. Such a set can be easily constructed from cardboard, tiles, or wooden blocks. They are used to evaluate knowledge of equivalent fractions, decimals, and percents.



MAGIC SQUARES. These squares can be designed using decimal fractions.

.8	1.8	.4
.6	1.0	1.4
1.6	.2	1.2

GO METRIC

"...The Metric System is taught naturally in connection with decimals, and is easily learned. Only the units employed furnish any difficulty. Only the units employed furnish any difficulty. The great number of problems given under the Metric System is to familiarize the learner with the units of the system, to show the simplicity of the system in its application to everyday problems, and at the same time to give practice in operations involving decimals. This system is used in the laboratories of science and in international transactions. Though not yet adopted by the United States in the common affairs of life, it has certainly forced its way to a position requiring recognition in all secondary schools of the country..." - G.A. Wentworth

Exeter, New Hampshire
June 1898

(from the Preface to his book *Advanced Arithmetic*, Ginn & Company, Publishers, 1901).

Metric Manuscripts Sought

Mathematics education should play a significant role in the shift to the metric system of measurement in the United States and Canada. The vital factor in that shift is education, not only in the schools but also for the public at large. Teaching materials and ideas of all kinds will be needed. Toward this end, NCTM's Publications Committee requests manuscripts or ideas in the following areas particularly:

1. Activities for elementary children in learning the metric system as the primary system for measuring.
2. Activities on the metric system for students who have already learned the English system.
3. Materials for teacher education on using and teaching the metric system.
4. Materials for teachers and laymen on the reasons for changing to the metric system and brief explanations of that system.
5. Materials for in-service education of teachers.
6. Publicity material, such as bulletin-board displays, flyers to be sent to parents, slogans, posters.
7. Scripts or ideas for instructional programs and short publicity spots on TV and radio.

Send your materials to the NCTM Publications Committee, P.O. Box 462, Herndon, Virginia 22070. The committee will carefully evaluate all materials received and seek avenues of use or publication for ideas and materials considered appropriate. Manuscripts suitable for pamphlets of 25 to 100 pages would be particularly welcome.

Reprinted from *NCTM Bulletin for Leaders*, December 1972.

STUDENT MATHEMATICS

a new Canadian publication

This publication's aim is to report and encourage student initiative in mathematics, pure and applied, at all levels. The first issue appeared in April 1970.

Some of the articles could be read at Grade IV or V level— for example, a study of goal scoring in hockey. Suitable for a somewhat more advanced level is a problem in number theory, sent to us from Geelong, Australia, for which a Toronto student devised a computer program. An article describes the independent reading a student did which enabled him to start university work in second-year mathematics. A first-year university student offers advice and experiences to those still in high school. There has been an article on optimization and rocketry. There are a number of puzzles, investigation proposals and a number of brief items, such as a report by a Grade XI student who has been reading about relativity in his spare time.

This eight-page bulletin is offered to students at the price of 10¢ per copy. The editors are trying to find ways of making its existence known to students in secondary schools, and of distributing it economically. They would be most grateful to any school or education authority who would take a parcel and be responsible for selling copies to students. In the outlying parts of the country, there may be schools in which only a single student wants a copy. If such a student would send a stamped, self-addressed envelope, at least 9 x 4 inches large, and 10¢, he would receive a copy by mail. One of the aims of the publication is to assist such isolated students.

All orders of *Student Mathematics* should be sent to: The Secretary, *Student Mathematics*, Rm. 373 - College of Education, 371 Bloor Street W., Toronto 181.

We hope this publication will spread out beyond Toronto and Ontario and get the cooperation of mathematics students throughout Canada.

MATHEMATICS MEETING

sponsored by

The National Council of Teachers of Mathematics

and

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- The Teacher's Contribution to the Mathematics Curriculum -

- Motivating Number Fumblers - Motion Geometry - What are Really the Basics? -

- Kid's Lib in Mathematics -

Do these titles catch your interest? Do any of them sound like "just the thing to help with my class"? Does "Kid's Lib" make you say, "What about teacher's lib?"

If so, you will be happy to know you can personally interact with the ideas and the persons behind them at the NCTM Edmonton meeting, October 4-6, at the Chateau Lacombe. There will be speakers from all parts of Canada and the United States. The past president of NCTM, Eugene Smith, will be with us, as well as the two current candidates for NCTM President, Glenadine Gibb and David Wells. Old friends of many Alberta mathematics teachers, John Del Grande from North York in Ontario and Gene Nichols from Florida, will return to share their exciting ideas with us.

There will be over 60 sessions to choose from with equal emphasis on elementary, junior and senior high schools. If workshops are what you like, you will find many of them. If you are interested in individualized instruction and what you can do to make it work, there will be numerous sessions devoted to you. Materials and activities are, of course, a major focal point, extending from elementary to senior high sessions. As you look at the speakers list, you will see that the program committee has found excellence at home and many of your friends and colleagues will be guiding you through new ideas and experiences.

If you want an opportunity to see what's new and good in mathematics teaching and you want an opportunity to evaluate these ideas and materials first-hand, *don't miss the Edmonton meeting of NCTM!* If you can, come for Thursday night and Friday. If not, come for Saturday; there will be good things going every hour of all three days. Your Mathematics Council has worked hard to bring this important professional opportunity to Alberta. Make it a personal success by attending.

Chateau Lacombe ***Edmonton*** ***October 4—6, 1973***