giving counselling and financial assistance upon request to any member of the council executive as listed in each *Delta-K*. We will help obtain speakers for your meetings. (If you have need for resource personnel for any activity involving mathematics, we will assist whether you are a regional or a single school wanting to improve your knowledge and teaching ability in a problem area.)

FORMAT

This issue has been put together by a new editor utilizing a new philosophy as to what a newsletter should contain and what it should present to the membership. Is this better suited to your needs and desires? How would you change the format to better meet your needs? This publication is only as good as its useability. It costs all of us a portion of our membership dues and must justify the expense and time involved. Your editor can only make improvements as you react. Letters to the editor are solicited and will be published when the topic is addressed to all math teachers and used as a guideline for changes in publication where indicated and feasible. Manuscripts are desirable when you have a strong pertinent message that requires more than a letter for proper presentation. Long manuscripts will be considered for monographs. You have your chance to publish, and if you don't take it, others will.

NCTM Conference Report

J. F. Woloshchuk
Mathematics Department Head
Dr. E. P. Scarlett High School
Calgary

I. What is important in preparatory mathematics for university?

Jack Macki,
University of Alberta
Edmonton.

Entering students are faced with problems, such as:

- lack of good study habits,
- pace of programs, especially the beginning lectures,
- lack of supervision (often) of homework,
- required appointments for meetings with lectures,
- testing procedures: work for solutions must be shown,
- social and cultural shock,
- resistance by students to using proofs,
- student literacy, reasoning.

Results of a survey of approximately 700 students and professors demonstrated that, (a) high school curriculum covers various mathematical areas quite well, and (b) students' backgrounds are generally satisfactory. About 70 percent appear well prepared.

II. Process objectives in geometery and algebra teaching.

Sol E. Sigurdson,
University of Alberta,
Edmonton.

Why have process objectives?

- survival purpose in society,
- make mathematics a human endeavor,
- stimulate people to invent their own problems,
- demonstrate the power of definition and assumption,
- demonstrate applicability of mathematics.

Six processes were demonstrated by videotape:

- making a model,
- posing the question,
- perceiving the mathematics,
- establishing the theorem,
- generalizing the result, that is, inventing the formulae, symbols, and such,
- systematizing the result.

III. What is the nature of teaching a great lesson when no learning takes place?

Walter P. Krepak,
Mathematics Department Head,
Western Canada High School
Calgary.

I am going to get a copy of the entire speech which I will lend on request.

IV. Student needs and subject requirements

Wallace S. Manning,
School District #91,
Idaho Falls, Idaho.

1. Arrive at realistic objectives for the students in the classroom.
2. Use media in most effective ways.
3. Staff involvement is the key to program development.
4. Individualized instruction is not for everyone.
5. Have program make adjustment to the outside world.
6. Examine the traditional versus individualized intruction which is similar to the Bowness High School Program.

V. Self-correction feedback, or how not to make dependent students

Dr. Alton T. Olson,
University of Alberta,
Edmonton.

In essence, self-correcting feedback implies immediate knowledge of the worth of a course of action for the purpose of correcting that course of action in order to maximize its worth. A servo-mechanism would be the prime example of an object which had a self-correcting feedback capability.

Why is this an important concept for me? What does it have to do with the teaching of high school mathematics? It is simply that I have seen far too much mathematics taught in such a way that a student had little recourse but to go to an answer book or a teacher for feedback. I am not denying that that is a viable source of feedback. However, I am proposing that we open up and explore other provisions for feedback. These provisions, I think, hold promise for more independence and more mathematical understanding for students. Furthermore, if one acknowledges that a significant amount of mathematics is learned by conditioning then positive reinforcement schedules demand a temporal contiguity of response and reinforcement. Self-correcting feedback can provide that contiguity.

Schematically, in very simple terms, feedback can be depicted as below:

With this picture in mind, consider the student who is given the problem $P - Q$ where P and Q are polynomials. He is told that $P - Q = P + (-Q)$. That prescribes a course of action. From this action, certain results are obtained. Where and how does the student obtain immediate feedback? If he depends on an answer book or a teacher the feedback may be long delayed which largely destroys its effectiveness. I would propose that the student be required to find R where $P = Q + \underline{R}$, rather then $P - Q = P + (-Q)$. The feedback is obviously more immediate. Also, students always find that the addition of polynomials is easier than subtraction, and emphasizes the relational aspects of mathematics.

For the next example, consider the student who is introduced to complex numbers or ordered pairs of real numbers. Then multiplication is defined as follows: $(a,b) \cdot (c,d) = (ac-bd, ad+bc)$. This prescribes a course of action. Where does the student get feedback concerning the worth of his results? He is made dependent on external sources for feedback. I would propose that concurrently the multiplication of complex numbers should be taught as follows: $(a_1 b) \cdot (c_1 d) = (ac-bd, ad+bc)$ and $(a+bi) \cdot (c+di) = ac+bdi^2+bci+adi^2 = (ac-bd) + (bc+ad)i$
In this way either one of these algorithms becomes a feedback channel for the other.

In the same manner, division of complex numbers is defined as follows:

$$z_1 \div z_2 = z_1 \cdot \frac{1}{z_2} \text{ where } z_1 = (a,b),\ z_2 = (c,d) \text{ and } \frac{1}{z_2} = \left(\frac{c}{c^2+d^2}\ \ \frac{-d}{c^2+d^2}\right)$$

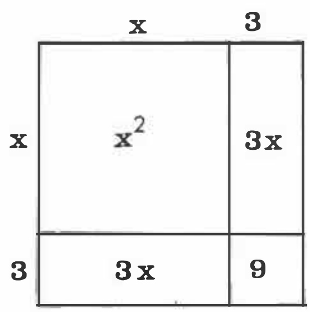
Where does a student obtain feedback for this course of action? I propose that if a student has obtained a result (e,f) where $(a,b) \div (c,d) = (a,b) \cdot \frac{1}{(c,d)} = (e,f)$ then feedback should be sought as follows: $(a,b) \div (c,d) = (c,f) =$

$(a,b) = (c,d) \cdot (e,f)$. In other words does $ce - df = a$ and does $cf + dc = b$?

Consider an example from the algebra of polynomials. A student squares a binomial and gets the following: $(x+3)^2 = x^2+6x+9$. In how many ways can he get feedback for his course of action? Let us consider them now:

1. relate it to reversing the distributive property, $x^2+6x+9 = x^2+3x+3x+9 = x(x+3)+3(x+3)$, etc.,

2. relate it to area,



3. relate it to place value,

$$(x + 3) \cdot (x + 3) = x^2 + 6x + 9$$
$$(10+3) \cdot (10+3) = 100 + 60 + 9$$
$$13 \cdot 13 = 169$$

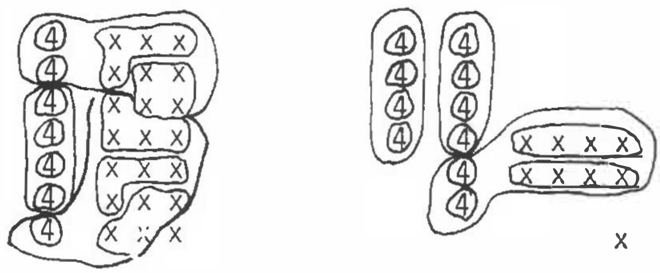
The statements above are true for other number bases as well. For instance, let x be 15 but write it in base 15 notation, then:

$$\left.\begin{array}{l}(x+3)\ (x+3) = x^2 + 6x + 9 \\ (10+3)\ (10+3) = 100+60+9 \\ 13 \cdot 13 = 169\end{array}\right\}\ \text{base 15}$$

$$(x+3)\ (x+3) = x^2 + 6x + 9$$
$$(15+3)\ (15+3) = 15^2 + 6 \cdot 15 + 9$$
$$18 \cdot 18 = 225 + 90 + 9$$
$$324 = 324$$

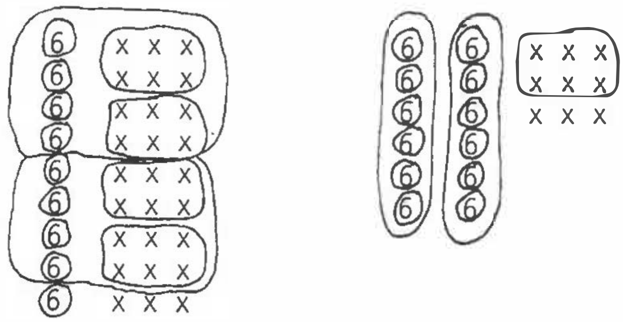
4. Relate it to grouping,
   This idea is very similar to that of place value but is perceptually different.

   For $(x+3)\,(x+3) = x^2 + 6x + 9$ let x be 4. Then we have:



An additional conclusion can be made here: $(x+3)\,(x+3)$ and $x^2 + 6x + 9$ are both equal to $3x^2 + 1$ when $x=4$.

When x is 6 then we have:



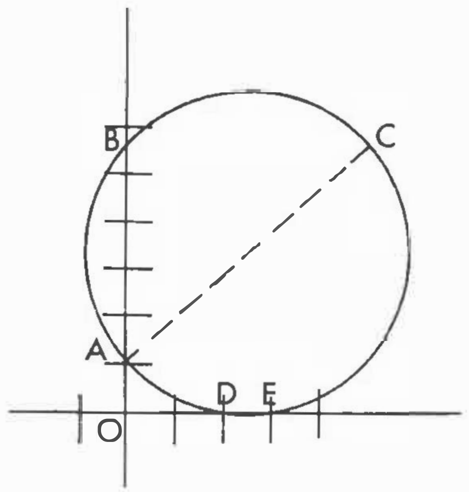
We can then conclude that $(x+3)\,(x+3) = x^2 + 6x + 9 = 2x^2 + x + 3$ when $x=6$.

5. Relate it to the use of the following equations, $mx^2 + ax + p = (ax+b)\,(cx+d)$
   where $ac = m$

   $$bc + ad = 6$$
   $$bd = 9$$

   from which b and d must be 3 and 3 respectively, and a and c must each be 1, and so on.

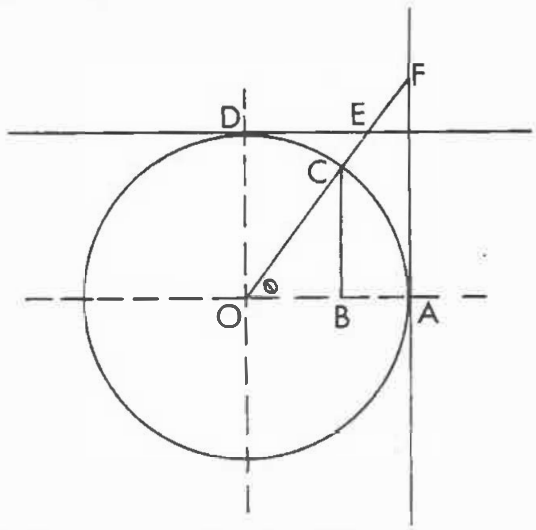
In a topic related to factoring quadratics, consider the problem of finding the roots for the quadratic equation $x^2 - 5x + 6 = 0$. If the student obtains the roots 2 and 3 they can be substituted into this equation to obtain immediate feedback. In another method of obtaining feedback, an effective link between algebra and geometry can be exploited.



$m(\overline{OA}) = 1$
$m(\overline{OB}) = 6$
$m(\overline{BC}) = 5$
$m(\overline{OD}) = 2$
$m(\overline{OE}) = 3$

The measures of OD and OE must be the roots of $x^2 - 5x + 6 = 0$ for the sum of these measures must be 5 and their product must be 6.

A final example will be taken from trigonometry. For feedback purposes, the trigonometric functions can be related to the measures of certain lines on the unit circle.



$m(\overline{BC}) = \sin \theta$
$m(\overline{OB}) = \cos \theta$
$m(\overline{AF}) = \tan \theta$
$m(\overline{OE}) = \csc \theta$
$m(\overline{DF}) = \sec \theta$
$m(\overline{DE}) = \text{ctn } \theta$

The procedure has the advantage of not requiring any quotients, and the behavior of the functions can be readily discerned.

These are some of my thoughts concerning an alternate approach to self-correcting feedback. I hope that these ideas may be useful to you.

VI. A systematic approach to the design of units of instruction for a general mathematics course at high school level.

Barry Eshpeter,
Media Consultant
Calgary Public Schools
Calgary.

Discussion revolved around the systems model reproduced below.

A SYSTEMS MODEL

1. PROBLEM
   - Identify Need for Content
   - Analyze Learners
2. OBJECTIVES
   - Terminal objectives (performance)
   - Prerequisite objectives
3. PLAN INSTRUCTION
   - Plan evaluation
   - Sequence objectives
   - Analyze objectives
   - Examine alternative teaching methods
   - Specify content
   - Specify needed resources
4. IMPLEMENT
5. EVALUATE

VII. Basic mathematical concepts and skills required by high school chemistry and physics.

William Tanasichuk,
Queen Elizabeth High School,
Edmonton.

The speaker identified the following mathematics skills and concepts required in the two sciences:

- significant digits
- rounding off
- exponents
- scientific notation
- fractions and decimals
- use of units
- metric units
- addition, subtraction, multiplication, division
- approximation and estimation
- formulae and equations
- ratio, proportion and variation
- trigonometric ratios
- vectors and scolars
- graphics

Note: Mr. Tanasichuk had also developed a 40-page unit entitled, "Math review for high school science". I have two copies available for anyone which I will lend on request.