Mathematical Competencies and Skills Essential for Enlightened Citizens

Reprinted from *The Mathematics Teacher*, November 1972, Volume 65, Number 7.

The Committee on Basic Mathematical Competencies and Skills
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In March 1970 the Instructional Affairs Committee proposed that an ad hoc committee write a set of objectives to define minimum competence in mathematics. Recognizing its responsibility to the profession as well as to the public to establish guidelines for the mathematical needs of all citizens, the NCTM Board of Directors accordingly appointed a committee to draw up a list of basic mathematical competencies, skills, and attitudes essential for enlightened citizenship in contemporary society.[1] A report containing such guidelines was painstakingly prepared by the committee and is presented here. Even though the report does not represent NCTM official policy and may contain points with which individual members may disagree, it unquestionably contains much food for thought on a subject of great importance to society in general and to the mathematics-education community in particular. The entire report is therefore presented here for the consideration of the membership.

Following World War II, the crucial importance of mathematics in the technological and other aspects of life in the complex modern society has been unquestionably recognized. With it the question, How much mathematics is a "must" for every citizen? reappeared with new urgency.

The National Council of Teachers of Mathematics appointed the Commission on Post-War Plans and charged it with the task of answering this question. After completing its deliberations, the commission issued a checklist of 29 questions which constituted a basis for assessing one's basic competence in mathematics. "Guidance Report of the Commission on Post-War Plans" was published in 1947 in volume 40, number 7, of the *Mathematics Teacher*.

Since that time many changes have been introduced into mathematics. The very nature of the subject has changed. The introduction of new areas such as abstract algebra, functional analysis, and topology has resulted in what has been called a "revolution in mathematics".

[1]The reader should recognize that this is not a list of minimum competencies, skills, and attitudes to be attained by all students with an IQ of 70 or above; unfortunately, some of our students will not become enlightened citizens in contemporary society.

Not until the 1950s, however, did these changes in the very nature of mathematics have an impact on secondary and elementary school curricula. Because of the revolutionary work of the University of Illinois Committee on School Mathematics, the School Mathematics Study Group, the Commission on Mathematics, and other such groups, the curriculum underwent some drastic changes. New concepts appeared in textbooks and new approaches to teaching old concepts found their way into classrooms. Thousands of teachers returned to the universities and colleges to update themselves in the new concepts and methods. Writers of commercial textbooks, benefiting by the suggestions from experimental groups, introduced new concepts and approaches.

The realization in the late 1950s that the technology leadership of the United States had been challenged by others has given an additional impetus to the emphasis on the study of mathematics and the natural sciences. The demand for increased competence in mathematics has become a reality. New competencies and skills have been introduced into the required repertoire of the day-to-day activities of the individual.

The National Council of Teachers of Mathematics found itself with a challenge to define what might be considered the minimal mathematical competencies that are expected of the enlightened citizenship in contemporary society. Although it is impossible to find a universal agreement on such things, it is nevertheless the duty of this professional organization of mathematics teachers to propose what might be considered a reasonable approximation to such a statement of competencies. It is with this intent that this report is being offered.

Mathematics is one thing to some, another thing to others. Some enjoy it; others do not. Some seek to use it; others avoid it. Some individuals can get by with very little mathematical knowledge. In other words, the amount of mathematical skills that some individuals must have to "barely get by" in society may be very nearly zero. Present-day society collectively, however, needs a great deal of the most sophisticated kind of mathematics for its functioning. It is essential for its very survival. The highly complex problems of the technological society require complex mathematics to solve them. The demand, therefore, for increased competencies in mathematics has become a reality for many.

There are three basic ways to view mathematics:

1. Mathematics as a tool for effective citizenship and personal living
2. Mathematics as a tool for the functioning of the technological world
3. Mathematics as a system in its own right.

Each of these three aspects of mathematics is of significance. The first aspect is important for all. The second is essential to the scientist, the engineer - generally to those who make their living through professions that use mathematics as a tool. The third aspect is of concern to the professional mathematician. He sees in it a fascination and beauty that usually escapes the mathematically untrained mind.

In isolation, no one of the three aspects is of greater significance than another. In the total complex of society, each aspect is of significance.

Few educators will deny that past efforts in mathematics education have produced significant gains for many pupils. There are too many cases, however,

where many pupils leave our schools without the necessary skills to make them employable and to allow them to apply mathematics to help them solve the problems of daily life.

Some decisions are desperately needed to give guidance and direction to teachers and administrators concerning

1. what mathematics every pupil must master to "barely get by" in contemporary society;
2. what mathematics is essential for full participation of an individual in contemporary society;
3. what teaching techniques and processes not only assure the acquisition of mathematical skills and competencies deemed essential but also convince pupils to be willing to use these skills once acquired.

The following listing of competencies and skills has been separated into three main categories.

In the first section, an attempt has been made to include examples of those skills and competencies that are felt to be necessary for the majority of adults in our society. This is not to imply, however, that every individual must master each concept listed. It is the hope of the NCTM that the reader will consult these guidelines and interpret them in light of his own situation. The listing should give some guidance to teachers of mathematics in establishing priorities. It is hoped that teachers will attempt to convince pupils by appropriate and reasonable objectives, activities, and other techniques that the acquisition of essential mathematical skills and competencies is necessary for effective functioning in today's society.

In the second section, a few of the basic characteristics of the nature of mathematics as a system in its own right are presented. It is done with the hope that the individual curious about this aspect of mathematics as a discipline will find it helpful as he seeks to comprehend the basic nature of mathematics.

The third section should appeal to those individuals with the kind of inclinations discussed in the second section. These individuals are usually more inclined to seek a deeper understanding of the role of mathematics in society than those who are not mathematically inclined.

Skills and Competencies

Individuals often find the need to use mathematics in everyday life and in many jobs that frequently require some technical application of mathematics. The following outline of content gives some indication, under each heading, as to what minimum "doing" skills are needed by the enlightened citizen.

1. Numbers and numerals
   a) Express a rational number using decimal notation
   b) List the first 10 multiples of 2 through 12
   c) Use the whole numbers in problem solving
   d) Recognize the digit, its place value, and the number represented through billions
   e) Describe a given positive rational number using decimal, percent, or fractional notation
   f) Convert to Roman numerals from decimal numerals and conversely, for

example, date translation
g) Represent very large and very small numbers using scientific notation

2. Operations and properties
   a) Write equivalent fractions for given fractions, such as 1/2, 2/3, and 3/5
   b) Use the standard algorithms for the operations of arithmetic of positive rational numbers
   c) Recognize and use properties of operations (grouping, order, and such) and properties of certain numbers with respect to operations ($a \cdot 1 = a$; $a + 0 = a$; etc.)
   d) Solve addition, subtraction, multiplication, and division problems involving fractions
   e) Solve problems involving percent
   f) Perform arithmetic operations with measures
   g) Estimate results
   h) Judge the reasonableness of answers to computational problems

3. Mathematical sentences
   a) Construct a mathematical sentence from a given verbal problem
   b) Solve simple linear equations such as $a + 3 = 12$; $16 - n = 4$; $n/3 = 7$; and $4a - 2 = 18$
   c) Translate mathematical sentences into verbal problems.

4. Geometry
   a) Recognize horizontal lines, vertical lines, parallel lines, perpendicular lines, and intersecting lines
   b) Classify simple plane figures by recognizing their properties
   c) Compute perimeters of polygons
   d) Compute the areas of rectangles, triangles, and circles
   e) Be familiar with the concepts of similarity and congruence of triangles

5. Measurement
   a) Apply measures of length, area, volume (dry or liquid), weight, time, money, and temperature
   b) Use units of length, area, mass, and volume in making measurements
   c) Use standard measuring devices to measure length, area, volume, time, and temperature
   d) Round off measurements to the nearest given unit of the measuring device (ruler, protractor, thermometer) used
   e) Read maps and estimate distances between locations

6. Relations and functions
   a) Interpret information from a graphical representation of a function
   b) Apply the concepts of ratio and proportion to construct scale drawings and to determine percent and other relations
   c) Write simple sentences showing the relations $=$, $<$, $>$, and $\neq$ for two given numbers

7. Probability and statistics
   a) Determine mean, median, and mode for given numerical data
   b) Analyze and solve simple probability problems such as tossing coins or drawing one red marble from a set containing one red and four white marbles
   c) Estimate answers to computational problems
   d) Recognize the techniques used in making predictions and estimates from samples

8. Graphing
   a) Determine measures of real objects from scale drawings
   b) Construct scale drawings of simple objects
   c) Construct graphs indicating relationships of two variables from given sets of data
   d) Interpret information from graphs and tables

9. Mathematical reasoning
   a) Produce counter examples to test the validity of statements
   b) Detect and describe flaws and fallacies in advertising and propaganda where statistical data and inferences are employed
   c) Gather and present data to support an inference or argument

10. Business and consumer mathematics
   a) Maintain personal bank records
   b) Plan a budget including record keeping of personal expenses
   c) Apply simple interest formulas to installment buying
   d) Estimate the real cost of an article
   e) Compute taxes and investment returns
   f) Use the necessary mathematical skills to appraise insurance and retirement benefits

The Nature of Mathematics

An individual who is inclined toward the study of the structure of mathematics would do well to consider those attributes of mathematics that deal with this aspect of mathematics. Some of these attributes and the abilities associated with them are discussed below.

1. The deductive nature of mathematics. Here the concern is with the question of what follows by force of logic from known truths. The derivations of new conclusions take the form of "if ... then" statements. The central issue is whether the statement following "then" is a logical consequence of the statement, assumed to be true, following "if".
2. In relation to attribute one, an individual should be able to carry through a consistent argument. Proofs of theorems constitute a very important part of mathematics. It has been demonstrated that some very young children are capable of proving simple theorems. Those individuals who display this ability should be provided with opportunities to experience success in this kind of mathematical activity. For example, establishing in a logical way that the sum of two even numbers is an even number would be a very satisfying experience to a child with such inclinations.
3. An individual with an inclination toward this aspect of mathematics should be able to differentiate between a valid argument and an invalid one. Theorems are established by proceeding from assumptions to conclusions by means of a series of logical implications. It is important to be able to recognize arguments that have fallacies in them. One should appreciate the fact that liking a conclusion does not imply that it has been established validly.
4. An individual of this type should be familiar with the basic properties of operations on numbers. For example, such properties as commutative and associative of addition of real numbers should be a second nature to him. For those individuals, it is not sufficient to be skillful in the use of basic operations - they should be able to achieve economy in computations

by making use of basic properties. The distributive property, for example, is very useful in simplifying computations.

5. These individuals should be able to verify whether or not a given system possesses given properties. They should study all the properties that a set of numbers under given operations should possess in order to constitute a particular system. For example, they should be able to verify whether a given set of objects is a field under two given operations.

6. Such individuals should be able to recognize that various concepts and operations are related to each other; for example, subtraction and addition are inverse operations. Having some idea of the ways in which concepts and operations are related to each other eliminates the necessity of memorizing many isolated facts. Experience and training in observing relations between entities aids retention and enhances the development of observational skills.

7. Such individuals should be able to perceive patterns displayed in sequences. Observations of patterns is a skill that finds applications in many areas of human endeavor. Mathematics is an ideal subject in which to develop this skill.

The Role of Mathematics in Society

The individual with the kinds of inclinations discussed in the previous section is also inclined to seek a deeper understanding of the role of mathematics in society than is usually the case with those who are not mathematically inclined. This implies the following:

1. *Knowing the ways in which computers are used in sciences, technology, business, and government.* Computers are indispensible for the functioning and the very survival of present-day society. The complexity of the present problems requires the use of computers. They are useful because of their speed and their ability to store a multitude of facts. The speed with which millions of facts can be processed is essential in complex decision-making situations. One must guard, however, against the belief that computers are capable of solving all problems. They should not be attributed the complex ability of independent thinking and decision making.

2. *Recognizing the evolutionary development of mathematics by noting the historical milestones in the development of mathematical ideas.* These mathematical ideas served man in solving his problems and aided him in the control of his environment. Beginning with the invention of numbers for the purposes of counting through the invention of irrational numbers for describing lengths, man has conceptualized mathematical things which at first served exclusively his utilitarian purposes. Both the invention of a specific mathematical tool and its conceptualization into an abstract system are essential ingredients of the development of mathematics.

3. *Being aware of the great frequency with which mathematical skills are used by individuals in their daily lives.* To appreciate the importance of mathematics in daily life, one might pose a question as to what would happen if mathematics disappeared. If number or geometry disappeared, would man be able to go about completing successfully his daily chores?

4. *Recognizing that there are problems that, by their nature, do not lend themselves to solution by mathematical methods.* Individuals at times make decisions that are primarily based on feelings and emotions, and they should recognize that such bases for decisions, though they may be valid, are by their very nature not amenable to mathematical analysis.

5. *Recognizing that some professions require knowledge of the most sophisticated and complex mathematical techniques.* A young individual who, because of a demonstrated high aptitude for mathematics, may desire to train for a profession that has high requirements of mathematical competence should apprise himself of these requirements and acquire the skills necessary for the given profession. He should realize that many years of mathematical training may be necessary.

6. *Being aware that mathematics finds direct applications not only in the natural sciences but in the behavioral and social sciences and arts as well.* Many psychologists as well as individuals concerned with humanities make increasingly greater use of mathematical techniques in developing their disciplines. For example, computers are used to analyze and understand the factors that influence human behavior. Since there is a large number of variables involved in such situations, this analysis and understanding could not be achieved without the electronic computer. Environmental problems, because of their complexity, will demand more and more sophisticated mathematical models for their solution.

Conclusion

The basic mathematical competencies and skills essential for enlightened citizens are determined by the needs of society at a given time. In our rapidly changing technical society, many factors bear upon what are and what will be the required competencies and skills. Educators charged with maintaining a contemporary basic mathematics program must be aware of these factors and give them consideration in their efforts to design effective programs.

Ours is the age of technology. A lessening need for individual computational skills has resulted from the universal availability of calculators. Computers have largely obviated the need for paper and pencil computations, but they have increased the need for formulating careful and reasoned generalizations. As other devices become available, their influence on society must be reflected in basic mathematical competencies and skills identified as those needed by enlightened citizens.

The educational level of our society is constantly changing. A larger percent of the school-age population is enrolled in school; the number of years students remain in school is on the increase. The needs of citizens, therefore, will change as the educational level compounds itself.

Ingenuity in devising individual means of checking one's conclusions for "reasonableness" is a most valuable aid to effective living and is a skill that mathematical training can greatly sharpen. An enlightened citizen must logically and judiciously cope with the constantly increasing bombardment of statistics, facts, and figures. As all teachers and students become more aware of the demands for responsible, self-reliant judgments, the central role of mathematics in making such judgments becomes more apparent.

An enlightened citizen is qualified for employment. Employment opportunities are changing and will change both in nature and in requirements of personnel. Surely the degree to which an occupation is mathematically oriented will determine the extent of the basic mathematical skills and competencies required of individuals employed in that occupation. The aggregate degree of mathematical orientation of the job market, in turn, influences what is considered a basic level for all citizens.

Technology, the educational level of society, and occupational requirements are examples of factors that influence the basic mathematical competencies and skills needed by enlightened citizens. Some other factors are probably manifested; still others will evolve from their influence. It is therefore critical that we continually examine the programs designed to improve basic mathematical competencies and skills in future years. It is also the hope of the National Council of Teachers of Mathematics that professional groups concerned with mathematical education for all citizens will constantly strive to interpret the factors influencing change, seeing these in relation to their implications for the mathematics curriculum.