More to it than You Think

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The New Math is supposed to teach concepts rather than just facts and operations, but I rather doubt if the "Seeing Through Arithmetic" series is doing this well enough in the primary grades. There are cases where the expression of concepts is too confusing.

Consider the sentence, "Five is greater than four." To master this, the child must learn five different concepts:

1. *Vocabulary* The word "greater" is not in the average Grade I child's vocabulary. Even the word "less" is not very often used, as these children tend to say, "He's got more than me," rather than, "I've got less than him." So the meaning of the words "greater" and "less" have to be taught.

2. *Word Recognition* Grade I children at this stage are only just starting to read, so they will not know any of the words in the sentence by sight.

3. *Grammar* The word "greater" applies to the first word in the sentence which is the subject. We take this for granted, but it is not so easy for children. I have found some Grade II children who didn't have this clear, as was shown by the type of mistakes they made. Whenever they had a sentence of the type, "____ is one greater than 7", they would write the correct numeral; but if the sentences had the missing number at the end, as in "7 is one greater than ____", they made mistakes. It appears that their thinking was something like, "Here is 7, and here is 'greater', so I must make 7 greater", rather than thinking, "The first number must be greater." This is even more difficult for Grade I students.

4. *Reading from left to right*. Children at this stage have not all grasped firmly the concept of expressing the order of spoken words by a left-to-right progression, but this is necessary if they are to put in writing the grammatical concept described above.

5. *Number* Finally we come to the mathematical concept that the number five is greater than the number four.

When I finish analyzing in this fashion, I am filled with admiration for the children who do succeed in mastering all this!

I have found it easier to use the symbols $>$ and $<$ first, because whichever way round you look at them, the point is always toward the lesser number, and therefore the "left to right" concept is not needed, so at first you just teach concepts (1) and (5). Then the children read what they have written, and so start to learn concepts (3) and (4). When they can read better the words may be substituted for the symbols.

Another source of confusion is in the type of picture used to teach the concepts of "greater" and "less". This reinforces an error common among young children - that of thinking that the child who has four small candies has more than the child who has three large ones. Three bears are greater than two bears, but they are not greater than two elephants, which is what some of the pictures teach. To teach this is, in effect, to teach that three dimes are greater than two quarters. To teach the correct concepts the pictures should be of objects that are the same, or at least of the same size.

The pictures illustrating subtraction are confusing also. I noticed that several Grade II children, who could complete straight equations in addition and subtraction satisfactorily, made many mistakes when writing a mathematical sentence from a picture. An examination of the pictures showed why. First they are taught addition with pictures such as this  They are taught to perceive two sets and to write first how many are in each set, then to complete the equation, 2 + 3 = 5. When they see a picture representing subtraction, such as  , they perceive the same two sets, so they tend to write 3 - 2. A picture representing subtraction should show the first number rather than the remainder. I found that the most effective way of clearing up this confusion was to put up this chart:

That this was helpful was shown by the fact that when I removed it the children asked me to put it up again.

But this is just a way of enabling children to do what the book requires them to do. I don't think subtraction can be efficiently taught through pictures. It needs manipulation of real objects and oral problems.

This confusion may be partly due to a failure to recognize that the operations of addition and subtraction are not the same as the equations expressing them. Two different concepts are involved. The operation is active; the equation is static. The operation involves change, motion, time; the equation is a universal statement of fact.

Because the numbers are the same, we are apt to think that the concepts are the same; so we need to look at this more closely. In the operation of addition there is one set of objects at the beginning; then another set moves toward it; then the two merge, forming a final resulting set which is greater than the first set. But an equation does not express change from a smaller number to a greater - it states that the two smaller numbers together are the same as one greater number. This can be expressed on a balance: a two-ounce weight and a one-ounce weight on one side will balance a three-ounce weight. But the operation of addition cannot be expressed on the two sides of a balance.

These are two different concepts, and I think the modern practice unsound, which insists on using the language of the equation and making children say, "Three plus two is equal to five", while they are observing a series of pictures to represent an operation in time. The old-fashioned teacher who permits the simpler sayings, "Three and two more make five," or, "Five take away two leaves three", does in fact teach the operational concept better. When children have performed the operation several times they may realize the basic rule expressed in the equation.

It is difficult to present the concepts of change, motion and time in pictures. Grade I children tend to look at all the pictures at once, rather than starting at the top left corner. This doesn't matter so much in addition which can be expressed in one picture; but subtraction requires at least two. It might be better if Grade I students had no text book, but did all their work with real objects.

This may seem too detailed a criticism, but if the proponents of New Math want us to teach concepts, let us think clearly what they are, and the most efficient way of teaching them. We must think in a more specialized way about Grade I. It is no use just planning a logical beginning to a series. The logical beginning may not be suited to Grade I. And it is there that we are laying the foundation; if that is not solid, what happens to the superstructure?

Distributor Cap Mathematics

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This article deals with problems that involve permutations and cyclic permutations. These problems are adaptable to the discovery method of teaching and include opportunities for making conjectures that can be followed by informal proof.

It is interesting to watch a skilled mechanic install a new distributor cap in a modern automobile. How quickly he connects the spark plug wires without much hesitation. It appears that he plugs the ends into the distributor cap connectors in any convenient order but this is not really what he does. There is one and only one correct arrangement that will work properly. How does he know the correct one? Ask any experienced mechanic and he'll probably tell you.

Let us suppose that we don't know the trick and that we start plugging in at random. How many arrangements are possible? The standard Delco-Remy distributor cap for an eight-cylinder car is shown in Figure 1. We note that the connectors for the eight spark plug wires are arranged in a circle and that there is a center connector for the power source. At the base are two clip mechanisms for fastening the cap. We shall examine the problem for motors with different numbers of spark plug wires. Figure 2 is a diagrammatic representation of a distributor cap for a four-cylinder motor. The spark plug

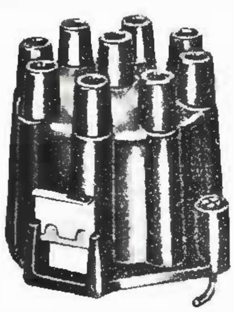

Figure 1

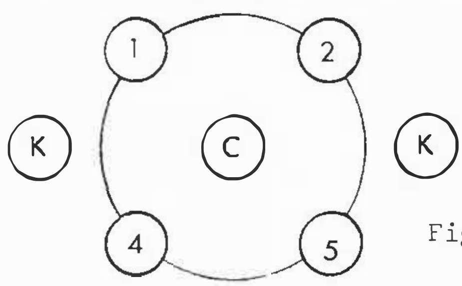

Figure 2

wire connectors are shown as 1, 2, 3, and 4, "c" is the center connector and "k" is a clip. It must be noted that, once installed, the cap is rigid. We consider the number of possible ways to plug in the spark plug wires. Suppose that the first wire that we choose is plugged into position 1. Then there are 3! ways of plugging in the other three wires for each position of the first wire. But the first wire could have been plugged in at four different places. Thus there are 4(3!) or 4! possible arrangements, a total of 24 ways to plug in the four wires.