This may seem too detailed a criticism, but if the proponents of New Math want us to teach concepts, let us think clearly what they are, and the most efficient way of teaching them. We must think in a more specialized way about Grade I. It is no use just planning a logical beginning to a series. The logical beginning may not be suited to Grade I. And it is there that we are laying the foundation; if that is not solid, what happens to the superstructure?

Distributor Cap Mathematics

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This article deals with problems that involve permutations and cyclic permutations. These problems are adaptable to the discovery method of teaching and include opportunities for making conjectures that can be followed by informal proof.

It is interesting to watch a skilled mechanic install a new distributor cap in a modern automobile. How quickly he connects the spark plug wires without much hesitation. It appears that he plugs the ends into the distributor cap connectors in any convenient order but this is not really what he does. There is one and only one correct arrangement that will work properly. How does he know the correct one? Ask any experienced mechanic and he'll probably tell you.

Let us suppose that we don't know the trick and that we start plugging in at random. How many arrangements are possible? The standard Delco-Remy distributor cap for an eight-cylinder car is shown in Figure 1. We note that the connectors for the eight spark plug wires are arranged in a circle and that there is a center connector for the power source. At the base are two clip mechanisms for fastening the cap. We shall examine the problem for motors with different numbers of spark plug wires. Figure 2 is a diagrammatic representation of a distributor cap for a four-cylinder motor. The spark plug

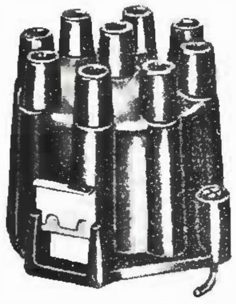

Figure 1

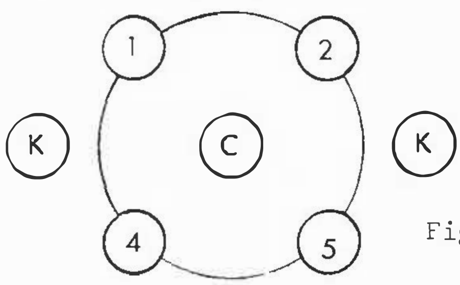

Figure 2

wire connectors are shown as 1, 2, 3, and 4, "c" is the center connector and "k" is a clip. It must be noted that, once installed, the cap is rigid. We consider the number of possible ways to plug in the spark plug wires. Suppose that the first wire that we choose is plugged into position 1. Then there are 3! ways of plugging in the other three wires for each position of the first wire. But the first wire could have been plugged in at four different places. Thus there are 4(3!) or 4! possible arrangements, a total of 24 ways to plug in the four wires.

Consider, now, caps for six- and eight-cylinder motors as diagrammed in Figures 3 and 4. For the six-cylinder motor we plug into position 1 again and permute the other five to obtain 5! ways of plugging in the other five wires.

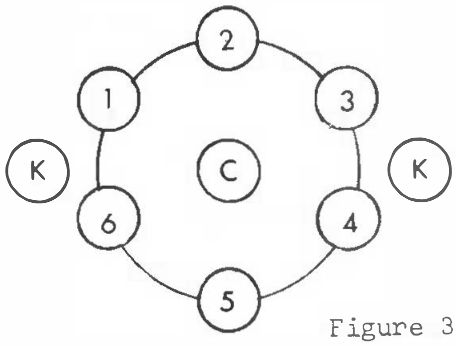

Figure 3

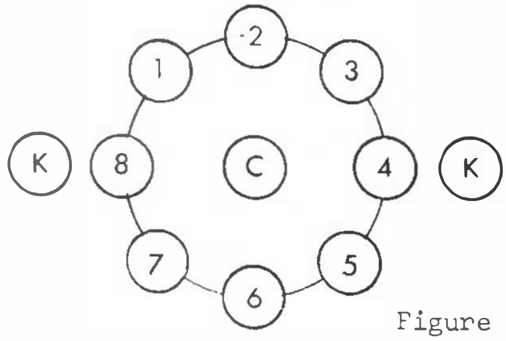

Figure 4

If we allow the first wire to be plugged into any of the six places we obtain 6(5!) or 6!, that is, 720 ways to plug in the six wires. Proceeding in the same way for the eight-cylinder motor we find that there are 8(7!) or 8!, that is, 40,320 ways to plug in the eight wires - obviously far too many for a trial and error approach.

Suppose further that the wire to the center connector is included with the spark plug wires and that all arrangements are determined with center wire and spark plug wires considered interchangeable. From Figure 2 we see that there are now 5 ways of plugging into the center position and for each of these there are 4! ways of plugging into the spark plug connectors, a total of 5(4!) or 5!, that is, 120 ways in all. From Figure 3 we similarly obtain 7! or 5040 ways and from Figure 4 we get 9! or 362,880 ways.

Remove the clip pins from a Delco-Remy distributor cap and it can be used as a pencil or crayon holder for your desk. If preferred, the black color can be changed to suit any decor by spray painting. An examination of Figures 2, 3, and 4 reveal that we have, respectively, 7, 9, and 11 places for pencils when we include the clip positions "k".

Suppose that all positions, except the center position, are to be filled with different colored pencils and that we do not allow the holder to rotate. We consider the number of possible arrangements of the pencils in each case, excluding the center position. From Figure 2 we see that there are 6(5) ways of filling positions "k" and for each of these there are 4! ways of filling positions 1, 2, 3, and 4. Hence there are 6(5)(4!) or 6!, that is, 720 ways of arranging the pencils. From Figures 3 and 4 we find similarly that there are 8! (that is, 40,320) and 10! (that is, 3,628,800) ways, respectively.

If we include the center position "c" in Figure 2, we find that there are 7(6)(5) ways of filling positions "k" and "c" together and for each of these there are 4! ways of filling positions 1, 2, 3, and 4. This yields a total of 7(6)(5)(4!) or 7!, that is, 5040 ways of arranging the pencils. From Figures 3 and 4 we find similarly that there are 9! (that is, 362,880) and 11! (that is, 39,916,800) ways, respectively.

The following table summarizes our results. If rotations had been permitted, the cyclic permutations would have been computed using (n - 1)! instead of n! as has been done.

n	p	p + c	p + 2k	p + c + 2k
4	4! = 24	5! = 120	6! = 720	7! = 5040
6	6! = 720	7! = 5040	8! = 40,320	9! = 362,880
8	8! = 40,320	9! = 362,880	10! = 3,628,800	11! = 39,916,800

n - number of spark plug wires.
p - number of cyclic permutations of the spark plug wire connections to the distributor cap (including rotations of the wires).
c - center connection on the distributor cap.
k - clip points of the distributor cap.

Solving Simple Equations

by *Howard L. Larson*

The author is a retired mathematics teacher and school superintendent with many years experience teaching and training others in the art of teaching.

Much classroom teaching is confusing for students until they are forced to learn without understanding for the sake of self-preservation. A good example of that type of teaching is the method usually, or often, offered to the student for finding the solution of equations.

An example of this is found in solving 3N = 12.

To solve for "N" we divide both sides by 3 (or multiply by 1/3: the choice is irrelevant to the discussion following). When we do this we get N=4. Simple isn't it! Why not subtract 3 from both sides and get N=9? Well, why not indeed? Many of your students will have this question in mind and may even dare to ask it.

The basic problem stems from the fact that we assume too much at the beginning levels. Here, for example, we assume that the student understands that 3N means 3•N or that 3 and N are factors. Again, we assume that the student understands that when we divide 3N by 3 the threes disappear in thin air! We neglect to teach that the meaning of identity and inverse or 3/3 = 1 have some operational principles which require conscious teaching.

Le us examine a more rational approach.

If $\overset{f}{3}\times\overset{f}{4} = \overset{p}{12}$ then $\overset{p}{12}\div\overset{f}{3} = \overset{f}{4}$ or $\overset{p}{12}\div\overset{f}{4} = \overset{f}{3}$. With a little practice, the truth of this statement becomes self-evident to the student and he can be