THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

delta-k

Volume XIII, Number 3, May 1974

OOPS!!

In the last issue of *Delta-K* (Vol. XIII, No. 2), we failed to give credit to Dr. Gerardus Vervoort, Lakehead University, Thunder Bay, Ontario for "Inching Our Way Through the Metric System". Please correct this oversight in your copy and also note that this article is copywrited by Dr. Vervoort. Our apologies have been extended to Dr. Vervoort and are hereby made publicly.

Math Kits - Metrication Materials

Three math kits are now in circulation and everyone who contacted Stu McCormick requesting a kit has had a reply. The response has been so great that some of us will not have our request filled until next school year. However, we are now preparing a new set of kits, consisting of only metric materials. They will be ready for distribution in the fall. Stu will be contacting us as soon as he and his committee are ready to start delivery. Ideas and materials you have that we can use should be sent to Stu as soon as possible.

NCTM - CUPM Project Request

With the financial support of the National Science Foundation, the National Council of Teachers of Mathematics and the Mathematical Association of America through its Committee on the Undergraduate Program in Mathematics, are engaged in producing resource materials in all the various applications of mathematics suitable for use by both teacher and student in mathematics instruction for Grades VII to XII, that is, the last six years of secondary school. Application of arithmetic, elementary and advanced algebra, geometry, computing, and other more advanced topics are being worked on. In addition to the uses of mathematics in other disciplines, applications of mathematics in daily life and to skilled trades will be especially emphasized. The readership of this journal is hereby requested

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to send suggestions regarding this project, sample problems, references, or any other suitable materials ranging from simple exercises to extended model building and mathematical development to the committee at:

CUPM
P.O. Box 1024
Berkeley, California 94701
U.S.A.

Our readers are reminded that through hobbies or previous employment they may know of special applications that might otherwise escape notice.

Annual Meetings - NCTM and MCATA

The NCTM meeting in Atlantic City is now history. A report will be printed in either the May or September issue from one of our members who attended. To those of you who did attend, please send in your own report so that we can have more complete coverage.

MCATA annual meeting is October 25-26 at Jasper. Registration fees are $5 for members and $10 for non-members which includes a membership for the 1974-1975 year. Accommodations are $12 for individual registration and $18 for double occupancy. Our program chairman, Denis Baudin, has announced that confirmation of some speakers is imminent, but we can still use other suggestions. Your editor and your chairman are both available for assistance regarding suggestions and questions.

One item of business that is coming at this meeting is a recommendation to increase the membership fee to $6 per member as of January 1, 1975. This fee increase will require a majority vote of those attending the business session. We are requesting approval for this modest increase in order that we can continue the services we are now giving our membership. Costs are going higher and we have not had an increase since 1961.

NCTM Edmonton "Name of Site"

The tapes include the following:

1. The Teacher's Contribution to the Curriculum, Dr. Eugene Smith, NCTM president

Is the teacher's role in education diminishing with the emphasis on individualized instruction, teacher-proof materials, computer-assisted instruction and other innovations?

2. Through the Eyes of our Students, Dr. E. Glenadine Gibb, NCTM president-elect

Observations of children in our mathematics classes provide basis for concern. Do we have the courage to get at the sources of evidence of frustration, resentment, "care-less" attitudes, and complacencies and then take action on our feelings?

3. *Motivating Number Fumblers,* Dr. Stanley Bezuska, Boston College, Boston, Mass.

The acquisition of arithmetical skills often depends on the students' interest and motivation. To get the students interested, tasks must be appealing. The session gives some of the potential solutions.

4. *Student Needs and Subject Requirements - Can One Be Met Without Sacrificing the Other?* Walter S. Manning, Idaho Falls, Idaho.

The real issue of the '70s is to improve teaching strategies to effectively use the materials that are presently available and adapt them to meet the needs and interests of the students.

5. *When is the Thing the Thing?,* Dr. George Immerzeel, University of Northern Iowa

It is important to recognize what things can do and what things cannot do. Examples from the elementary classroom were used to illustrate the use of things to aid the teacher.

6. *Mathematics and the Low Achiever,* Dr. Oscar F. Schaaf, University of Oregon

The results of studies gathering information on low achievers were reported with some operational procedures that may be effective in assisting them to improve.

7. *Transformations and School Mathematics,* John Del Grande, North York Board of Education, Toronto, Ontario.

Transformations is one of the main themes in contemporary mathematics programs. Applications in the study of school mathematics programs are suggested to the teachers of geometry and related areas.

8. *Secondary School Mathematics from a Piagetian Point of View,* Dr. Bruce D. Harrison, University of Calgary

A case for rethinking how mathematics is presented in secondary schools in the light of insights that have emerged from Piaget-related research with practicable alternative approaches to specific topics described.

9. *What About Drill,* Robert C. Clary, Roanoke Rapids City Schools, Roanoke Rapids, North Carolina

Principles used to determine when, where, why, and how drill is to be used in teaching.

10. *What Really are the Basics?,* Louis S. Cohen, Thomas Jefferson High School, Bloomington, Minnesota

Strategies other than "redipping" process for teaching fundamentals are presented that have worked in levels III-XII.

11. *The Last Twenty-Five Years - What Have We Learned?* David W. Wells, Oakland Schools, Pontiac, Michigan

A discussion of what improvements in mathematics teaching have occurred in the last 25 years with suggestions as to how we may continue to improve our programs.

The above listed tapes are available from the Department of Education upon request with no charge other than postage for return. We trust that you will find many of them useful in refreshing your memory on some of the sessions you attended and enlightening for those that you could not attend.

More to it than You Think

Marion Loring
Tangent School
Tangent, Alberta

The New Math is supposed to teach concepts rather than just facts and operations, but I rather doubt if the "Seeing Through Arithmetic" series is doing this well enough in the primary grades. There are cases where the expression of concepts is too confusing.

Consider the sentence, "Five is greater than four." To master this, the child must learn five different concepts:

1. *Vocabulary* The word "greater" is not in the average Grade I child's vocabulary. Even the word "less" is not very often used, as these children tend to say, "He's got more than me," rather than, "I've got less than him." So the meaning of the words "greater" and "less" have to be taught.

2. *Word Recognition* Grade I children at this stage are only just starting to read, so they will not know any of the words in the sentence by sight.

3. *Grammar* The word "greater" applies to the first word in the sentence which is the subject. We take this for granted, but it is not so easy for children. I have found some Grade II children who didn't have this clear, as was shown by the type of mistakes they made. Whenever they had a sentence of the type, "____ is one greater than 7", they would write the correct numeral; but if the sentences had the missing number at the end, as in "7 is one greater than ____", they made mistakes. It appears that their thinking was something like, "Here is 7, and here is 'greater', so I must make 7 greater", rather than thinking, "The first number must be greater." This is even more difficult for Grade I students.

4. *Reading from left to right.* Children at this stage have not all grasped firmly the concept of expressing the order of spoken words by a left-to-right progression, but this is necessary if they are to put in writing the grammatical concept described above.

5. *Number* Finally we come to the mathematical concept that the number five is greater than the number four.

When I finish analyzing in this fashion, I am filled with admiration for the children who do succeed in mastering all this!

I have found it easier to use the symbols $>$ and $<$ first, because whichever way round you look at them, the point is always toward the lesser number, and therefore the "left to right" concept is not needed, so at first you just teach concepts (1) and (5). Then the children read what they have written, and so start to learn concepts (3) and (4). When they can read better the words may be substituted for the symbols.

Another source of confusion is in the type of picture used to teach the concepts of "greater" and "less". This reinforces an error common among young children - that of thinking that the child who has four small candies has more than the child who has three large ones. Three bears are greater than two bears, but they are not greater than two elephants, which is what some of the pictures teach. To teach this is, in effect, to teach that three dimes are greater than two quarters. To teach the correct concepts the pictures should be of objects that are the same, or at least of the same size.

The pictures illustrating subtraction are confusing also. I noticed that several Grade II children, who could complete straight equations in addition and subtraction satisfactorily, made many mistakes when writing a mathematical sentence from a picture. An examination of the pictures showed why. First they are taught addition with pictures such as this  They are taught to perceive two sets and to write first how many are in each set, then to complete the equation, 2 + 3 = 5. When they see a picture representing subtraction, such as  , they perceive the same two sets, so they tend to write 3 - 2. A picture representing subtraction should show the first number rather than the remainder. I found that the most effective way of clearing up this confusion was to put up this chart:

That this was helpful was shown by the fact that when I removed it the children asked me to put it up again.

But this is just a way of enabling children to do what the book requires them to do. I don't think subtraction can be efficiently taught through pictures. It needs manipulation of real objects and oral problems.

This confusion may be partly due to a failure to recognize that the operations of addition and subtraction are not the same as the equations expressing them. Two different concepts are involved. The operation is active; the equation is static. The operation involves change, motion, time; the equation is a universal statement of fact.

Because the numbers are the same, we are apt to think that the concepts are the same; so we need to look at this more closely. In the operation of addition there is one set of objects at the beginning; then another set moves toward it; then the two merge, forming a final resulting set which is greater than the first set. But an equation does not express change from a smaller number to a greater - it states that the two smaller numbers together are the same as one greater number. This can be expressed on a balance: a two-ounce weight and a one-ounce weight on one side will balance a three-ounce weight. But the operation of addition cannot be expressed on the two sides of a balance.

These are two different concepts, and I think the modern practice unsound, which insists on using the language of the equation and making children say, "Three plus two is equal to five", while they are observing a series of pictures to represent an operation in time. The old-fashioned teacher who permits the simpler sayings, "Three and two more make five," or, "Five take away two leaves three", does in fact teach the operational concept better. When children have performed the operation several times they may realize the basic rule expressed in the equation.

It is difficult to present the concepts of change, motion and time in pictures. Grade I children tend to look at all the pictures at once, rather than starting at the top left corner. This doesn't matter so much in addition which can be expressed in one picture; but subtraction requires at least two. It might be better if Grade I students had no text book, but did all their work with real objects.

This may seem too detailed a criticism, but if the proponents of New Math want us to teach concepts, let us think clearly what they are, and the most efficient way of teaching them. We must think in a more specialized way about Grade I. It is no use just planning a logical beginning to a series. The logical beginning may not be suited to Grade I. And it is there that we are laying the foundation; if that is not solid, what happens to the superstructure?

Distributor Cap Mathematics

by *William J. Bruce*
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Edmonton, Alberta

This article deals with problems that involve permutations and cyclic permutations. These problems are adaptable to the discovery method of teaching and include opportunities for making conjectures that can be followed by informal proof.

It is interesting to watch a skilled mechanic install a new distributor cap in a modern automobile. How quickly he connects the spark plug wires without much hesitation. It appears that he plugs the ends into the distributor cap connectors in any convenient order but this is not really what he does. There is one and only one correct arrangement that will work properly. How does he know the correct one? Ask any experienced mechanic and he'll probably tell you.

Let us suppose that we don't know the trick and that we start plugging in at random. How many arrangements are possible? The standard Delco-Remy distributor cap for an eight-cylinder car is shown in Figure 1. We note that the connectors for the eight spark plug wires are arranged in a circle and that there is a center connector for the power source. At the base are two clip mechanisms for fastening the cap. We shall examine the problem for motors with different numbers of spark plug wires. Figure 2 is a diagrammatic representation of a distributor cap for a four-cylinder motor. The spark plug

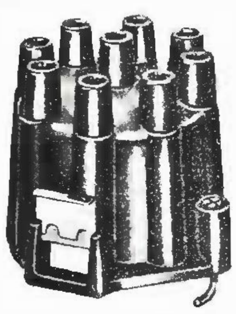

Figure 1

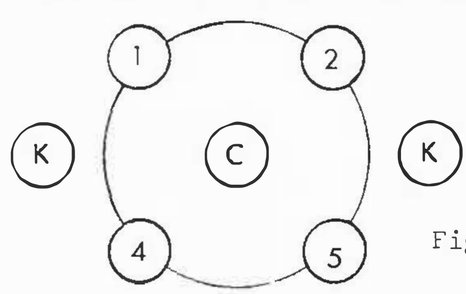

Figure 2

wire connectors are shown as 1, 2, 3, and 4, "c" is the center connector and "k" is a clip. It must be noted that, once installed, the cap is rigid. We consider the number of possible ways to plug in the spark plug wires. Suppose that the first wire that we choose is plugged into position 1. Then there are 3! ways of plugging in the other three wires for each position of the first wire. But the first wire could have been plugged in at four different places. Thus there are 4(3!) or 4! possible arrangements, a total of 24 ways to plug in the four wires.

Consider, now, caps for six- and eight-cylinder motors as diagrammed in Figures 3 and 4. For the six-cylinder motor we plug into position 1 again and permute the other five to obtain 5! ways of plugging in the other five wires.

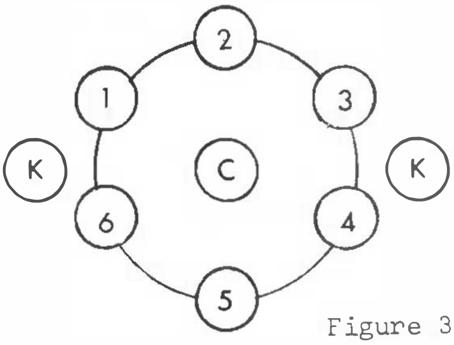

Figure 3

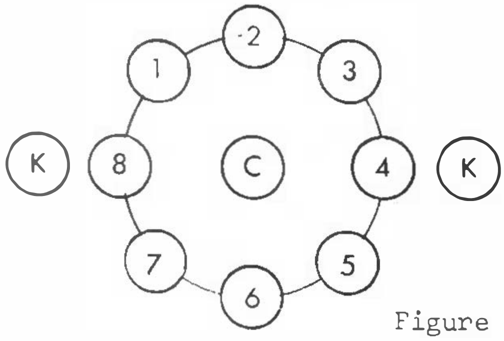

Figure 4

If we allow the first wire to be plugged into any of the six places we obtain 6(5!) or 6!, that is, 720 ways to plug in the six wires. Proceeding in the same way for the eight-cylinder motor we find that there are 8(7!) or 8!, that is, 40,320 ways to plug in the eight wires - obviously far too many for a trial and error approach.

Suppose further that the wire to the center connector is included with the spark plug wires and that all arrangements are determined with center wire and spark plug wires considered interchangeable. From Figure 2 we see that there are now 5 ways of plugging into the center position and for each of these there are 4! ways of plugging into the spark plug connectors, a total of 5(4!) or 5!, that is, 120 ways in all. From Figure 3 we similarly obtain 7! or 5040 ways and from Figure 4 we get 9! or 362,880 ways.

Remove the clip pins from a Delco-Remy distributor cap and it can be used as a pencil or crayon holder for your desk. If preferred, the black color can be changed to suit any decor by spray painting. An examination of Figures 2, 3, and 4 reveal that we have, respectively, 7, 9, and 11 places for pencils when we include the clip positions "k".

Suppose that all positions, except the center position, are to be filled with different colored pencils and that we do not allow the holder to rotate. We consider the number of possible arrangements of the pencils in each case, excluding the center position. From Figure 2 we see that there are 6(5) ways of filling positions "k" and for each of these there are 4! ways of filling positions 1, 2, 3, and 4. Hence there are 6(5)(4!) or 6!, that is, 720 ways of arranging the pencils. From Figures 3 and 4 we find similarly that there are 8! (that is, 40,320) and 10! (that is, 3,628,800) ways, respectively.

If we include the center position "c" in Figure 2, we find that there are 7(6)(5) ways of filling positions "k" and "c" together and for each of these there are 4! ways of filling positions 1, 2, 3, and 4. This yields a total of 7(6)(5)(4!) or 7!, that is, 5040 ways of arranging the pencils. From Figures 3 and 4 we find similarly that there are 9! (that is, 362,880) and 11! (that is, 39,916,800) ways, respectively.

The following table summarizes our results. If rotations had been permitted, the cyclic permutations would have been computed using (n - 1)! instead of n! as has been done.

n	p	p + c	p + 2k	p + c + 2k
4	4! = 24	5! = 120	6! = 720	7! = 5040
6	6! = 720	7! = 5040	8! = 40,320	9! = 362,880
8	8! = 40,320	9! = 362,880	10! = 3,628,800	11! = 39,916,800

n - number of spark plug wires.
p - number of cyclic permutations of the spark plug wire connections to the distributor cap (including rotations of the wires).
c - center connection on the distributor cap.
k - clip points of the distributor cap.

Solving Simple Equations

by *Howard L. Larson*

The author is a retired mathematics teacher and school superintendent with many years experience teaching and training others in the art of teaching.

Much classroom teaching is confusing for students until they are forced to learn without understanding for the sake of self-preservation. A good example of that type of teaching is the method usually, or often, offered to the student for finding the solution of equations.

An example of this is found in solving 3N = 12.

To solve for "N" we divide both sides by 3 (or multiply by 1/3: the choice is irrelevant to the discussion following). When we do this we get N=4. Simple isn't it! Why not subtract 3 from both sides and get N=9? Well, why not indeed? Many of your students will have this question in mind and may even dare to ask it.

The basic problem stems from the fact that we assume too much at the beginning levels. Here, for example, we assume that the student understands that 3N means 3•N or that 3 and N are factors. Again, we assume that the student understands that when we divide 3N by 3 the threes disappear in thin air! We neglect to teach that the meaning of identity and inverse or 3/3 = 1 have some operational principles which require conscious teaching.

Le us examine a more rational approach.

If $\overset{f}{3}\times\overset{f}{4} = \overset{p}{12}$ then $\overset{p}{12}\div\overset{f}{3} = \overset{f}{4}$ or $\overset{p}{12}\div\overset{f}{4} = \overset{f}{3}$. With a little practice, the truth of this statement becomes self-evident to the student and he can be expected to rewrite many product equations in the form of a quotient by stressing the ffp language with an understanding of what he is doing.

f f p p f f p f f
If 2xN = 10 then 10÷N = 2 or 10÷2 = N. Having introduced an unknown factor in this sequence comes as no mystery to the student. He has only to make a choice regarding which option is most productive for his use, but it is now a simple choice to make.

Let us try another inverse situation to illustrate our approach.
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If 3+4 = 7 then 7-4 = 3 or 7-3 = 4. Again, using the language of aas the student can learn to rewrite all the addition facts or equations in their inverse form.

a a s s a a s a a
If 3+N = 8 then 8-3 = N or 8-N =3. The student can easily decide which of the equivalent forms is most useful for him and of course arrive at the solution N = 5 without difficulty. We have not added or subtracted from either side. This latter procedure or technique is a cause of considerable difficulty for students in junior high and/or senior high.

Suppose next we wish to solve a problem combining both of the above
a a s s a a s a a
forms. Our example is 3N+5 = 14. The inverse forms are 14-5 = 3N or 14-3N =5 The student's choice for finding a solution will be 14-5 = 3N. The result is 9 = 3N. This equation will be rewritten in the form 9÷3 = N and 3 = N is the student's solution.

If the student is given this kind of background in whole numbers he will be able to adapt readily to other number systems.

s a a a a s s a a
For example, if 3-N = 7 then N+7 = 3 and 3-7 = N. Note that the inverse form can work both ways. Since 3-7 is an impossible operation in the set of whole numbers, we use the set of integers for a solution ($^{+}3-^{+}7=N$). At this time we have assumed the student has been taught to subtract any pair of numerals in set 1. Our solution then becomes $^{+}3 + ^{-}7 = N$ or $^{-}4 = N$. This latter step of course comes at the conclusion of the teaching sequence and can be readily understood at this juncture.

If 2/3N = 6, then 6÷2/3 = N or 6÷N = 2/3. Assuming now that the student has learned to divide fractions the rest will be readily self-evident.

The next step will be to add the rational numerals. For example,
s a a
3/4 x N + 3 = 6. The inverse form for the first step is 6 - 3 = 3/4 x N, and
p f f
rewritten becomes 3 = 3/4N. This problem is then rewritten 3÷ $^{-}3/4$ = N.

We suggest this method of teaching begin at the elementary level so that the mystery of solving equations at the junior high level disappears.

Yes Senior High Mathematics Can Use the Library

by *Jake Woloshchuk*
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Dr. E.P. Scarlett High School
Calgary, Alta.

Traditionally there are several subject areas of the school program that do not typically use the school library. This does not mean, of course, that the teachers in these areas don't want to utilize the library but tradition doesn't permit them to do as social studies teachers might. Mathematics is a good example of a subject area that has found library utilization difficult. Considering the number of items usually found in the pure mathematics collection it is easy to see why these teachers have shied away from exploratory units of work. However, after searching the collection, we discovered that the general collection contains a great deal of material useful for projects in mathematics. Health sciences are useful for statistical work dealing with blood pressure, EEGs and ECGs. The automotives collection lends itself to studies of gear-ratios, firing orders and rates of speed. The literature collection is useful for looking at meter, Haiku and concrete poetry. The sports and games collection lends itself to studies of the measurement of human achievement, the permutation and combinations of chess maneuvers and the mechanics of muscle-building. Economics, statistics, and criminal patterns of behavior are also covered in this mathematical serendipity.

According to mathematics teachers at Dr. E.P. Scarlett High School, the library plays an important role for increasing the relevancy of mathematics for students keenly interested in the problems of today's world. Students may be encouraged to write mathematics reports on a wide variety of student-selected topics. The range of topics can be wide indeed. Following is a list of actual topics researched by the students: investments, taxation, consumer credit, business mathematics, banking services, types of insurance, probability, history of mathematicians or a certain aspect of mathematics, budgeting, mathematics in mechanics, mathematics in sports, mathematics in home economics, the slide rule, geometrical designs, mathematics in construction, mathematics in map-making, mathematics in engineering, mathematics in urban planning, the role of mathematics in an occupation or student interest area, structural designs, mathematics and psychology, mathematical designs, mathematical models, mathematics in everyday life, pure mathematics, relationship of mathematics to population study, computers, computer programming, mathematical experiments, mathematics in music, and a mathematics topic of your interest.

Such reports based upon library research in mathematics have encouraged, as well as demonstrated, the applicability of mathematics in various disciplines. The librarian and members of the English Department have provided necessary guidelines for researching, footnoting and providing bibliographical data for these reports. The librarian insured that students go beyond the traditional to the unusual for sources of information.

And not all of the "reports" were written - some were models of geodesic domes, urban renewal maps, and pictures of flood plain studies. One student interested in campanology showed how bell-ringing is formula-math!

The Mathematics Department and librarian should (and in this school do) cooperate in a two-way flow of information and idea exchange. In addition, there is no reason why the school librarian cannot be an active participant at departmental meetings in order to inform teachers about new reference books in mathematics; new materials related to mathematics; audio-visual equipment and services; professional reading; new ideas gleaned from other department meetings; innovative projects, such as videotape usage.

Librarians may wish to assess teachers' needs in the above areas and develop programs geared to professional needs - such as our Annual Media Workshop. Because of this, the library at Dr. E.P. Scarlett High School has, with the aid and example of the Mathematics Department, embarked upon an innovative project that will hopefully have implications for all Calgary schools. The Integrated Resources Project will catalog all school resources into one central card catalog indicating the location of all school-owned materials. Ordering procedures will be established to ensure the arrival of catalog information at approximately the same time as the material ordered. Hopefully, interdepartmental usage will increase and maximum utilization of resources will be effected. Duplication of exact and similar items between departments should be minimized.

The library has many and varied uses for mathematics education in a school. Improvement of the mathematics programs is possible through an appropriate and necessary relationship with school librarians who can utilize a variety of resources and be a generative source of ideas.

Mathematics teachers should run, not walk, to their school libraries. Who knows - a Mobius Strip could always be a new program for topless research!

This article was reprinted from *Moccasin Telegraph,* Vol. 16, No. 1, November 1973, the newsletter of the Canadian School Library Association.

Mathematical Competencies and Skills Essential for Enlightened Citizens

The Committee on Basic Mathematical Competencies and Skills
E.L. Edwards, Jr., Chairman
Eugene D. Nichols
Glyn H. Sharpe

In March 1970 the Instructional Affairs Committee proposed that an ad hoc committee write a set of objectives to define minimum competence in mathematics. Recognizing its responsibility to the profession as well as to the public to establish guidelines for the mathematical needs of all citizens, the NCTM Board of Directors accordingly appointed a committee to draw up a list of basic mathematical competencies, skills, and attitudes essential for enlightened citizenship in contemporary society.[1] A report containing such guidelines was painstakingly prepared by the committee and is presented here. Even though the report does not represent NCTM official policy and may contain points with which individual members may disagree, it unquestionably contains much food for thought on a subject of great importance to society in general and to the mathematics-education community in particular. The entire report is therefore presented here for the consideration of the membership.

Following World War II, the crucial importance of mathematics in the technological and other aspects of life in the complex modern society has been unquestionably recognized. With it the question, How much mathematics is a "must" for every citizen? reappeared with new urgency.

The National Council of Teachers of Mathematics appointed the Commission on Post-War Plans and charged it with the task of answering this question. After completing its deliberations, the commission issued a checklist of 29 questions which constituted a basis for assessing one's basic competence in mathematics. "Guidance Report of the Commission on Post-War Plans" was published in 1947 in volume 40, number 7, of the *Mathematics Teacher*.

Since that time many changes have been introduced into mathematics. The very nature of the subject has changed. The introduction of new areas such as abstract algebra, functional analysis, and topology has resulted in what has been called a "revolution in mathematics".

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[1]The reader should recognize that this is not a list of minimum competencies, skills, and attitudes to be attained by all students with an IQ of 70 or above; unfortunately, some of our students will not become enlightened citizens in contemporary society.

Not until the 1950s, however, did these changes in the very nature of mathematics have an impact on secondary and elementary school curricula. Because of the revolutionary work of the University of Illinois Committee on School Mathematics, the School Mathematics Study Group, the Commission on Mathematics, and other such groups, the curriculum underwent some drastic changes. New concepts appeared in textbooks and new approaches to teaching old concepts found their way into classrooms. Thousands of teachers returned to the universities and colleges to update themselves in the new concepts and methods. Writers of commercial textbooks, benefiting by the suggestions from experimental groups, introduced new concepts and approaches.

The realization in the late 1950s that the technology leadership of the United States had been challenged by others has given an additional impetus to the emphasis on the study of mathematics and the natural sciences. The demand for increased competence in mathematics has become a reality. New competencies and skills have been introduced into the required repertoire of the day-to-day activities of the individual.

The National Council of Teachers of Mathematics found itself with a challenge to define what might be considered the minimal mathematical competencies that are expected of the enlightened citizenship in contemporary society. Although it is impossible to find a universal agreement on such things, it is nevertheless the duty of this professional organization of mathematics teachers to propose what might be considered a reasonable approximation to such a statement of competencies. It is with this intent that this report is being offered.

Mathematics is one thing to some, another thing to others. Some enjoy it; others do not. Some seek to use it; others avoid it. Some individuals can get by with very little mathematical knowledge. In other words, the amount of mathematical skills that some individuals must have to "barely get by" in society may be very nearly zero. Present-day society collectively, however, needs a great deal of the most sophisticated kind of mathematics for its functioning. It is essential for its very survival. The highly complex problems of the technological society require complex mathematics to solve them. The demand, therefore, for increased competencies in mathematics has become a reality for many.

There are three basic ways to view mathematics:

1. Mathematics as a tool for effective citizenship and personal living
2. Mathematics as a tool for the functioning of the technological world
3. Mathematics as a system in its own right.

Each of these three aspects of mathematics is of significance. The first aspect is important for all. The second is essential to the scientist, the engineer - generally to those who make their living through professions that use mathematics as a tool. The third aspect is of concern to the professional mathematician. He sees in it a fascination and beauty that usually escapes the mathematically untrained mind.

In isolation, no one of the three aspects is of greater significance than another. In the total complex of society, each aspect is of significance.

Few educators will deny that past efforts in mathematics education have produced significant gains for many pupils. There are too many cases, however, where many pupils leave our schools without the necessary skills to make them employable and to allow them to apply mathematics to help them solve the problems of daily life.

Some decisions are desperately needed to give guidance and direction to teachers and administrators concerning

1. what mathematics every pupil must master to "barely get by" in contemporary society;
2. what mathematics is essential for full participation of an individual in contemporary society;
3. what teaching techniques and processes not only assure the acquisition of mathematical skills and competencies deemed essential but also convince pupils to be willing to use these skills once acquired.

The following listing of competencies and skills has been separated into three main categories.

In the first section, an attempt has been made to include examples of those skills and competencies that are felt to be necessary for the majority of adults in our society. This is not to imply, however, that every individual must master each concept listed. It is the hope of the NCTM that the reader will consult these guidelines and interpret them in light of his own situation. The listing should give some guidance to teachers of mathematics in establishing priorities. It is hoped that teachers will attempt to convince pupils by appropriate and reasonable objectives, activities, and other techniques that the acquisition of essential mathematical skills and competencies is necessary for effective functioning in today's society.

In the second section, a few of the basic characteristics of the nature of mathematics as a system in its own right are presented. It is done with the hope that the individual curious about this aspect of mathematics as a discipline will find it helpful as he seeks to comprehend the basic nature of mathematics.

The third section should appeal to those individuals with the kind of inclinations discussed in the second section. These individuals are usually more inclined to seek a deeper understanding of the role of mathematics in society than those who are not mathematically inclined.

Skills and Competencies

Individuals often find the need to use mathematics in everyday life and in many jobs that frequently require some technical application of mathematics. The following outline of content gives some indication, under each heading, as to what minimum "doing" skills are needed by the enlightened citizen.

1. Numbers and numerals
   a) Express a rational number using decimal notation
   b) List the first 10 multiples of 2 through 12
   c) Use the whole numbers in problem solving
   d) Recognize the digit, its place value, and the number represented through billions
   e) Describe a given positive rational number using decimal, percent, or fractional notation
   f) Convert to Roman numerals from decimal numerals and conversely, for

example, date translation
g) Represent very large and very small numbers using scientific notation

2. Operations and properties
   a) Write equivalent fractions for given fractions, such as 1/2, 2/3, and 3/5
   b) Use the standard algorithms for the operations of arithmetic of positive rational numbers
   c) Recognize and use properties of operations (grouping, order, and such) and properties of certain numbers with respect to operations ($a \cdot 1 = a$; $a + 0 = a$; etc.)
   d) Solve addition, subtraction, multiplication, and division problems involving fractions
   e) Solve problems involving percent
   f) Perform arithmetic operations with measures
   g) Estimate results
   h) Judge the reasonableness of answers to computational problems

3. Mathematical sentences
   a) Construct a mathematical sentence from a given verbal problem
   b) Solve simple linear equations such as $a + 3 = 12$; $16 - n = 4$; $n/3 = 7$; and $4a - 2 = 18$
   c) Translate mathematical sentences into verbal problems.

4. Geometry
   a) Recognize horizontal lines, vertical lines, parallel lines, perpendicular lines, and intersecting lines
   b) Classify simple plane figures by recognizing their properties
   c) Compute perimeters of polygons
   d) Compute the areas of rectangles, triangles, and circles
   e) Be familiar with the concepts of similarity and congruence of triangles

5. Measurement
   a) Apply measures of length, area, volume (dry or liquid), weight, time, money, and temperature
   b) Use units of length, area, mass, and volume in making measurements
   c) Use standard measuring devices to measure length, area, volume, time, and temperature
   d) Round off measurements to the nearest given unit of the measuring device (ruler, protractor, thermometer) used
   e) Read maps and estimate distances between locations

6. Relations and functions
   a) Interpret information from a graphical representation of a function
   b) Apply the concepts of ratio and proportion to construct scale drawings and to determine percent and other relations
   c) Write simple sentences showing the relations $=$, $<$, $>$, and $\neq$ for two given numbers

7. Probability and statistics
   a) Determine mean, median, and mode for given numerical data
   b) Analyze and solve simple probability problems such as tossing coins or drawing one red marble from a set containing one red and four white marbles
   c) Estimate answers to computational problems
   d) Recognize the techniques used in making predictions and estimates from samples

8. Graphing
   a) Determine measures of real objects from scale drawings
   b) Construct scale drawings of simple objects
   c) Construct graphs indicating relationships of two variables from given sets of data
   d) Interpret information from graphs and tables

9. Mathematical reasoning
   a) Produce counter examples to test the validity of statements
   b) Detect and describe flaws and fallacies in advertising and propaganda where statistical data and inferences are employed
   c) Gather and present data to support an inference or argument

10. Business and consumer mathematics
   a) Maintain personal bank records
   b) Plan a budget including record keeping of personal expenses
   c) Apply simple interest formulas to installment buying
   d) Estimate the real cost of an article
   e) Compute taxes and investment returns
   f) Use the necessary mathematical skills to appraise insurance and retirement benefits

The Nature of Mathematics

An individual who is inclined toward the study of the structure of mathematics would do well to consider those attributes of mathematics that deal with this aspect of mathematics. Some of these attributes and the abilities associated with them are discussed below.

1. The deductive nature of mathematics. Here the concern is with the question of what follows by force of logic from known truths. The derivations of new conclusions take the form of "if ... then" statements. The central issue is whether the statement following "then" is a logical consequence of the statement, assumed to be true, following "if".

2. In relation to attribute one, an individual should be able to carry through a consistent argument. Proofs of theorems constitute a very important part of mathematics. It has been demonstrated that some very young children are capable of proving simple theorems. Those individuals who display this ability should be provided with opportunities to experience success in this kind of mathematical activity. For example, establishing in a logical way that the sum of two even numbers is an even number would be a very satisfying experience to a child with such inclinations.

3. An individual with an inclination toward this aspect of mathematics should be able to differentiate between a valid argument and an invalid one. Theorems are established by proceeding from assumptions to conclusions by means of a series of logical implications. It is important to be able to recognize arguments that have fallacies in them. One should appreciate the fact that liking a conclusion does not imply that it has been established validly.

4. An individual of this type should be familiar with the basic properties of operations on numbers. For example, such properties as commutative and associative of addition of real numbers should be a second nature to him. For those individuals, it is not sufficient to be skillful in the use of basic operations - they should be able to achieve economy in computations

by making use of basic properties. The distributive property, for example, is very useful in simplifying computations.

5. These individuals should be able to verify whether or not a given system possesses given properties. They should study all the properties that a set of numbers under given operations should possess in order to constitute a particular system. For example, they should be able to verify whether a given set of objects is a field under two given operations.

6. Such individuals should be able to recognize that various concepts and operations are related to each other; for example, subtraction and addition are inverse operations. Having some idea of the ways in which concepts and operations are related to each other eliminates the necessity of memorizing many isolated facts. Experience and training in observing relations between entities aids retention and enhances the development of observational skills.

7. Such individuals should be able to perceive patterns displayed in sequences. Observations of patterns is a skill that finds applications in many areas of human endeavor. Mathematics is an ideal subject in which to develop this skill.

The Role of Mathematics in Society

The individual with the kinds of inclinations discussed in the previous section is also inclined to seek a deeper understanding of the role of mathematics in society than is usually the case with those who are not mathematically inclined. This implies the following:

1. *Knowing the ways in which computers are used in sciences, technology, business, and government.* Computers are indispensible for the functioning and the very survival of present-day society. The complexity of the present problems requires the use of computers. They are useful because of their speed and their ability to store a multitude of facts. The speed with which millions of facts can be processed is essential in complex decision-making situations. One must guard, however, against the belief that computers are capable of solving all problems. They should not be attributed the complex ability of independent thinking and decision making.

2. *Recognizing the evolutionary development of mathematics by noting the historical milestones in the development of mathematical ideas.* These mathematical ideas served man in solving his problems and aided him in the control of his environment. Beginning with the invention of numbers for the purposes of counting through the invention of irrational numbers for describing lengths, man has conceptualized mathematical things which at first served exclusively his utilitarian purposes. Both the invention of a specific mathematical tool and its conceptualization into an abstract system are essential ingredients of the development of mathematics.

3. *Being aware of the great frequency with which mathematical skills are used by individuals in their daily lives.* To appreciate the importance of mathematics in daily life, one might pose a question as to what would happen if mathematics disappeared. If number or geometry disappeared, would man be able to go about completing successfully his daily chores?

4. *Recognizing that there are problems that, by their nature, do not lend themselves to solution by mathematical methods.* Individuals at times make decisions that are primarily based on feelings and emotions, and they should recognize that such bases for decisions, though they may be valid, are by their very nature not amenable to mathematical analysis.

5. *Recognizing that some professions require knowledge of the most sophisticated and complex mathematical techniques.* A young individual who, because of a demonstrated high aptitude for mathematics, may desire to train for a profession that has high requirements of mathematical competence should apprise himself of these requirements and acquire the skills necessary for the given profession. He should realize that many years of mathematical training may be necessary.

6. *Being aware that mathematics finds direct applications not only in the natural sciences but in the behavioral and social sciences and arts as well.* Many psychologists as well as individuals concerned with humanities make increasingly greater use of mathematical techniques in developing their disciplines. For example, computers are used to analyze and understand the factors that influence human behavior. Since there is a large number of variables involved in such situations, this analysis and understanding could not be achieved without the electronic computer. Environmental problems, because of their complexity, will demand more and more sophisticated mathematical models for their solution.

Conclusion

The basic mathematical competencies and skills essential for enlightened citizens are determined by the needs of society at a given time. In our rapidly changing technical society, many factors bear upon what are and what will be the required competencies and skills. Educators charged with maintaining a contemporary basic mathematics program must be aware of these factors and give them consideration in their efforts to design effective programs.

Ours is the age of technology. A lessening need for individual computational skills has resulted from the universal availability of calculators. Computers have largely obviated the need for paper and pencil computations, but they have increased the need for formulating careful and reasoned generalizations. As other devices become available, their influence on society must be reflected in basic mathematical competencies and skills identified as those needed by enlightened citizens.

The educational level of our society is constantly changing. A larger percent of the school-age population is enrolled in school; the number of years students remain in school is on the increase. The needs of citizens, therefore, will change as the educational level compounds itself.

Ingenuity in devising individual means of checking one's conclusions for "reasonableness" is a most valuable aid to effective living and is a skill that mathematical training can greatly sharpen. An enlightened citizen must logically and judiciously cope with the constantly increasing bombardment of statistics, facts, and figures. As all teachers and students become more aware of the demands for responsible, self-reliant judgments, the central role of mathematics in making such judgments becomes more apparent.

An enlightened citizen is qualified for employment. Employment opportunities are changing and will change both in nature and in requirements of personnel. Surely the degree to which an occupation is mathematically oriented will determine the extent of the basic mathematical skills and competencies required of individuals employed in that occupation. The aggregate degree of mathematical orientation of the job market, in turn, influences what is considered a basic level for all citizens.

Technology, the educational level of society, and occupational requirements are examples of factors that influence the basic mathematical competencies and skills needed by enlightened citizens. Some other factors are probably manifested; still others will evolve from their influence. It is therefore critical that we continually examine the programs designed to improve basic mathematical competencies and skills in future years. It is also the hope of the National Council of Teachers of Mathematics that professional groups concerned with mathematical education for all citizens will constantly strive to interpret the factors influencing change, seeing these in relation to their implications for the mathematics curriculum.

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