THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

delta-k

Volume XIII, Number 4, June 1974

Important Notice

The Executive Committee recommends the following constitutional changes to the membership. These recommendations will be voted on at the annual meeting, October 25 and 26, in Jasper.

1. It is recommended that the "Reporting" clause be changed to read:

   "This council shall submit annually a written report of its activities and an audited financial statement to The Alberta Teachers' Association by October 1 of each year. The activities reported shall be for the preceding year."

2. It is recommended that the "Executive Committee' definition be changed to read:

   "The Executive Committee shall consist of the officers, one member from the Faculty of Education from a university in Alberta, one member from the Department of Mathematics of a university in Alberta, one member from the Department of Education, the editor of the newsletter and six directors to be appointed by the officers from the following: editor of the monograph, the chairmen of committees, the presidents of regional councils, members at large, provided that each university representative be appointed for a two-year term and also that the two university representatives not be from the same university and provided that the directors be appointed to ensure that the executive committee includes at least two representatives of each of elementary, junior high and senior high school teachers.

In addition, the Executive Committee recommends a change in the fee structure, as follows -

Regular membership	$6
Subscription	$6
Student membership	$3

Nominations for 1974-75 MCATA Executive

President - Dr. W. George Cathcart of the University of Alberta

Vice-President - Francis Somerville of Calgary

Secretary - Dennis Treslan of Calgary

Treasurer - Donald Hinde of Lacombe

As only one person was nominated for each office, the nominees are declared elected by proclamation and will assume office following the annual meeting.

THINK JASPER - OCTOBER 25-26, 1974

MCATA members and other mathematics teachers will gather at Jasper Park Lodge to hear speakers present ideas and challenges to help us improve our classroom presentation of materials and increase our ability to help our students to gain mathematical competency. Among the speakers will be Robert Eicholtz of Addison-Wesley (Canada) Ltd.; Ed Murin, NCTM national representative; Emery Dosdall of the Edmonton Public School Board; Glen Kauffman of Parkland Composite High School in Edson; Thomas D. Baker, retired teacher; Bill Easton of Moyers Publishing; and Dorothy Burton of Bowness High School in Calgary. Details not previously announced will be in *The ATA News* and the *Delta-K* September issue, and in at least one other mailing.

THINK DENVER - APRIL 23-26, 1975

NCTM national meeting will be in Denver. We are planning to travel as a group to take advantage of reduced air fares so that many of us can have an opportunity often available to only a few. This gathering will have all the advantages of a name-of-site meeting, plus many others. We also want to support our own Dr. Joan Kirkpatrick, MCATA member and director, whom we congratulate on her election at the 1974 annual meeting, as a director.

NUMBER ONE ON THE METRIC HIT PARADE

What can a music teacher do when his school decides to "go metric" for a month? This was the question confronting Charles Rinehart at the Campus School in Oswego, New York. His response was to use rhyme and rhythm to teach the new metric vocabulary. His song, "Halve Your Meter", has become an instant hit due to its catchy melody and painless introduction to the metric system. From the standpoint of a music teacher looking at the metric system, Charles says, "How pleasant it is to have a spoonful of sugar to help the metric go down."

The words and music of the tune appear in the May, 1974 issue of *School Science and Mathematics*. If you would like a reprint of the article—which includes the words and music to the tune—simply send a stamped, self-addressed envelope to:

School Science and Mathematics Association
P.O. Box 1614
Indiana University of Pennsylvania
Indiana, Pennsylvania 15701

MONOGRAPHS NOS. 2 AND 3 IN THE WORKS

Two monographs are being prepared for publication: Monograph No. 2 - MATHEMATICS TEACHING: The Site of the Art - is a report on the 1973 name-of-site, presenting the general interest speeches. It is expected to be ready for distribution in July. Monograph No. 3, on metrication, edited by Dr. K. Allen Neufeld, will not be ready for release until next year. The monographs will be free to members.

A LETTER FROM THE EDITOR OF MONOGRAPH NO. 3.

Dear MCATA member,

I have been named editor of the Monograph No. 3, to be published by the Mathematics Council of The Alberta Teachers' Association. The topic for the monograph is metrication.

The purpose of this letter is to solicit articles for the publication. I would like the major emphasis to be on activities which teachers could use with students from kindergarten through Grade XII. At the elementary level, there is a need for separate sets of activities covering each of the aspects of linear, area, capacity, temperature, and mass measurement. At the secondary level, separate sets of activities could be devoted to topics such as the measurement of electric current, substance, luminous intensity, plane and solid angles.

It would also be desirable to have several articles on broader themes such as history, scope and sequence, societal implications, interrelationships among subject areas, and so on.

If you are interested in submitting an article for this monograph, please let me know by July 15. Tell me your proposed topic and I will let you know if several others have decided to cover the same subject of metrication.

Manuscripts must be in my hands by September 1, 1974. They should be typed (double-space) and appropriately referenced.

I look forward to hear of your interest in this project. My address is:

Dr. Allen Neufeld
Associate Professor
Department of Elementary Education
University of Alberta
Edmonton, Alberta T6G 2G5

OOPS!

An oversight in our *Delta-K* issue No. 2 released in February resulted in our omitting to mention Art Jorgensen, principal of Jubilee High School in Edson, as the reviewer of the books in that issue. Our apologies are extended for this oversight.

CANADIAN MATHEMATICAL CONGRESS
1974 Alberta High School Prize Examination

N O T E

We were asked by the Department of Mathematics, University of Alberta, to publish the following CORRECTION TO PART I of the Alberta High School Prize Examination in Mathematics:

The answer "Key" to Part I of the above exam is to be amended to read 1. = "C" and 5. = "E".

The Mathematics Council congratulates the following for outstanding achievement in the 1974 Alberta High School Prize Examination in Mathematics:

Place	Name	Grade	School
1.	Paranjape, Manu B.	12	Strathcona Composite High School, Edmonton, Alberta
2.	Whitney, Arthur T.	12	Strathcona Composite High School, Edmonton, Alberta
3.	Graham, William R.	11	Harry Ainlay Composite High School, Edmonton, Alberta
4.	Smith, Robert J.	12	Frank Maddock High School, Drayton Valley, Alberta
5.	Mallet-Paret, Louis C.	11	Archbishop MacDonald High School, Edmonton, Alberta
6.	Romanycia, Marc H.	12	Ecole J.H. Picard, Edmonton, Alberta
7.	Gretton, Jeremy S.	12	Western Canada High School, Calgary, Alberta
8.	Reid, Douglas R.	12	Bonnie Doon Composite High School, Edmonton, Alberta
9.	Davenport, Michael R.	12	Lord Beaverbrook High School, Calgary, Alberta
10.	Stochinsky, Tamin	12	Jasper Place Composite High School, Edmonton, Alberta
11.	Elliott, John F.	12	Camrose Composite High School, Camrose, Alberta
12.	DeGroot, E. Thomas	12	Edmonton Christian High School, Edmonton, Alberta

13.(A)	Sande, Gregory J.	12	St. Francis Xavier High School, Edmonton, Alberta
13.(B)	Hwang, Helena	12	Bishop Grandin High (see below)
14.	Ady, Michael S.	12	Bishop Grandin High School Calgary, Alberta
15.	Steffler, Peter	12	Louis St. Laurent High School, Edmonton, Alberta
16.	Grant, Donald B.	12	Ernest Manning High School, Calgary, Alberta

We wish to extend our best wishes to all who participated and our heartfelt desire that they will continue to achieve and to contribute to society the results of their achievements.

As a service to the teachers of participating students, and for the students and the coordinating personnel, we include the solutions to Part II of the examination which were not available for distribution with Part I at the time of mailing the results.

Solutions to Part II

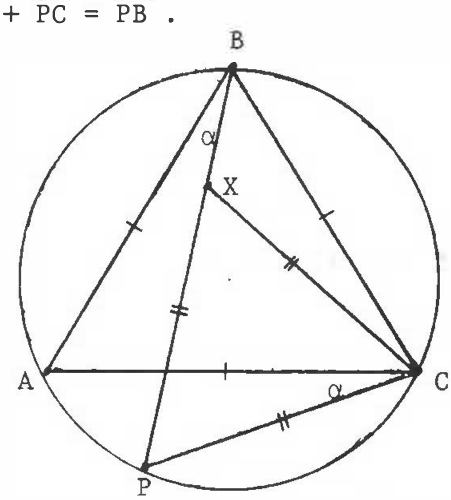
1. Suppose we divide the set of numbers $A = \{1,2,3,4,5\}$ into two sets C and D $(C \cup D = A)$. Prove that at least one of the sets C, D must contain two numbers and their difference.

   Solution: Let's try to show the conclusion need not be true. Put the number 1 into set C. Then the number 2 must go into D, since if $\{1,2\} \subset C$ then $1,2$ and $2-1$ would be in C. Then the number 4 must go into C, otherwise $2,4$ and $4-2$ would be in D. So far, $\{1,4\} \subset C$, $\{2\} \subset D$. The number 3 cannot go in C, since then, $4,3$ and $4-3$ would be in C. So $\{1,4\} \subset C$, $\{2,3\} \subset D$. No matter where you put 5, the conclusion follows.

2. Is the product of any n consecutive natural numbers divisible by $n!$ (with remainder 0)? Prove or give a counterexample.

Solution: Yes $(k+1)(k+2)\ldots(k+n) = \binom{n+k}{k} n!$, where $\binom{n+k}{k}$ is the binomial coefficient, which is always a natural number for n and k natural numbers.

3. Given an equilateral triangle ABC inscribed in a circle, let P be an arbitrary point on the arc AC . Show that $\overline{PA} + \overline{PC} = \overline{PB}$.



Solution: ABC is equilateral and P is an arbitrary point on AC . Prove $\overline{PA} + \overline{PC} = \overline{PB}$. Choose a point X on $\overline{PB}$ such that $|\overline{PX}| = |\overline{PC}|$. We will show that ΔCXB is congruent to ΔCPA , so that $|\overline{AP}| = |\overline{BX}|$ which in turn implies $|\overline{PB}| = |\overline{PX}| + |\overline{BX}| = |\overline{PC}| + |\overline{PA}|$. First $m(<BAC) = m(<BPC) = 60°$. Since ΔPXC is isosceles, $m(<PXC) = m(<XCP) = 60°$ and thus $|\overline{XC}| = |\overline{PC}|$. However $|\overline{BC}| = |\overline{AC}|$. Also $m(<XCA) = 60 - m(<ACP)$ so that $m(<BCP) = 60 - m(<XCA) = m(<ACP)$. Thus ΔACP is congruent to ΔBCX , and $|\overline{PB}| = |\overline{PC}| + |\overline{PA}|$.

4. Prove that $\sum\limits_{k=2}^{n} \frac{1}{k^2-1} = \frac{(3n+2)(n-1)}{4n(n+1)}$ for $n \geq 2$.

Solution: The proposition can be shown by induction. If $n = 2$, $\frac{1}{2^2-1} = \frac{1}{3}$ and $\frac{(3\cdot 2+2)(2-1)}{4\ 2(2+1)} = \frac{1}{3}$. Suppose now that $\sum\limits_{k=2}^{n} \frac{1}{k^2-1} = \frac{(3n+2)(n-1)}{4n(n+1)}$ where n is an arbitrary integer ≥ 2 . Then

$$\sum_{k=2}^{n+1} \frac{1}{k^2-1} = \sum_{k=2}^{n} \frac{1}{k^2-1} + \frac{1}{(n+1)^2-1} = \frac{(3n+2)(n-1)}{4n(n+1)} + \frac{1}{(n+2)n}$$

$$= \frac{1}{n} \cdot \frac{(3n+2)(n-1)(n+2) + 4(n+1)}{4(n+1)(n+2)}$$

$$= \frac{1}{n} \cdot \frac{(3n+5)\ n^2}{4(n+1)(n+2)} = \frac{(3(n+1)+2)((n+1)-1)}{4(n+1)((n+1)+1)}$$

which proves the formula.

Alternate Proof:

$$\sum_{k=2}^{n} \frac{1}{k^2-1} = \sum_{k=2}^{n} \left(\frac{1/2}{k-1} - \frac{1/2}{k+1}\right)$$

$$= \frac{1}{2}\left(\sum_{k=2}^{n} \left(\frac{1}{k-1}\right) - \sum_{k=2}^{n} \left(\frac{1}{k+1}\right)\right) = \frac{1}{2}\left(\sum_{k=2}^{n} \left(\frac{1}{k-1}\right) - \sum_{k=4}^{n+2} \left(\frac{1}{k-1}\right)\right)$$

$$= \frac{1}{2}\left(\frac{1}{2-1} + \frac{1}{3-1} - \frac{1}{(n+1)-1} - \frac{1}{(n+2)-1}\right) = \frac{(3n+2)(n-1)}{4n(n+1)} \quad .$$

5. Which is bigger, $(\sqrt{5})^{\sqrt{3}}$, $(\sqrt{3})^{\sqrt{5}}$? Give a complete proof.

Solution: If we raise $(\sqrt{5})^{\sqrt{3}}$ to the $2\sqrt{3}$ power we get

$$[(\sqrt{5})^{\sqrt{3}}]^{2\sqrt{3}} = (\sqrt{5})^6 = 5^3 = 125 \quad .$$

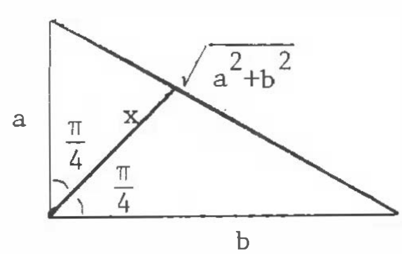
If we raise $(\sqrt{3})^{\sqrt{5}}$ to the $2\sqrt{3}$ power we get

$$[(\sqrt{3})^{\sqrt{5}}]^{2\sqrt{3}} = 3^{\sqrt{15}} < 3^{\sqrt{16}} = 3^4 = 81 \quad .$$

$$\therefore \quad (\sqrt{5})^{\sqrt{3}} > (\sqrt{3})^{\sqrt{5}}$$

6. A right triangle has legs of length a,b . Prove that the bisector of the right angle has length $\dfrac{ab\sqrt{2}}{a+b}$.

Solution:



$$\sqrt{a^2+b^2} = \{x^2 + a^2 - 2ax \cos \tfrac{\pi}{4}\}^{1/2} + \{x^2 + b^2 - 2bx \cos \tfrac{\pi}{4}\}^{1/2}$$

(law of cosines twice)

$$a^2 + b^2 = 2x^2 + a^2 + b^2 - \sqrt{2}\, x(a+b)$$

$$+ 2\{x^2 + a^2 - \sqrt{2}\, ax\}^{1/2} \{x^2 + b^2 - \sqrt{2}\, bx\}^{1/2} - 2x^2 + \sqrt{2}\, x(a+b)$$

$$= 2\{x^2 + a^2 - \sqrt{2}\, ax\}^{1/2} \{x^2 + b^2 - \sqrt{2}\, bx\}^{1/2}$$

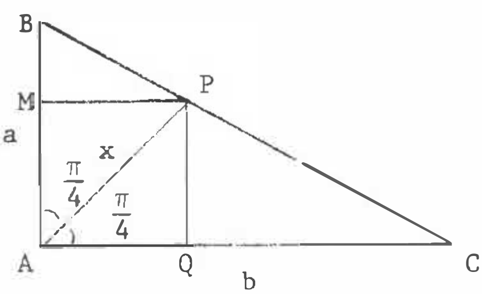
$$4x^4 - 4\sqrt{2}\, x^3(a+b) + 2x^2(a+b)^2 = 4\{x^2 + a^2 - \sqrt{2}\, ax\}\{x^2 + b^2 - \sqrt{2}\, bx\}$$

$$0 = 2x^2(a^2+b^2) + 4a^2b^2 - 4\sqrt{2}\, a^2bx - 4\sqrt{2}\, b^2\, ax + 4ab\, x^2$$

$$0 = (a+b)^2x^2 - 2\sqrt{2}\, ab(a+b)x + 2a^2b^2$$

$$x = \frac{2\sqrt{2}\, ab(a+b) \pm \{8a^2b^2(a+b)^2 - 8a^2b^2(a+b)^2\}^{1/2}}{2(a+b)^2} = \frac{\sqrt{2}\, ab}{a+b} .$$

Alternate Proof:



Draw line MP parallel to AC .

Draw line PQ parallel to BA .

Triangle MBP is similar to triangle QPC (all angles equal). Therefore,

$$\frac{\overline{MP}}{\overline{MB}} = \frac{QC}{QP} \quad , \quad \frac{x \sin \frac{\pi}{4}}{a - x \cos \frac{\pi}{4}} = \frac{b - x \cos \frac{\pi}{4}}{x \sin \frac{\pi}{4}}$$

$$\frac{x}{a - \frac{x}{\sqrt{2}}} = \frac{b - \frac{x}{\sqrt{2}}}{\frac{x}{\sqrt{2}}} \quad , \quad \frac{x^2}{2} = \left(b - \frac{x}{\sqrt{2}}\right)\left(a - \frac{x}{\sqrt{2}}\right)$$

$$0 = ba - \frac{(a+b)}{\sqrt{2}} x \quad , \quad x = \frac{\sqrt{2}\, ab}{a+b} \ .$$

7. At a party some people shake hands and some do not. Show that there must exist at least two people who shake hands with the same number of people.

Solution: If there are n people who shake hands, then the maximum number of possible handshakes for anyone is $n-1$ and the minimum number is 1 (since one does not shake hands with oneself). It follows therefore that there are only $n-1$ choices for the number of possible handshakes for anyone and hence two people (at least) must shake hands the same number of times.

8. Let $n-1$, n , $n+1$ be three consecutive natural numbers. Prove that the cube of the largest cannot equal the sum of the cubes of the other two.

Solution: Suppose, on the contrary, that we have $(n-1)^3 + n^3 = (n+1)^3$ for some natural number n . Then simplifying we get the equation $n^3 - 6n^2 - 2 = 0$, which implies $n^2(n-6) = 2$. Therefore the left hand side must be positive so $n > 6$ but also n^2 must divide 2, a contradiction. This proves the result.

9. How many natural numbers of n digits exist such that each digit is 1, 2 or 3 ? How many of these numbers use all three of the digits 1, 2 and 3 ?

Solution: 3^n .

$3^n - K$, where K is the number of such numbers that lack at least one of 1, 2, 3 . $K = M + N$

where M = numbers using only a single digit from $\{1,2,3\}$,

N = numbers using exactly two digits from $\{1,2,3\}$.

$M = 3$. $N = \binom{3}{2} \{2^n-2\}$. The $\binom{3}{2}$ represents choosing the two numbers to be used, the 2^n-2 represents all possible numbers of n digits formed from the two, less the pair of numbers that use exactly one digit. Thus the desired number is $3^n - 3 - \binom{3}{2} [2^n-2]$.

10. Let $x_1, x_2, \ldots, x_n$ be positive angles, with $\sum_{j=1}^{n} x_j \leq \frac{\pi}{2}$ (in degrees, $\sum_{j=1}^{n} x_j \leq 90°$). Show that

$$\sin(x_1+x_2+\ldots+x_n) \leq \sin x_1 + \sin x_2 + \ldots + \sin x_n .$$

Solution: The proof will be by induction on n . It is clear for $n = 1$. Suppose $\sin(x_1 + \ldots + x_n) \leq \sin x_1 + \ldots + \sin x_n$ for any positive angles $x_1, \ldots, x_n$ satisfying $\sum_{i=1}^{n} x_i \leq \frac{\pi}{2}$. If $x_1, \ldots, x_{n+1}$ are positive angles with $\sum_{i=1}^{n+1} x_i \leq \frac{\pi}{2}$ then $\sin((x_1 + \ldots + x_n) + x_{n+1}) = \sin(x_1 + \ldots + x_n) \cdot \cos x_{n+1} + \sin(x_{n+1}) \cos(x_1 + \ldots + x_n)$. However $0 < \cos\alpha < 1$ for $0 < \alpha < \frac{\pi}{2}$ so that $\sin(x_1 + \ldots + x_{n+1}) \leq \sin(x_1 + \ldots + x_n) + \sin x_{n+1} \leq \sin x_1 + \ldots + \sin x_n + \sin x_{n+1}$.

The Metric System: Effects on Teaching Mathematics

Eugene P. Smith
NCTM President

This article is reprinted, with permission of the author, from *Newsletter, The National Council of Teachers of Mathematics,* March 1974.

The move to the metric system is well under way, and many of you are already collecting aids and developing ideas for teaching the metric system. These are essential tasks, but have you also considered the implications of teaching mathematics *outside* the system of measurement?

Of the seven basic units in the International System, the meter and the kilogram - and the measures derived from them - will receive the greatest attention in teaching mathematics. Since subunits and multiples of these two units are related by powers of ten, the learner must understand the decimal numeration system, including decimal fractions, if he is to change from one related unit to another - for example, from centimeters to meters. The teacher must not only provide readiness experiences for learning the metric system but also consider whether children of differing abilities can understand decimal fractions well enough to change from one unit to another. Also, the teacher must help youngsters understand the new vocabulary.

Most of our needs for fractions arise from using the English system of measurement. After the metric system is in widespread use, what need will we have for fractions in daily life? If would seem that skill work with common fractions could be virtually eliminated. But for the algebra student, how much readiness for manipulating algebraic fractions is provided by the work with fractions in arithmetic? If such work does provide readiness, could (or *should*) the skill development be delayed until the student is ready for algebra? Can all common fractions be eliminated? This is not likely, for we shall still sometimes need them for describing ratios and parts of a whole.

And how about the stock exchanges? Will they convert to a decimal system for stock quotations instead of the more difficult multiples of 1/8? I see no reason why they could not even use multiples of 10¢ instead of the decimal equivalents of eighths.

Recently a home economist raised this problem: "Should measuring cups go metric? If so, should they be graduated in milliliters or in decimal fractions of a liter?" She argued that milliliters should be used, since whole numbers are more easily understood than decimal fractions. Should cookbooks of the future use metric measures rather than cups?

The problems of the stock market and the cookbook are typical of the wide-ranging questions that must be answered before we can make curriculum decisions about teaching measurement. In order to help eliminate the unnecessary use of common fractions, should we, as mathematics teachers, make proposals to the stock exchanges, the home economists, and other groups?

There is no longer any doubt *whether* Canada will move to the metric system—the only question is *when*. It is time for us to look, not merely at teaching the metric system, but at the effects this move will have on teaching *all* mathematics.

Stretch Your Sketching Skill

Wayne J.P. Turley
Western Canada High School
Calgary

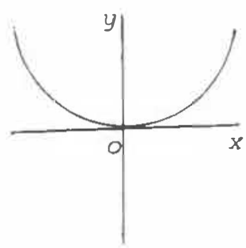
A number of elementary techniques are available for sketching graphs but, to our knowledge, they have not appeared in any definitive fashion in textbooks. This presentation is an attempt to ameliorate that situation.

The rules of thumb which follow are non-rigorous suggestions that usually assist the student in determining the nature of the graph.

1. DEGREE: The degree of the equation suggests the number of "bends" the resulting curve has. The degree of the equation is determined by the highest power of the independent variable (i.e., x). Therefore, in $y=x^2$ (degree two) there is one bend and in $y=x^3+3x$ (degree three) there are two bends, but in $y=2x - 3$ (first degree) there are no bends. It appears that the number of twists in the curve is one less than the highest power of the independent variable, that is, if $f(x)=ax^n + bx^{n-1} + ex^{n-2} + \ldots + wx$, then we should expect that there should be n-1 bends. Of course, if f(x) equals x to the three quarters, we cannot use the rule, so perhaps we should restrict the domain of n to the set of positive integers.

2. NUMERICAL COEFFICIENTS: The numerical coefficient of the highest power of the independent variable also helps determine the complexion of the graph. In the quadratic function we have a "smile" in the sketch of

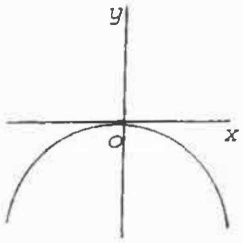
$y = x^2$



positive coefficient
concave upwards

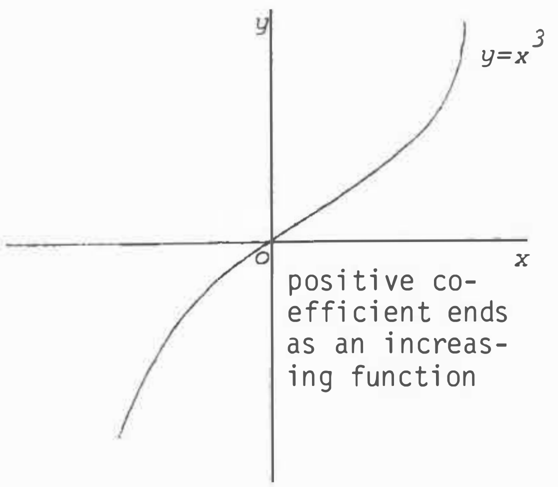
or a "frown" in

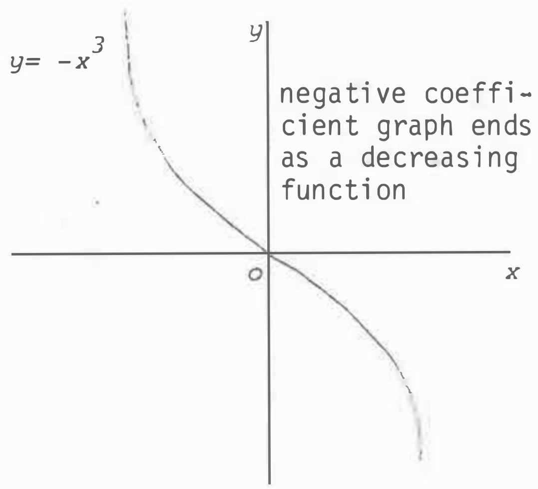
$y = -x^2$

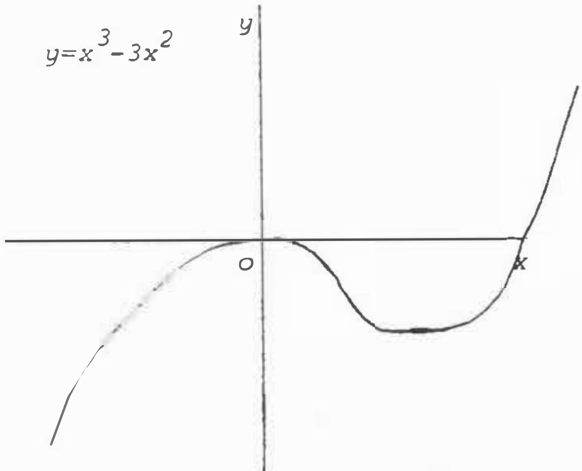


negative coefficient
concave downwards

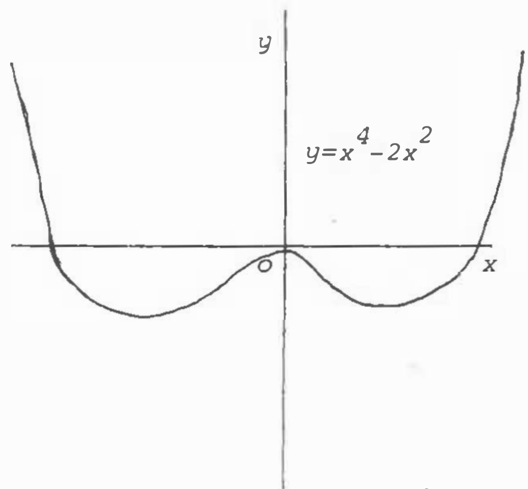
The use of 'smile' or 'frown' to indicate concave upwards and concave downwards is a useful memory device for the beginning student of the quadratic. But wait, there is more! When the coefficient of x is positive the graph ending is always increasing. We note that in $y = x^2$ the graph ends as an increasing function but in $y = -x^2$ the function ends as a decreasing function. The following examples are illustrative of this generalization.







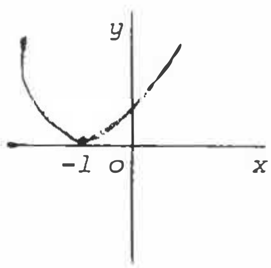
positive coefficient of x^3 means graph ends as increasing function



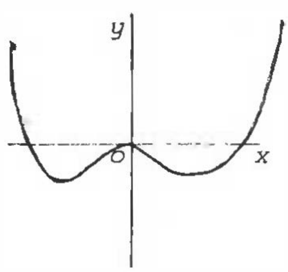
positive coefficient of x^4 means graph ends as increasing function

It is interesting and perhaps instructive to note also that, in $y=x^3$ we have an initially increasing graph, while in $y = -x^3$ we have an initially decreasing graph. The odd power and the positive coefficient combine to produce a graph whose genesis is always increasing, while the odd power and the negative coefficient produce an initially decreasing graph. By extrapolation, the even power and the positive coefficient initiate a decreasing graph while the increasing mode is born of an even power and a negative coefficient.

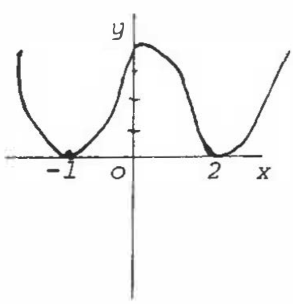
3. SQUARED FACTOR: If f(x) can be expressed as a factor squared, that is, if $f(x) = (x-a)^2$, the graph will be tangent to the x-axis at x=a. Consider the following examples.



$y=(x+1)^2$ the graph touches the x-axis at x= -1



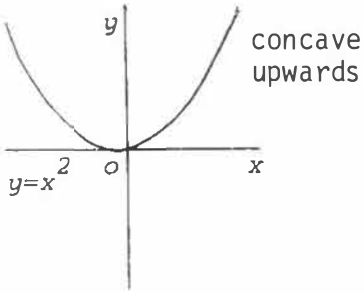
$y=x^4 -2x^2 = x^2(x - 2)$
since x is squared,
at x=0 the graph touches the x-axis

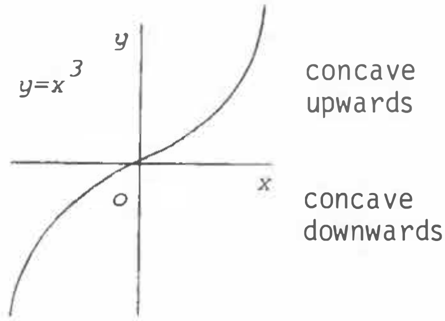


$y=(x-2)^2 \ (x+1)^2$
graph is tangent to x-axis
at x=2
x= -1

4. INFLECTION POINTS: In order to properly sketch a function, it is essential to know the inflection points. An inflection point is a point on the graph

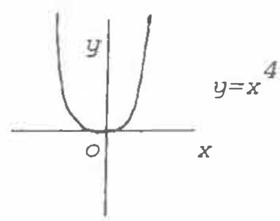
at which the concavity of the graph changes. Since $y=x^2$ is concave upward for $x \in R$ but $y=x^3$ has one inflection point at $x=0$,





seemingly the number of inflection points in $f(x) = ax^n + bx^{n-1} + cx^{n-3} +...+ wx$ would be $(n - 2)$ (two less than the highest power of the independent variable). If this is so, then let us consider $f(x) = x^4$. We would predict that it should have two inflection points. Before we propose a solution to this problem, let us consider an analogous but simpler problem, vis., the evaluation of the x-intercepts. To find these we solve $x^4 = 0$. Gauss showed that there are four roots to this equation, albeit they are all the same but there are still FOUR roots. This means that there are four x-intercepts all at the same point $x = 0$. Hence, when we solve $f''(x) = 0$ to find the inflection points we get $f'(x) = 4x^3$ and $f''(x) = 12x^2$. Therefore, if $f''(x) = 0$

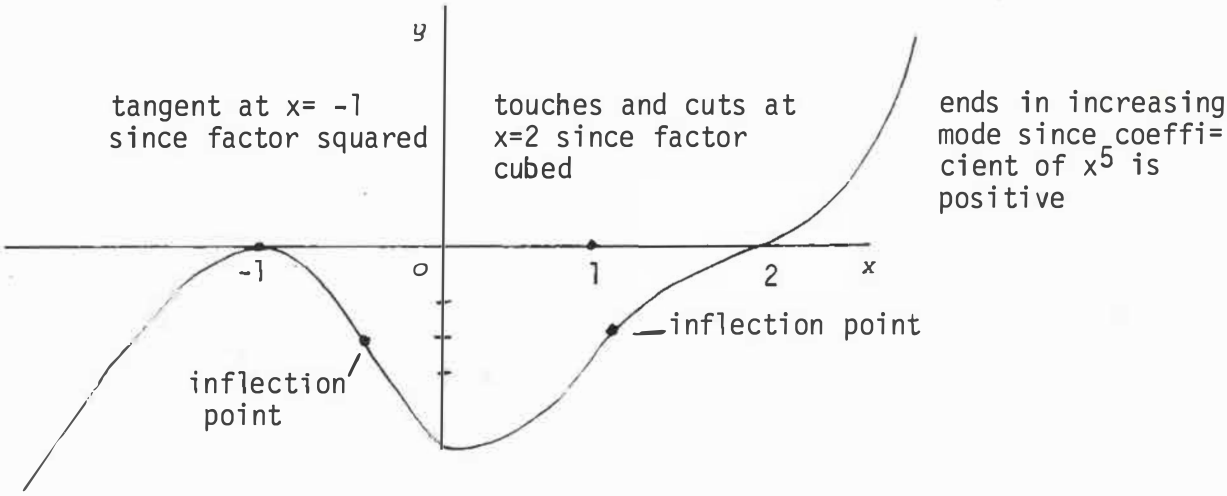
$$12x^2 = 0$$
$$\text{and } x^2 = 0$$
$$\text{so } x = 0 \text{ or } x = 0$$



Hence we have two inflection points at the origin. The inflection points are coincidental just as the roots are. This explanation for the phenomenon displayed by $f(x) = x^4$ is intuitively more satisfying to students than the explanation that this function just breaks the rules thus established. Further, this explanation offers a consistent theoretical position not heretofore deductible.

5. CUBED FACTOR: If $f(x)$ can be expressed as a factor cubed, that is, if $f(x) = (x-a)^3$, the graph will be tangent to the x-axis at $x=a$ AND the graph will cut the x-axis at $x=a$. For example, consider the curve of $y=x^3$. Here we have a touch and a cut at $x=0$.

In summary we should like to consider the complex function, $f(x) = (x - 2)^3 (x + 1)^2$ and observe how these generalizations are useful in determining its graph.



In graphing this function we can make use of more advanced techniques of calculus to determine the critical values of x. Setting the differential coefficient equal to zero, that is $f'(x)=0$ gives us $2(x-2)^3\,(x-1) + 3(x-1)^2\,(x-2)^2 = 0$, which yields $x = 2$ or $x = -1$ or $x = 1/5$, and these are the x values for which the slope of the tangent line is zero. This gives us a relative maximum at $x=-1$, a relative minimum at $x=1/5$ but neither of these at $x=2$. The geometrical interpretation of the function at $x=2$ is that the function is horizontal at $x=2$. This means that, if we were to describe the x values for which the function were increasing, they would be $\{x/\ x<-1,\ x > 1/5,\ x \neq 2,\ x \in R\}$ since at $x=2$ the function is neither increasing nor decreasing.

degree	number of bends	number of inf.pts if $n\geq 2$	POSITIVE COEFFICIENT begins	ends	NEGATIVE COEFFICIENT begins	ends
1	0					
2	1	0				
3	2	1				
4	3	2				
5	4	3				
⋮	⋮	⋮				

THE FOLLOWING REPRINTS ARE AVAILABLE TO YOU FOR REPRODUCTION

From the *Arithmetic Teacher,* January 1974

IDEAS

Activities that contribute to the student's personal understanding of key concepts in mathematics.

Prepared by **George Immerzeel** *and* **Don Wiederanders,** *Malcolm Price Laboratory School, University of Northern Iowa, Cedar Falls, Iowa.*

Each IDEAS presents activities that are appropriate for use with students at the various levels in the elementary school. After you have chosen the activities that are most appropriate for your students, remove the activity sheets and reproduce the copies you need. After a sheet has been used, add your own comments and file the materials for future use.

IDEAS for this month utilizes a paper geoboard to provide a variety of experiences with basic geometric concepts. The paper geoboard provides an excellent model for focusing on the relation between the end points and the line segments that form polygons. Any geoboard sequence can be taught using "paper and pencil" geoboards.

IDEAS

For Teachers

Objective: Experience in drawing on a paper geoboard to develop the non-metric concept of congruence.

Levels: 1, 2, or 3

Directions for teachers:

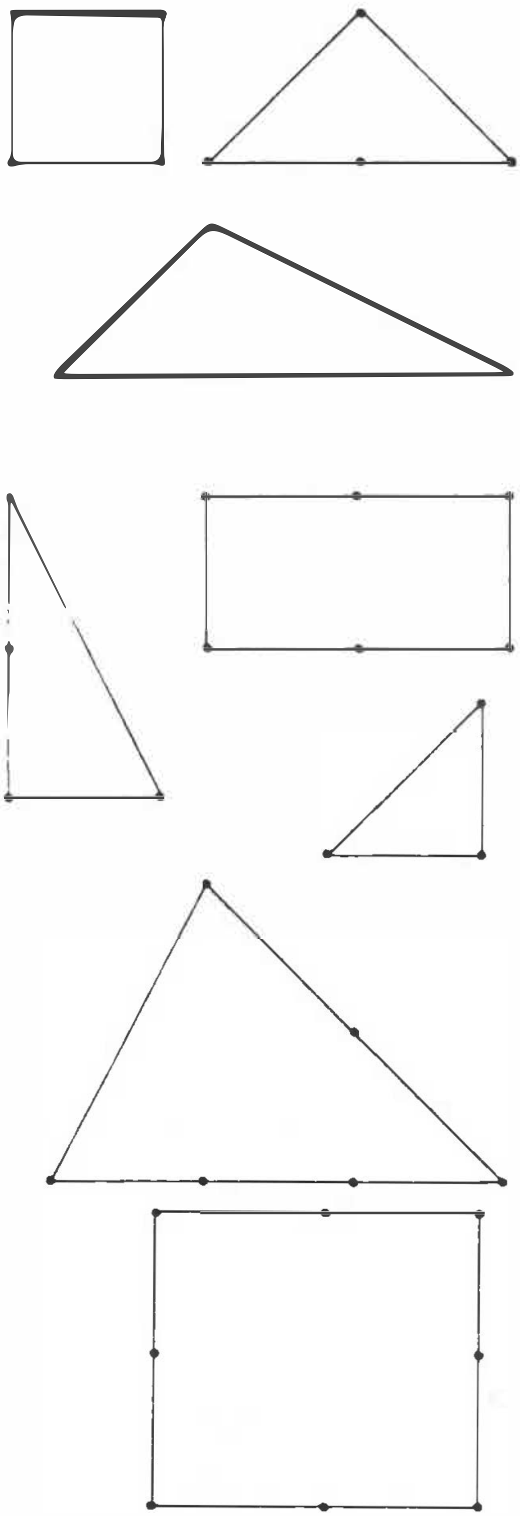
1. Remove the activity sheet and reproduce a copy for each student.
2. Give each student a straight edge but do not require that he use it.
3. Be sure that the student understands that each polygon is to be drawn on the geoboard to the right.

Comments: Other paper geoboards can be used to have the students respond to such directions as: "Draw a '3-point' triangle." "Draw the largest '3-point' triangle you can." "Draw a '4-point' square with a point inside." [•] and questions such as: "How many squares can you draw with corner points on the geoboard?" (10)

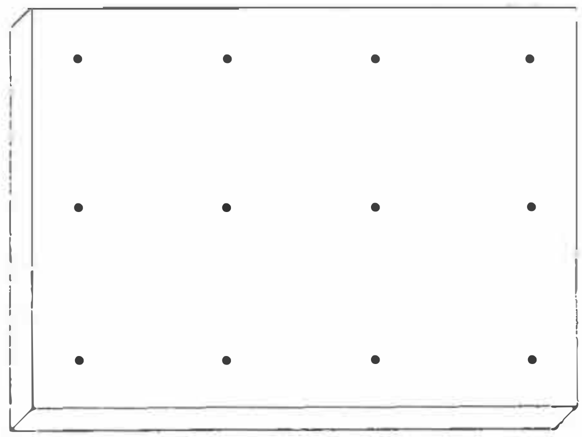
IDEAS

Name ____________

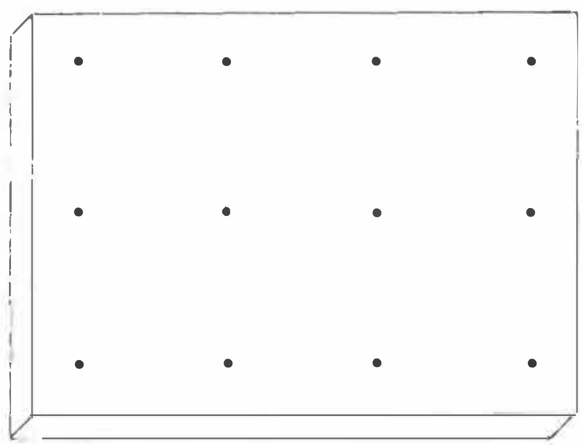
Draw these figures on the geoboards.



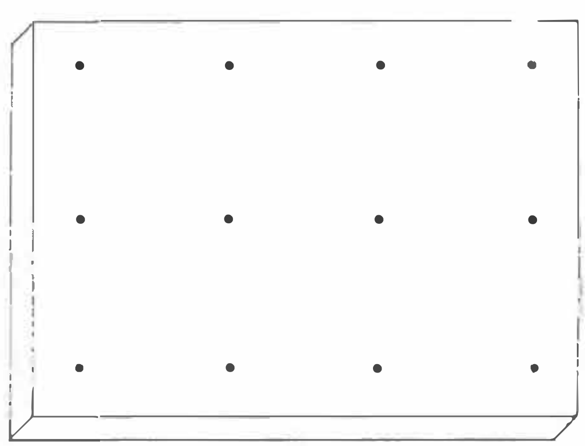
Geoboards



A



B



C

IDEAS

For Teachers

Objective: Experiences with basic geometric shapes and the standard technique for naming the shapes.

Levels: 3, 4, or 5

Directions for teachers:

1. Remove the activity sheet and reproduce a copy for each student.
2. Encourage students to visualize the triangles without actually drawing them. (Expect that some students will have to draw the triangles before they can see them.)
3. There are many 3-letter words on geoboard A that name the corners of a triangle. Some students will list more than the number of blanks shown.
4. The letters naming the corners of a square must be in clockwise or counterclockwise order. (MOLE is a word but the square would not be properly named with the letters in that order.)
5. The student who knows that all squares are rectangles may fill the last blanks with names for squares.

Comments: You may wish to hand out 4-by-4 arrays of dots challenging the students to place letters on them to form squares, rectangles, or other polygons.

Key:

A: ROD, CAN, NOD, DUG, DOG

B: SICK, HUMP, FLAY, DAUB

IDEAS

Name ____________________

Ring those words that name the corners of a triangle :

ROD	RED
RAN	DUG
CAN	YOU
NOD	DOG

A

R	E	D	U
A	C	I	K
N	O	W	S
Y	B	F	G

List other words that name corners of a triangle.

______ ______ ______ ______ ______ ______ ______

Ring those words that name the corners of a square.

SICK	FOAM
HUMP	FLAY
SULK	HACK
MODE	DAUB

B

S	H	F	K
Y	B	E	P
U	O	D	L
I	A	M	C

List words that name corners of a rectangle.

____________________ ____________________

IDEAS

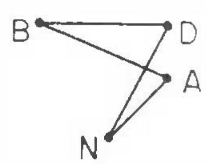
For Teachers

Objectives: Experience in visualizing special polygons

Levels: 5, 6, 7

Directions for teachers:

1. Remove the activity sheet and reproduce a copy for each student
2. Encourage students to visualize the polygons without actually drawing them.
3. Annouce that a key will be posted at a specific time.
4. Polygons are lettered clockwise or counterclockwise. (Normally BAND is not considered a polygon.)



Comments: Hand out 4-by-4 dot arrays that do not have letters. Challenge your language oriented students to make up similar sets of questions.

Key:

A: 1. AXE 2. DOG 3. JOHN 4. CORN
5. RAKE 6. CAKE

B: 1. BAT 2. MAT 3. SIT 4. BIT
5. MAST 6. BOAST

IDEAS

Name ____________

Find a polygon for each condition:

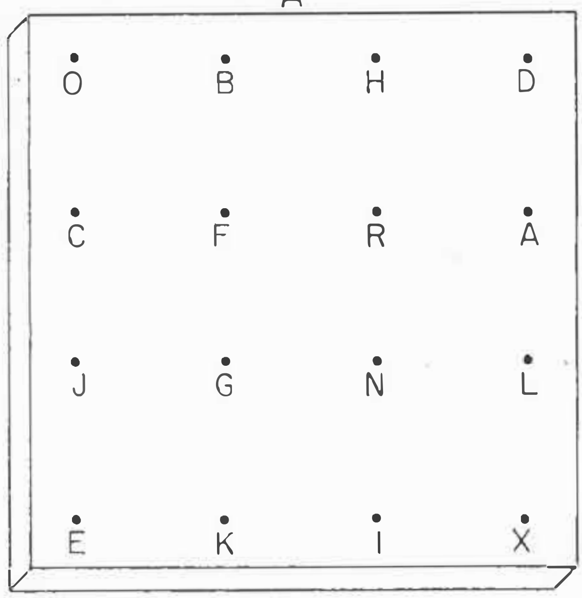
1. A right triangle that chops wood. ___ ___ ___

2. A triangle that names man's best friend. ___ ___ ___

3. A square that names a man. J___ ___ ___

4. A parallelogram that grows on ears. ___ ___ ___ ___

A



5. A parallelogram that is used to gather leaves. ___ ___ ___ ___

6. A trapezoid that usually has frosting. ___ ___ ___ ___

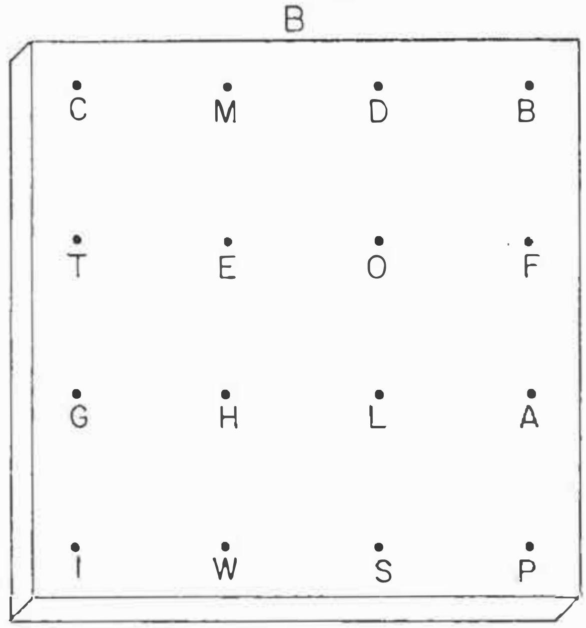
Find a polygon for each condition:

1. An isosceles triangle that flies at night. ________

2. A right triangle to wipe your shoes on. ________

3. An isosceles right triangle that is done on a chair. ________

B



4. An obtuse triangle that is placed in a horse's mouth. ________

5. A rectangle that holds a sail. ________

6. A concave polygon that means: to brag. ________

IDEAS

For Teachers

Objectives : Experience in visualizing and drawing composite polygons that requires a concept of congruence.

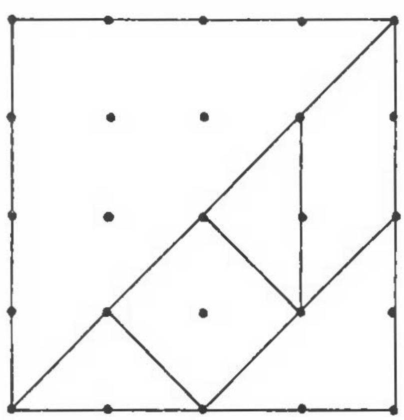
Levels : 6, 7, or 8

Directions for teachers :

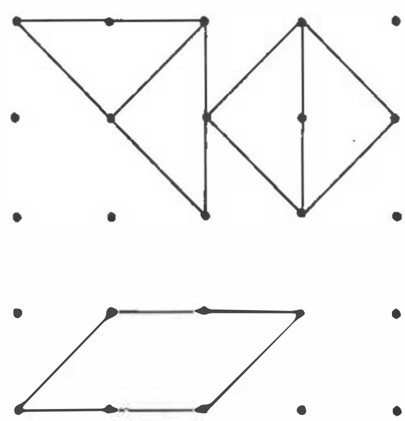
1. Remove the activity sheet and reproduce a copy for each student.
2. Be sure each student has a straight edge but do not require that he use it.
3. Present this activity as a challenge. Don't expect a high level of success.
4. Announce that all correct solutions will be posted at the end of one week.

Comments : These experiences are similar to but considerably more challenging than their counterpart using tangrams. You may wish to extend this idea into the study of different shapes with the same area.

Key :



A.



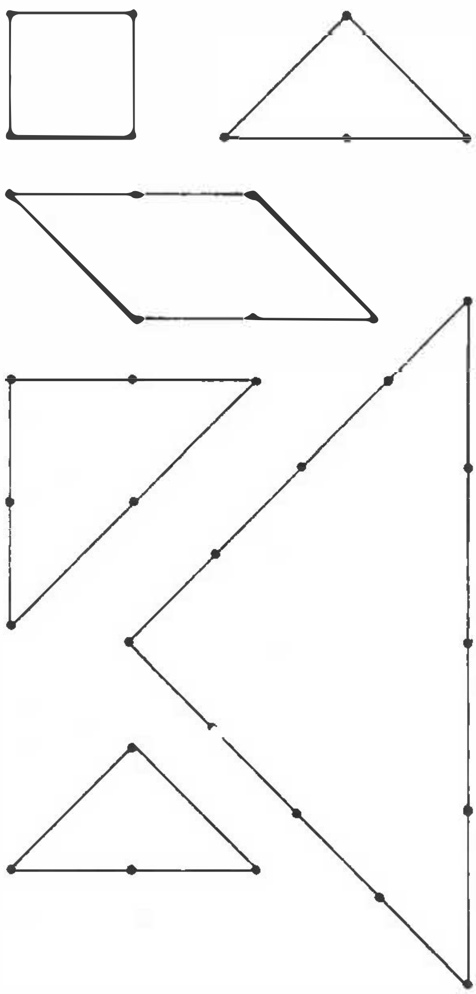
B.

IDEAS

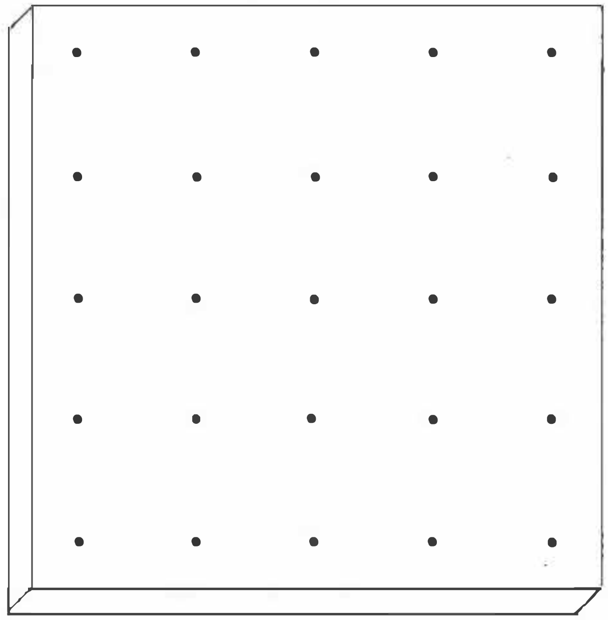
Name ______________________

On geoboard A :

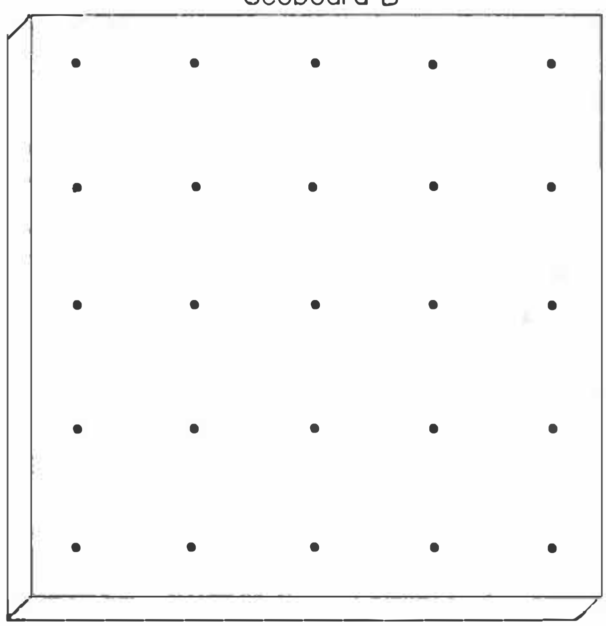
Draw all the polygons.
Use all the dots on the geoboard.



Geoboard A



Geoboard B



On geoboard B:

a) Draw one of the above polygons twice to form a right triangle.
b) Draw one of the polygons twice to form a square.
c) Draw one of the polygons twice to form a parallelogram.

From the *Mathematics Teacher*, January 1974

ACTIVITIES

Edited by
L. CAREY BOLSTER
Baltimore County Public Schools
Towson, Maryland
and
EVAN M. MALETSKY
Montclair State College
Upper Montclair, New Jersey

DISCOVERY WITH CUBES

By Robert E. Reys, University of Missouri—Columbia Columbia, Missouri

Teacher's Guide

Grade level: 6–12

Materials: Student worksheets

Objectives: Students will visualize three-dimensional figures, construct a table, discover patterns in the table, and use patterns to make predictions.

Directions: Make copies of the tear-out pages for students. Divide the class into groups of two each, and let them work together to solve this exercise. It would be quite helpful if the teacher had a set of cubes that were colored as stated in activities 1–3. In this way, students could verify their results.

After completing activity 4, students should record their results in the table (activity 6) found on worksheet 3. Check the table with the class to insure that all students have the correct values, since predictions will be made on the basis of their data. Students should then sketch or construct a 6 × 6 cube as indicated in activity 5 and add its data to the table.

Few students will be able to complete the table for a 10 × 10 × 10 cube unless some patterns have been identified. Ask, "Are there any constants in a column? Any multiples?" Encouraging pupils to keep track of the factors used in the table aids pattern recognition. For example, 0, 6, 24, 54, and 96 are the first five values for one of the columns. A pattern is more discernible when these values are written as 0, 6 × 1, 6 × 4, 6 × 9, and 6 × 16.

Here is a question that might be used to culminate this activity: "Let the length of one side of the cube be N. When you complete this row of the table, is the sum of these values N^3?"

Solutions:

Activity 1: a. 8; b. 8; c. 0; d. 0; e. 0; f. 8; g. equal
Activity 2: a. 27; b. 8; c. 12; d. 6; e. 1; f. 27; g. equal
Activity 3: a. 64; b. 8; c. 24; d. 24; e. 8; f. 64; g. equal
Activity 4: a. 125; b. 8; c. 36; d. 54; e. 27; f. 125; g. equal

SHEET 1 Name________________

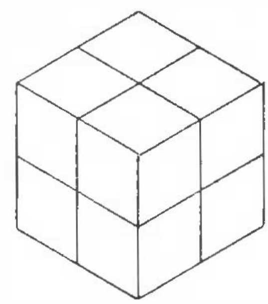
DISCOVERY WITH CUBES

Small cubes have been stacked and glued together to form these larger cubes.

Activity 1

a. How many small cubes are in the large cube? _____

If this large cube is dropped into a bucket of paint and completely submerged:



b. How many of the smaller cubes are painted on three sides? _____

c. How many on only two sides? _____

d. How many on only one side? _____

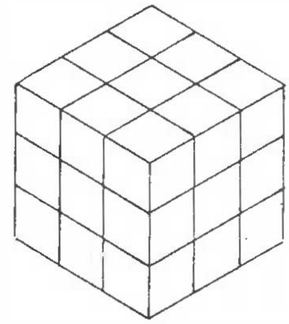
e. How many on zero sides? _____

f. What is the sum of your answers in b, c, d, and e? _____

g. How does your answer to f compare to a? _____

Activity 2

a. How many small cubes are in the large cube? _____



b. How many of the smaller cubes are painted on three sides? _____

c. How many on only two sides? _____

d. How many on only one side? _____

e. How many on zero sides? _____

f. What is the sum of your answers in b, c, d, and e? _____

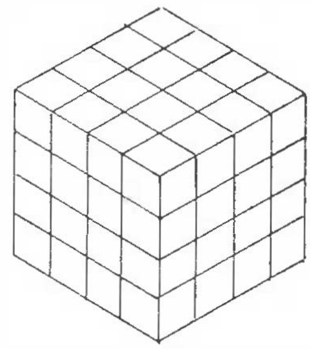
g. How does your answer to f compare to a? _____

SHEET 2

Activity 3

a. How many small cubes are in this large cube?_____

If this large cube is dropped into a bucket of paint and completely submerged:



b. How many of the smaller cubes are painted on three sides? _____

c. How many on only two sides?_____

d. How many on only one side?_____

e. How many on zero sides?_____

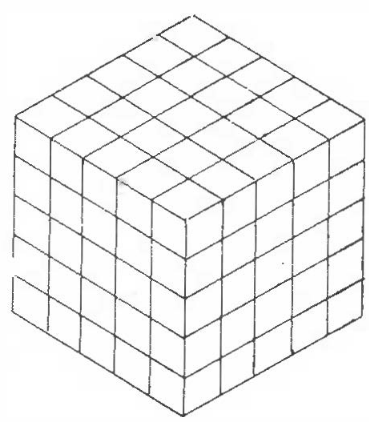
f. What is the sum of your answers in b, c, d, and e?_____

g. How does your answer to f compare to a?_____

Activity 4

a. How many small cubes make up this large cube?______

If this large cube is dropped into a bucket of paint and completely submerged:



b. How many of the smaller cubes are painted on three sides?_____

c. How many on only two sides?_____

d. How many on only one side?_____

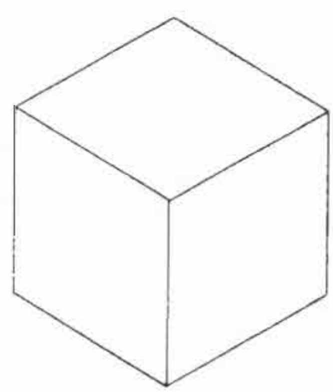
e. How many on zero sides?_____

f. What is the sum of your answers in b, c, d, and e?_____

g. How does your answer to f compare to a? _____

Activity 5

Suppose your cube was 6x6x6. Complete this model by sketching a 6x6x6 cube. Use it to determine the total number of cubes as well as the number of faces with zero, one, two, three and four sides painted.



SHEET 3

Activity 6

Now that you have solved several problems with the cubes, record this information in the Table:

Length of Side of Cube	Number of Painted Sides 0	1	2	3	4	Total Number of Cubes
2						
3						
4						
5						
6						

Do you observe any patterns? __________ If so, complete the Table for a 7x7x7 cube. If not, sketch or construct a cube and then complete the Table.

Have you really got the idea? If you think so, try to complete the Table for a 10x10x10 cube.

● ●

We will be pleased to have any teacher submit material for reproduction. Your reward will be credit for articles used, and colleagues will benefit from sharing your ideas and experiences.

Two publications of *The National Council of Teachers of Mathematics*—the *Arithmetic Teacher* and the *Mathematics Teacher*—are available, by subscription, to its members. Issues of each contain similar inserts. NCTM membership application forms will be obtainable at the MCATA annual conference, as well as through your *Delta-K* editor and other members of the executive.

THE ALBERTA
TEACHERS'
ASSOCIATION
MATHEMATICS
COUNCIL

delta-k

Supplement to June 1974

MCATA ANNOUNCEMENTS

Metric Missionary Workshop

1. A four-hour program will include activities and ideas for teaching the use of devices for measuring length, weight, volume, and temperature. These devices will include tapes, meter sticks, rulers, scales, thermometers, and various containers for capacity and volume measurement.

2. Tentative subject areas are length, area, temperature, weight (using balances and scales), capacity, volume of solids, displacement method of measuring volume, everyday measuring devices, and enrichment ideas.

3. Contact personnel for detailed program outline and to reserve a date in September, October or November, are -

- in northern Alberta, Dr. Joan Kirkpatrick, Department of Elementary Education, University of Alberta, Edmonton,
- in central Alberta, Brian Chapman, Box 1525, Lacombe,
- in southern Alberta, Dr. Mary Beaton, 657 Northmount Drive, N.W. Calgary

4. Workshop fee: $5 for members
$8 for non-members.

Non-members wishing to become active members will be able to do so at the workshop. Members receive *Delta-K*, which is our newsletter, monographs on various subjects (a 1974-75 project is on metrication), and advantages in workshop, convention and other activities. So, for a total of $10, a non-member attending the Metric Missionary Workshop may become an active member.

New MCATA Project

We wish to announce receipt of a grant of $1,000 from The National Council of Teachers of Mathematics, toward development of a teacher resource center. The center will be located in the Education Building at the University of Alberta. Selection of Edmonton as the first center was made for various reasons, one of the most important being its accessibility to the greatest number of people with the least amount of travel. Our Metric Missionary program is one way we are planning to help extend to our MCATA teachers the availability of resource materials beyond the basic text. More information on the center will be released at the annual meeting.

ANNUAL MEETING MCATA
October 25-26, 1974 Jasper, Alberta

(basic information has been in an earlier *Delta-K*)

DEPARTURE - 3.30 p.m. from Chinook-Ridge Shopping Center, Greyhound Bus Depot and North Hill Shopping Center

ARRIVAL - Jasper Park Lodge at 8.00 p.m.

COST - approximately $12.50 return (includes coffee and sandwiches enroute)

FOR ADVANCE REGISTRATION - send $5 ($8 non-members), by October 12th, to

Francis Somerville
Bishop Pinkham Junior High School
3304 - 63 Avenue S.W.,
Calgary

FOR INFORMATION ON TRAVEL ARRANGEMENTS FROM EDMONTON - contact an Edmonton member of the council executive