

Integers in the Elementary School:

An Annotated Bibliography

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As children explore the world of whole numbers and then of fractions, they often discover incidentally the need for negative integers. This topic has already been enjoyed by many elementary school children who have not been forced into abstractions and definitions before they have gained an intuitive understanding of this "new" kind of number.

Often an intuitive understanding of negative integers is obtained through out-of-school experiences. Many games which children play involve "going in the hole" and it is amazing how easily children can perform the necessary operations in these games without prior instruction in negative numbers.

Educational journals contain many articles on teaching integers to elementary school children. In preparing the following annotated bibliography, many articles were rejected because of overloading with abstract notions and/or an overdependence on definitions in order to justify (usually most unsuccessfully) to children the nature of integers and the operations upon them. The following, then, is a select bibliography which teachers should find very helpful in teaching negative numbers.

- ▶ Ashlock, Robert B., and Tommie A. West. "Physical Representation for Signed-number Operations", *The Arithmetic Teacher*, 14 (October 1967), 549-554.

This survey covers both the measurement and counting representations of signed numbers, and suggests a variety of procedures for developing understanding of the operations on integers. An extensive annotated bibliography is included.

- ▶ Bates, John Hyde. "Positive and Negative Numbers", *Mathex*, Level 5, Teacher's Bulletin No. 14. Toronto: Encyclopaedia Britannica, 1967.

This practical introduction to integers through associations with rocket launching and oil wells encourages the children to record their explorations on the number line. The pupils can establish their own patterns using both a matrix and a coordinate graph.

- ▶ Calandra, Alexander. "Teaching Signed Numbers in Grade 8", *The Arithmetic Teacher*, V (November 1959), 259-260.

The teaching strategy described depends on definitions rather than understanding. However, the use of $\overset{\rightarrow}{3}$ and $\overset{\leftarrow}{3}$ to indicate, on a number line, positive three and negative three respectively, may be a helpful idea for teachers who wish to avoid initial confusion between value and direction.

- ▶ Cochran, Beryl. "Children use Signed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 587-588.

The author, a member of the Madison Project team, describes two classroom situations in which the teacher has capitalized on a child's intuitive understanding of mathematics and its logical notations to introduce the notion of negative integers.

- ▶Cohen, Louis S. "A Rationale in Working with Signed Numbers, *The Arithmetic Teacher*, 12 (November 1965), 563-567.

The "Postman Stories" model, devised by the Madison Project, uses the delivery of checks and bills to develop an understanding of integers in a most enjoyable way. Verbal questions and descriptions are followed by symbolization using the conventional positive and negative signs.

- ▶Cohen, Louis S. "A Rationale in Working with Signed Numbers Revisited", *The Arithmetic Teacher*, 13 (November 1966), 564-567.

The author continues to use the Postman model (see Cohen, 1965) to develop a convincing explanation of why "a negative times a negative is a positive".

- ▶Cotter, Stanley. "Charged Particles: a Model for Teaching Operations with Directed Numbers", *The Arithmetic Teacher*, 16 (May 1969), 349-353.

The concepts of positive and negative particles, amount of electrical charge (including the zero charge) and neutralization provide a logical, consistent and satisfying approach to integers. The four operations are dealt with. The way in which the model explains the sometimes dreaded multiplication of two negatives is particularly imaginative.

- ▶Frank, Charlotte, "Play Shuffleboard with Negative Numbers", *The Arithmetic Teacher*, 16 (May 1969), 395-397.

The game of shuffleboard, adapted for classroom play, incorporated negative integers into its rules. By recording their scores on a number line, pupils have semi-abstract experience with integers.

- ▶Fremont, Herbert. "Pipe Cleaners and Loops - Discovering How to Add and Subtract Directed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 568-572.

Loops, initially made of pipe cleaners but later drawn, allow pupils to physically represent positive and negative integers. Children use the knowledge that the combination of -1 and $+1$ is in fact zero to develop an understanding of the addition and subtraction of integers.

- ▶Hannon, Herbert. "A Device for Teaching Addition and Subtraction on Integers", *Mathematics Teacher*, 60 (December 1967), 860-861.

Although elementary school teachers may consider the reasoning behind the described device too obscure for their pupils, children would certainly enjoy exploring the resultant table for number patterns and relationships.

- ▶Havenhill, Wallace P. "Though This Be Madness", *The Arithmetic Teacher*, 16 (December 1969), 606-608.

The interpretations of the + and - signs as both

(a) direction, and

(b) reversal and non-reversal

allow the four operations, including division, to be clearly represented on the number line.

- ▶ Luth, Lois. "A Model for Arithmetic of Signed Numbers", *The Arithmetic Teacher*, 14 (March 1967), 220-222.

The feature of this teaching strategy is an enjoyable story line about Hy who lives in a house in the sky. He buys and frees balloons (+) and sandbags (-) in order to alter his altitude. The model, which can be represented on a vertical number line, accommodates all four operations of both positive and negative integers.

- ▶ Magnuson, Russell C. "Signed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 573-575.

The use of pattern is one of the practical suggestions for the introduction of the notion of negative integers. Unfortunately, the multiplication of two negatives is presented as a fait-accompli, without any justifying explanation whatsoever.

- ▶ Mauthe, Albert H. "Climb the Ladder", *The Arithmetic Teacher*, 16 (May 1969), 354-357.

A simple game to reinforce addition of integers uses a ladder marked -12 to $+12$ and a spinner marked from -4 to $+4$.

A ten-faced die appropriately marked could replace the spinner.

- ▶ Milne, Ester. "Disguised Practice for Multiplication and Addition of Directed Numbers", *The Arithmetic Teacher*, 16 (May 1969).

The familiar game of combining the numbers that come up on two spinners is adapted to include integers (and powers, if desired).

- ▶ The Nuffield Mathematics Project. *Computation and Structure 4*. London: W. & R. Chambers and John Murray, 1969.

The idea of the integers is built up in terms of ordered pairs before the number line and other applications are introduced. Thus, a sound foundation for operations on integers is laid.

- ▶ Pratt, Edna M. "A Teaching Aid for Signed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 589-591.

This simple device for making by the pupils is described in detail. The many variations suggested will certainly reinforce the traditional approaches and in addition provide enjoyment, as claimed by the author. This is not, however, a suitable activity to introduce the concept.

- ▶ Sherzer, Laurence. "Adding integers using only the Concepts of One-to-one Correspondence and Counting", *The Arithmetic Teacher*, 16 (May 1969), 360-361.

The addition of integers is seen as the pairing of negative ones with positive ones, followed by the counting of the balance. Ordered pairs of whole numbers are used to justify this procedure mathematically.