THE ALBERTA TEACHERS' ASSOCIATION MATHEMATICS COUNCIL

delta-k

Volume XIV, Number 1, October 1974

Welcome back

We welcome you back and hope that your teaching experience will be a joyful one throughout this school year. AND we want you to know that MCATA is here to help you in the field of mathematics.

This issue of *Delta-K* is going to all schools to give teachers and administrators an idea of what MCATA is doing for its members. Of course, in one issue we cannot possibly show everything that is offered - only a small sampling.

In many editions of *Delta-K* we include special activities which might be useful for reinforcement and/or enrichment in specific areas. Volume XIII, Number 4, June 1974, contains 10 pages of such activities. We will be very pleased to receive similar ideas from YOU - ideas which you have used in your classes and would like to share with your fellow teachers. We will accept any worthwhile idea for print even if it is still in rough draft. Your editor is more than willing to help you make *Delta-K* a publication for Alberta teachers from Alberta teachers.

Important Notice – *Reminder*

The Executive Committee recommends the following constitutional changes to the membership. These recommendations will be voted on at the annual meeting, October 25 and 26, in Jasper.

1. It is recommended that the Reporting clause be changed to read:

> "This council shall submit annually a written report of its activities and an audited financial statement to The Alberta Teachers' Association by October 1 of each year. The activities reported shall be for the preceding year."

2. It is recommended that the Executive Committee definition be changed to read:

> "The Executive Committee shall consist of the officers, one member from the Faculty of Education of a university in Alberta, one member from the Department of Mathematics of a university in Alberta, one member from the Department of Education, the editor of the newsletter, and six directors to be appointed by the officers from the following: editor of the monograph, the chairmen of committees, the

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presidents of regional councils, members at large, provided that each university representative be appointed for a two-year term and also that the two university representatives not be from the same university and provided that the directors be appointed to ensure that the executive committee includes at least two representatives of each of elementary, junior high and senior high school teachers."

In addition, the Executive Committee recommends a change in the fee structure as follows -

Regular membership	$6
Subscription	$6
Student membership	$3

Nominations for 1974-75 MCATA Executive

President - Dr. W. George Cathcart of the University of Alberta

Vice-President - Francis Somerville of Calgary

Secretary - Dennis Treslan of Calgary

Treasurer - Donald Hinde of Lacombe

As only one person was nominated for each office, the nominees are declared elected by proclamation and will assume office following the annual meeting.

THINK JASPER - OCTOBER 25-26, 1974

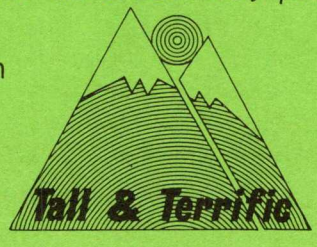
MCATA members and other mathematics teachers will gather at Jasper Park Lodge to hear speakers present ideas and challenges to help us improve our classroom presentation of materials and increase our ability to help our students to gain mathematical competency. Among the speakers will be Robert Eicholtz of Addison-Wesley (Canada) Ltd.; Ed Murin, NCTM national representative; Emery Dosdall of the Edmonton Public School Board; Glen Kauffman of Parkland Composite High School in Edson; Thomas D. Baker, retired teacher; Bill Easton of Moyers Publishing; and Dorothy Burton of Bowness High School in Calgary. Detailed information about this conference was mailed to MCATA members and was also published in the September 15 issue of *The ATA News*.

THINK DENVER - APRIL 23-26, 1975

Plan now to attend the National Council of Teachers of Mathematics annual meeting in Denver, Colorado, April 23-26, 1975.

Your Mathematics Council is arranging a group flight to Denver. The discount on air fare for a group is 30%. For more details, please contact:

Dr. George Cathcart
Department of Elementary Education
Faculty of Education
University of Alberta
Edmonton, Alberta T6G 2G5



Math Games With a Purpose

- *Mary Beaton*

One of the goals we try to achieve with our mathematics program is to make students learn to think creatively about mathematics. To achieve this goal, students should have opportunities to play mathematical games. There are a number of games available that foster critical thinking.

Commercially made games can be used to greater advantage in a school if they are stored in the materials center rather than in individual classrooms. The games can be brought to each classroom to enrich specific topics. Games increase understanding of mathematical concepts and add variety to a unit of work.

Three different games are based on the formation of equations. In order of difficulty, these games are *Heads Up*, *TUF*, and *Equations*. All of these games can be played at various levels. They all require cubes marked with numerals or operational signs and one or two timers. *Heads Up* requires the players to form equations in a stated time. The game of *TUF* can be played at many levels of difficulty according to how many and what color of cubes are used. Cubes marked with square root signs and logarithmic signs may be used to increase difficulty level. Players compete to form the equation containing the greatest number of blocks or to use all of one's blocks. *Equations* is a game involving constant decision-making. It was invented by Layman Allen, a professor of law at the University of Michigan. This game is open-ended and changes rapidly as each player takes his turn. The level of difficulty is determined by the number of cubes used, the color of the cubes, and the skill of the players. Seven-year olds can play the game with the twelve red blocks. Twelve-year olds and older children can play a more difficult game with five blocks of each color. It is advisable to begin with the simplest possible version of the game as described in the book of instructions and gradually add the more sophisticated rules. This game can be challenging to students from Grade II level to adult level.

In every elementary mathematics classroom, practice of basic facts is essential. Two commercially made games, *Numo* and *Operations Bingo*, allow for practice which is fun. *Numo* contains twenty-four different games which give practice in the facts of addition, subtraction, multiplication, division. Two games are based on equivalent fractions and two games are based on factoring.

The new feature in *Operations Bingo* is the bingo sheet. It is made of eleven columns and fourteen rows of squares. Before a game begins, the players decide whether they wish to play on a 3 x 3 square, a 4 x 4 square, or a 5 x 5 square. Each player chooses his own square of the specified size within the master sheet. A number or fact may appear on a card more than once. The rules of each game are decided ahead of time. They may be any one or a combination of: any four squares in a row; any three squares in a row; the four corners; all the numbers; or center free.

Dr. Beaton is Associate Professor, Faculty of Education, University of Calgary, Calgary, Alberta.

Science Research Associates have produced sets of *Cross Number Puzzles* to give practice in all arithmetic operations with whole numbers, fractions, decimals, and also percent. The most recent box of cross-number puzzle cards provides practice with story problems. Humor and everyday situations add to the interest of the problems.

Operational Systems Games are challenging for junior high school students. Least common multiple, greatest common denominator, sets, clock arithmetic and modular arithmetic are the mathematical ideas on which the games are based.

One of the most popular mathematical games is *Numble*. This is a game which involves practice in addition, subtraction, multiplication, division, sequencing of numbers and in finding multiples of three. It is a game of strategy, for the playing board contains double and triple number squares as well as double sequence and triple sequence squares. *Numble* is a mathematical game which is analogous to the word game *Scrabble*. It can be enjoyed by all age levels.

A set of teacher references entitled *Let's Play Games in Mathematics* is available in seven volumes. The games are cross-referenced with content topics and with student behavioral objectives. This series and another single volume entitled *Math Games for Greater Achievement* would be useful references for elementary and junior high school teachers.

If an adequate supply of mathematical games is available in the Materials Center, teachers can use games as a valuable strategy to reinforce other methods of learning mathematical concepts. A corner of the Materials Center could be set aside for the playing of math games, and students could be encouraged to drop in for a game at recess or at noon. Layman Allen found that when children in Detroit schools became interested in the game of *Equations*, the rate of truancy dropped. Let's give students a chance to have fun with a purpose.

BIBLIOGRAPHY

Allen, Layman. *Equations* WFF 'N PROOF, New Haven, Connecticut.

Ernest, Robert L. *Cross Number Puzzles*, Science Research Associates, Chicago.

Heads Up, Creative Publications, Palo Alto, California.

Henderson, George. *Math Games for Greater Achievement*, National Textbook Co., Skokie, Illinois.

_____ and Lowell D. Gunn. *Let's Play Games in Mathematics*, Volumes I-VII, National Textbook Co., Skokie, Illinois, 1971.

Numble, Selchow and Righter Co., Bay Shore, New York, 1968.

Numo, Midwest Publications Co., Inc., Birmingham, Michigan, 1969.

Operations Bingo, Creative Publications Inc., Palo Alto, California, and Educational Tools Inc., Wichita, Kansas.

Sterner, Hans G., and Burt Kaufman. *Operational Systems Games*, McGraw-Hill, Toronto, 1969.

Teaching Fractions:
An Annotated Bibliography

- Bettina M. Blackall and W. George Cathcart

Fractions is a topic of concern to many teachers and a cause of considerable difficulty for many students. The following list of periodical articles should be helpful to the experienced teacher looking for meaningful instructional alternatives as well as for the beginner seeking guidance. The articles annotated below were selected from hundreds of articles on fractions as being the most mathematically and pedagogically sound.

The articles are divided into three categories. The first set deals primarily with basic mathematical principles. Recent trends in mathematics education have emphasized the teacher's need for a deeper knowledge of basic principles, in order to foster in children a similar understanding and appreciation of mathematical concepts.

The articles in the second group focus primarily upon alternative teaching strategies. The meaningfulness of children's learnings depends largely on the strategies and situations prepared by the teacher.

The final set of articles outlines some puzzles and games which can provide children with enjoyable practice needed for consolidation of ideas.

TEACHER INFORMATION

▸Bray, Claud J. "To Invert or Not to Invert", *The Arithmetic Teacher*, 10 (May 1963), 274-276.

A clear explanation of division of fractions by the reciprocal method. Numerals only are used to illustrate the procedure.

▸Dilley, Clyde, and Walter E. Rucker. "Division with Common and Decimal Fractional Numbers", *The Arithmetic Teacher*, 17 (May 1970), 438-441.

After explaining the importance of 1 in division of fractions, the authors develop an alternate method that depends on number pattern and an understanding of measurement division.

▸Filipek, Jerome T. "Common Denominators Made Easy", *Vector*, 11 (July 1970), 16-17.

An interesting method of finding Lowest Common Denominator through the use of number patterns. The approach could be used to extend a pupil's knowledge of equivalent fractions.

Miss Blackall is a graduate student in the Department of Elementary Education at the University of Alberta, and Dr. Cathcart is an Assistant Professor in the same department.

▸Hannon, Herbert. "Why Invert the Divisor?" *The Arithmetic Teacher*, IV (December 1957), 262-265.

The author approaches the problem of the title by concentrating on the inverse relationship between division and multiplication. Thus he provides, for at least some teachers, another line of reasoning, to broaden the basis of pupils' understandings of the former "mumbo-jumbo" recital.

▸Heddens, James W., and Michael Hynes. "Division of Fractional Numbers", *The Arithmetic Teacher*, 16 (February 1969), 99-103.

A summary of practical approaches to the learning of division of fractions, including regions, number lines, and sets of discrete objects.

▸Junge, Charlotte W. "Now Try This - Division of Fractions", *The Arithmetic Teacher*, 15 (February 1968), 177-178.

A number of surprisingly "modern" methods of division of fractions, gleaned from textbooks of the last century.

▸Koenker, Robert H. "Dividing by a Fraction", *The Arithmetic Teacher*, 12 (March 1965), 225-226.

Explanation of the lowest common denominator method, and two versions that lead to the generalization "To divide by a fraction, invert the divisor and multiply".

▸Larson, H. L. "The Structure of a Fraction", *The Arithmetic Teacher*. 13 (April 1966) 296-297.

A short article by an Albertan, who adds a further dimension to understanding by linking the unit fraction of the Egyptians to the child's knowledge of whole numbers.

▸Latino, Joseph J. "Take the Folly out of Fractions", *The Arithmetic Teacher*, 2 (November 1955), 113-118.

The author questions children's comprehension level of terms such as lowest common denominator and lowest terms, and gives some practical suggestions for sounder teaching practices that utilize concrete materials.

▸Morton, Robert L. "Fractional Numbers with a Sum of 1", *The Arithmetic Teacher*, 13 (December 1966), 647-655.

Children who have enjoyed exploring patterns in numbers will delight in exploring unit fractions, which we have inherited from the ancient Egyptians. Bonuses include lowest common multiples, factors, perfect and abundant numbers, and lots of practice in the four basic operations as applied to fractions.

▸Olberg, Robert. "Visual Aid for Multiplication and Division of Fractions", *The Arithmetic Teacher*, 14 (January 1967), 44-46.

A gridded rectangular region gives a concrete model for multiplication and division of fractions.

▸Vance, Irvin E. "A Natural Way of Teaching Division of Rational Numbers", *The Arithmetic Teacher*. 16 (February 1969), 91-93.

A clear explanation of division of fractions using the method

$$\frac{14}{9} \div \frac{2}{3} = \frac{14 \div 2}{9 \div 3} = \frac{7}{3}$$

SPECIFIC TEACHING STRATEGIES

▶Clarkson, David M. "A Number Pencil", *The Arithmetic Teacher*, 14 (November 1967), 557-559.

Inspired by Robert Davis of the Madison Project, this discovery lesson develops the notion of fractions as numbered pairs plotted on coordinate graphs.

▶Cunningham, George S., and David Raskin. "The Pegboard as a Fraction Maker", *The Arithmetic Teacher*, 15 (March 1968), 224-227.

The lengths between pegs are used to develop an understanding of simple fractions. The number line is shown as a "natural" way of recording these relationships.

▶Florence, Desmond. "Fractions in the Junior School", *Primary Mathematics*, 7 (April 1969), 41-49.

Having justified the use of structured materials in the development of mathematical ideas, the writer gives the specifications of a "Fraction Set", consisting of circle segments and a pack of cards bearing various fractional numerals in color. The games described give enjoyable practice in equivalence of fractions and the four operations involving fractions.

However, it would seem that (a) the children would need more experience of fractional relationships before being introduced to symbols, and (b) the exclusive use of one structured material is narrowing to a child's understanding.

▶Hillerby, D. J. "Multiplication and Division of Fractions", *Teaching Arithmetic*, 4 (Summer 1966), 52-53.

Children build up experience of the meaning of multiplication of fractions by exploring rectangular grids of varying proportions.

▶Jacobson, Ruth S. "Fun with Fractions for Special Education", *The Arithmetic Teacher*, 18 (October 1971), 417-419.

The learning of addition and subtraction of fractional numbers, both "like" and "unlike", is facilitated by the use of an acetate projectual and corresponding fractional parts in color. The children themselves can discover equivalent relationships and solutions to examples. Although designed for use on an overhead projector, the projectuals would be beneficial as a manipulative aid for the individual pupil.

▶Lansdown, Brenda C. "From Cake to Cancellation", *The Arithmetic Teacher*, IV (April 1957), 136-137.

Each pupil makes his own set of segments of circles - "fraction pies". After much exploratory play, the children are encouraged to play games that incorporate the idea of exchange, thus giving experience in equivalence of fractions and cancelling.

▶McMeen, George, "Division by a Fraction - a New Method", *The Arithmetic Teacher*, 9 (March 1962), 122-126.

A different approach to division of fractions by the use of number wheels, easily constructed by the child. The author presents a combination of experiment and telling, though the creative teacher will devise means of reducing the latter.

▶Ranucci, Ernest R. "Fractions without Friction", *Primary Mathematics*, 9 (June 1971), 27-30.

Geometric shapes are used to give experience in value relations, as a preparatory step to the manipulative stages of the development of fractions.

Unfortunately, the exercises are somewhat inconsistent in the use of area measure in determining the value of the various shapes; therefore, otherwise very useful assignments could confuse the pupils.

N.B. Teachers need to guard against the unfortunate tendency to depend on the use of one structured material as the solution to all problems, and to forget that children need a wide variety of experiences in order to generalize basic mathematical concepts.

PUZZLES TO REINFORCE

▶Hammond, Robert C. "A Device for Practice with Common Denominators and Addition of Unlike Fractions", *The Arithmetic Teacher*, VIII (November 1961), 373.

A magic square game designed to give practice in addition of fractions to the sum of 1. Pupils are encouraged to make up their own sets of rules.

▶Cook, Nancy. "Fraction Bingo", *The Arithmetic Teacher*, 17 (March 1970), 237-239.

An enjoyable activity to reinforce knowledge of equivalent fractions, after the concept has been developed. Children are encouraged to explore relationships and to devise feasible strategies.

▶Rode, Joann, "Make a Whole - a Game using Simple Fractions", *The Arithmetic Teacher*, 18 (February 1971), 116-117.

A set of simply-made circle segments are used to play a fractions game that practises equivalent fractions as well as the combinations of fractions that equal one. The writer leaves the way clear for the classroom teacher and/or the pupils to devise other sets of rules.

Graphing Experiences:

An Annotated Biblography

- *Bettina M. Blackall* and *W. George Cathcart*

Graphs are a vital part of the world of communication. They simplify masses of statistics and present them in concise form. All communication including graphs must be interpreted. Therefore, children should have ample opportunity to discuss and ask questions about their graphs.

Graphing is an important skill in the activity approach to mathematics. Graphs make the discovery of relationships and principles easier.

Since graphs are important in the child's everyday living and in his mathematical experiences, we should do more work with graphs in the elementary school program. The following selected references suggest many activities which you can use to give your students some interesting experiences with graphing.

▶Berry, Clifford. "Probability and Graphing", *Mathex*, Level 4, Teacher's Bulletin No. 12. Toronto: Encyclopaedia Britannica, 1967.

The usefulness of graphing, so that more information can be generated, is seen in this study of probability.

▶Bishop, Carolyn. "Graphical Illustration", *Mathex*, Level 3, Teacher's Bulletin No. 1. Toronto: Encyclopaedia Britannica, 1967.

A variety of materials is suggested for the concrete construction of column graphs. Situations that are real-life to young students and from which meaningful graphing can develop are described and illustrated.

▶Boxell, L. "A Graph by any other Name ...", *Mathematics Teaching*, 42 (Spring 1965), 14-15.

A plea for greater uniformity in the naming of the graphs commonly found in elementary schools, this short article clarifies some simple types of graphs and scales.

▶Fyffe, R. M. "Show Me", *Primary Mathematics*, 8 (December 1970), 152-162.

A number of practical examples suggest ways to develop an understanding of ordered pairs and how this kind of data can be graphed in a variety of forms.

▶France, Norman. "Counting with a Purpose", *Mathex*, Level 5, Teacher's Bulletin No. 1. Toronto: Encyclopaedia Britannica, 1967.

Numerous suggestions concerning the collection and graphing of statistical data are provided so that seven experience stations can operate profitably in a classroom.

▶Girard, Ruth A. "Development of Critical Interpretation of Statistics and Graphs", *The Arithmetic Teacher*, 14 (April 1967), 272-277.

An appeal for a planned program in critical interpretation of statistics and graphs is followed by a description of a sample unit. Relative error, sampling, incomplete information and average are some of the concepts discussed.

▶Hallam, Frances. "Recording Information", *Mathex*, Level 1, Teacher's Bulletin No. 15. Toronto: Encyclopaedia Britannica, 1967.

Finding ways of recording information leads children to utilize the number line and from there to move into bar and column graphs. Anecdotal descriptions of classroom activity illustrate the author's approach to this topic.

▶Heard, Ida Mae. "Making and Using Graphs in the Kindergarten Mathematics Program", *The Arithmetic Teacher*, 15 (October 1968), 504-506.

The experiences of small children who construct their own block, bar and picture graphs are described and illustrated.

▶Jorden, Janet. "Graphing Relationships", *Mathex*, Level 2, Teacher's Bulletin No. 12. Toronto: Encyclopaedia Britannica, 1967.

The "Steps and Stages in Graphing" are set out in model form. Classroom situations illustrate each aspect listed. The emphasis is on teacher guidance, pupil activity and relationships.

▶Junge, Charlotte. "Dots, Plots and Profiles", *The Arithmetic Teacher*, 16 (May 1969), 371-378.

The initial need to teach graphing, as well as the necessity to plan a sequential treatment of the topic, is argued clearly. Various types of graphs, with sample questions, emphasize the benefit to children who not only interpret others' graphs but also construct their own.

▶Lansdown, Brenda. "Exploring Rate Graphs with Gifted Ten-year-olds", *The Arithmetic Teacher*, 11 (March 1964), 146-149.

Children involved in graphing practical situations are motivated to play with concepts of straight line and parabolic graphs. These pupils are encouraged to relate practical, theoretical and graphical ideas to form a pattern of thinking.

▶Liedtke, Werner W. "Battleship", *Mathex*, Level 4, Teacher's Bulletin No. 14. Toronto: Encyclopaedia Britannica, 1967.

This popular game is an excellent example of mathematical ideas being reinforced in an enjoyable atmosphere. Children become familiar with the location of coordinate points, the transformation of geometric figures and positional estimation. This article sets out the rules very clearly and also suggests some possible adaptations.

▶Lowery, Lawrence F., and Donald Lundstrom. "Graphic Representation of Plant

Growth", *Science and Children*, 4 (April 1967), 15-17.

The practical application of graphing techniques at various grade levels shows the advantages of giving children numerous experiences in the construction of a variety of graphs.

▶Martin, J. R. "Graphical Representation", *Primary Mathematics*, 9 (June 1971), 3-14.

One situation, that of the books the children have read, is used to provide a survey of ways of recording data in an organized fashion, from simple arrow graphs to coordinate graphs. The factors to be considered in choosing a particular kind of graph for specific data are set out clearly. The use of graphical representation in order to make predictions is discussed.

▶McGlathery, Glenn. "Tic-Tac-Toe Graph - an Elementary Graphing Game", *Science and Children*, 8 (November 1970), 19-20.

This game, which practises the naming of intersections on a coordinate graph, can be played by a pair of children or a whole class.

▶Moulds, Doris M. "Graphs", *Mathex*, Level 1, Teacher's Bulletin No. 11. Toronto: Encyclopaedia Britannica, 1967.

Pictorial Representation is developed from one-to-one correspondence through to the stage of using "quadrille" or grid paper to record the data which children have gathered from their own environment. Some of the well-described suggestions for classroom activity are illustrated.

▶Nelson, L. D., and W. W. Sawyer (eds.) *Mathex: Matching and Graphing* (Teacher's Resource Book No. 1). Toronto: Encyclopaedia Britannica, 1970.

The reasons for the inclusion of graphing in primary grades are established initially. Numerous suggestions are given for meaningful situations suitable for recording graphically. The young pupils are encouraged to construct their own graphs in a variety of ways. A similar policy is suggested for older students who bring a greater sophistication to their organization of relationships and other data. The introduction of coordinates is given special consideration.

▶Nelson, L. D., and W. W. Sawyer (eds.) *Mathex: Graphing and Probability* (Teacher's Resource Book No. 6). Toronto: Enclopaedia Britannica, 1970.

Various types of graphs are dealth with in detail, with plenty of practical applications being suggested. The game "Battleship", based on coordinates, is especially well treated. The graphing of ordered pairs and equations can help children realize the connections between certain branches of mathematics. Activities are provided to give pupils a meaningful introduction to statistics, particularly to the notion of the spread of distributions.

▶The Nuffield Mathematics Project. *Pictorial Representation*. London: W & R Chambers and John Murray, 1967.

In this approach, graphs are seen as pictures of relationships. They communicate, and the communication must be interpreted. Many examples of children's construction and interpretation of various kinds of graphs are included. The emphasis is on meaningful data. This 40-page book is full of practical suggestions for teachers of Grade I through to Junior High School.

▶Pierson, Robert C. "Elementary Graphing Experiences", *The Arithmetic Teacher*,

16 (March 1969), 199-201.

An easy development of graphing places initial emphasis on one-to-one correspondence. A number of interesting topics are suggested to give practice in various types of graphs.

▶ Pincus, Morris, and Frances Morgenstern. "Graphs in the Primary Grades", *The Arithmetic Teacher*, 17 (October 1970), 499-501.

The ability to read and construct graphs is developed through a sequence of illustrated activities that proceed from concrete objects to symbolic representation.

▶ Pomeroy, David J. "Pictorial Representation - Is There a Progression?" *Primary Mathematics*, 9 (Summer 1971), 78-84.

It is argued that the development of abilities in graphing lies not in the type of graph nor in the content, but in the interpretation of the graphs constructed. An outline scheme presents a clear program. Examples of children's work on a single topic are included to show the increasing sophistication of interpretation.

▶ Schell, Leo M. "Horizontal Enrichment with Graphs", *The Arithmetic Teacher*, 14 (December 1967), 654-656.

The problem of plotted points which do not lie on a straight or a smooth line is raised in four sample lessons. Stimulating "thought" questions, rather than "solutions", are included in the self-administered units.

Integers in the Elementary School:
An Annotated Bibliography

- *Bettina M. Blackall* and *W. George Cathcart*

As children explore the world of whole numbers and then of fractions, they often discover incidentally the need for negative integers. This topic has already been enjoyed by many elementary school children who have not been forced into abstractions and definitions before they have gained an intuitive understanding of this "new" kind of number.

Often an intuitive understanding of negative integers is obtained through out-of-school experiences. Many games which children play involve "going in the hole" and it is amazing how easily children can perform the necessary operations in these games without prior instruction in negative numbers.

Educational journals contain many articles on teaching integers to elementary school children. In preparing the following annotated bibliography, many articles were rejected because of overloading with abstract notions and/or an overdependence on definitions in order to justify (usually most unsuccessfully) to children the nature of integers and the operations upon them. The following, then, is a select bibliography which teachers should find very helpful in teaching negative numbers.

▶Ashlock, Robert B., and Tommie A. West. "Physical Representation for Signed-number Operations", *The Arithmetic Teacher*, 14 (October 1967), 549-554.

This survey covers both the measurement and counting representations of signed numbers, and suggests a variety of procedures for developing understanding of the operations on integers. An extensive annotated bibliography is included.

▶Bates, John Hyde. "Positive and Negative Numbers", *Mathex*, Level 5, Teacher's Bulletin No. 14. Toronto: Encyclopaedia Britannica, 1967.

This practical introduction to integers through associations with rocket launching and oil wells encourages the children to record their explorations on the number line. The pupils can establish their own patterns using both a matrix and a co-ordinate graph.

▶Calandra, Alexander. "Teaching Signed Numbers in Grade 8", *The Arithmetic Teacher*, V (November 1959), 259-260.

The teaching strategy described depends on definitions rather than understanding. However, the use of $\overrightarrow{3}$ and $\overleftarrow{3}$ to indicate, on a number line, positive three and negative three respectively, may be a helpful idea for teachers who wish to avoid initial confusion between value and direction.

▶Cochran, Beryl. "Children use Signed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 587-588.

The author, a member of the Madison Project team, describes two classroom situations in which the teacher has capitalized on a child's intuitive understanding of mathematics and its logical notations to introduce the notion of negative integers.

▶Cohen, Louis S. "A Rationale in Working with Signed Numbers, *The Arithmetic Teacher*, 12 (November 1965), 563-567.

The "Postman Stories" model, devised by the Madison Project, uses the delivery of checks and bills to develop an understanding of integers in a most enjoyable way. Verbal questions and descriptions are followed by symbolization using the conventional positive and negative signs.

▶Cohen, Louis S. "A Rationale in Working with Signed Numbers Revisited", *The Arithmetic Teacher*, 13 (November 1966), 564-567.

The author continues to use the Postman model (see Cohen, 1965) to develop a convincing explanation of why "a negative times a negative is a positive".

▶Cotter, Stanley. "Charged Particles: a Model for Teaching Operations with Directed Numbers", *The Arithmetic Teacher*, 16 (May 1969), 349-353.

The concepts of positive and negative particles, amount of electrical charge (including the zero charge) and neutralization provide a logical, consistent and satisfying approach to integers. The four operations are dealt with. The way in which the model explains the sometimes dreaded multiplication of two negatives is particularly imaginative.

▶Frank, Charlotte, "Play Shuffleboard with Negative Numbers", *The Arithmetic Teacher*, 16 (May 1969), 395-397.

The game of shuffleboard, adapted for classroom play, incorporated negative integers into its rules. By recording their scores on a number line, pupils have semi-abstract experience with integers.

▶Fremont, Herbert. "Pipe Cleaners and Loops - Discovering How to Add and Subtract Directed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 568-572.

Loops, initially made of pipe cleaners but later drawn, allow pupils to physically represent positive and negative integers. Children use the knowledge that the combination of $^{-}1$ and $^{+}1$ is in fact zero to develop an understanding of the addition and subtraction of integers.

▶Hannon, Herbert. "A Device for Teaching Addition and Subtraction on Integers", *Mathematics Teacher*, 60 (December 1967), 860-861.

Although elementary school teachers may consider the reasoning behind the described device too obscure for their pupils, children would certainly enjoy exploring the resultant table for number patterns and relationships.

▶Havenhill, Wallace P. "Though This Be Madness", *The Arithmetic Teacher*, 16 (December 1969), 606-608.

The interpretations of the + and - signs as both
(a) direction, and
(b) reversal and non-reversal
allow the four operations, including division, to be clearly represented on the number line.

▶Luth, Lois. "A Model for Arithmetic of Signed Numbers", *The Arithmetic Teacher*, 14 (March 1967), 220-222.

The feature of this teaching strategy is an enjoyable story line about Hy who lives in a house in the sky. He buys and frees balloons ($^{+}$) and sandbags ($^{-}$) in order to alter his altitude. The model, which can be represented on a vertical number line, accommodates all four operations of both positive and negative integers.

▶Magnuson, Russell C. "Signed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 573-575.

The use of pattern is one of the practical suggestions for the introduction of the notion of negative integers. Unfortunately, the multiplication of two negatives is presented as a fait-accompli, without any justifying explanation whatsoever.

▶Mauthe, Albert H. "Climb the Ladder", *The Arithmetic Teacher*, 16 (May 1969), 354-357.

A simple game to reinforce addition of integers uses a ladder marked $^{-}12$ to $^{+}12$ and a spinner marked from $^{-}4$ to $^{+}4$.
A ten-faced die appropriately marked could replace the spinner.

▶Milne, Ester. "Disguised Practice for Multiplication and Addition of Directed Numbers", *The Arithmetic Teacher*, 16 (May 1969).

The familiar game of combining the numbers that come up on two spinners is adapted to include integers (and powers, if desired).

▶The Nuffield Mathematics Project. *Computation and Structure 4*. London: W. & R. Chambers and John Murray, 1969.

The idea of the integers is built up in terms of ordered pairs before the number line and other applications are introduced. Thus, a sound foundation for operations on integers is laid.

▶Pratt, Edna M. "A Teaching Aid for Signed Numbers", *The Arithmetic Teacher*, 13 (November 1966), 589-591.

This simple device for making by the pupils is described in detail. The many variations suggested will certainly reinforce the traditional approaches and in addition provide enjoyment, as claimed by the author. This is not, however, a suitable activity to introduce the concept.

▶Sherzer, Laurence. "Adding integers using only the Concepts of One-to-one Correspondence and Counting", *The Arithmetic Teacher*, 16 (May 1969), 360-361.

The addition of integers is seen as the pairing of negative ones with positive ones, followed by the counting of the balance. Ordered pairs of whole numbers are used to justify this procedure mathematically. ●

Probability in the Elementary School:

An Annotated Bibliography

- *Bettina M. Blackall* and *W. George Cathcart*

Probability is a versatile topic in elementary school mathematics. Many teachers see its study as an integral part of a program which has as a goal problem-solving and independent attack.

On the other hand, it can be regarded, at the elementary level, as an enrichment topic that provides desirable preliminary experiences for children prior to their move to junior high school.

Unfortunately, there are many texts currently available which presume that pupils already know what probability is all about, yet the authors claim they are presenting introductory instruction.

Much material has been sifted in order to produce the following list of references. Whether you view probability as an integral part of your elementary mathematics program or simply as an enrichment topic, you should find the following references valuable resource material.

▶Berry, Clifford. "Probability and Graphing", *Mathex*, Level 4. Teacher's Bulletin 12. Toronto: Encyclopaedia Britannica, 1968.

Emphasis on child activity provides a sound basis for discovering the fundamental ideas of probability. Children are encouraged to record their findings using graphs and to write their explanations. The author avoids "giving formal definitions and developing formal and often complicated notation".

▶Buxton, R. "Probability and its Measurement", *Mathematics Teaching*, 49 (Winter 1969), 4-12.

This readable account discusses clearly the meaning of probability without recourse to complicated formulae. Practical applications with emphasis on the concept of equally likely frequencies are suggested.

The second part of this article compares (a) objective and subjective theories, and (b) empirical and logical theories; therefore, it does not seem to be relevant to the elementary school teacher or pupil.

▶Cathcart, W. George. "What are the Chances?" *Mathex*, Level 6. Pupil Bulletin 10. Toronto: Encyclopaedia Britannica, 1968.

More advanced activities are used to establish the numerical relationships within a probability situation. The pupils are offered informal experiences leading to the graphing of the normal distribution curve. Those pupils who are mathematically sophisticated will be interested in learning to distinguish empirical from theoretical probability.

▶Engel, Arthur. "Mathematical Research and Instruction in Probability Theory", *The Mathematics Teacher*, 59 (December 1966), 771-782.

Geared to the teaching of junior high school students, this relatively short article describes experiments that children can do to develop intuitive notions of probability, prior to a study of the theory itself. Although mathematical discourse and explanation are provided in abundance, the non-math major teacher can ignore this aspect and still gain much from reading this article at his own level of conceptualization.

▶Engel, Arthur. "Teaching Probability in Intermediate Grades", *International Journal of Mathematical Education in Science and Technology*, 2 (July/September 1971), 243-294.

This long article is packed with teacher information as well as with an extremely large collection of activity ideas. The level extends from the very simple assignment to problems that would challenge many junior high school students. There is much theoretical explanation for the expert, which can be ignored by the non-mathematician without spoiling the usefulness of this major contribution to the topic of probability.

▶Johnson, Donovan, Viggo Hansen et al. *Activities in Mathematics - first course, Probability*. Glenview, Illinois: Scott, Foresman, 1971.

Some novel situations as well as the usual dice - and coin - throwing activities provide experiences of probability. Students are given much practice in completing one particular type of table, but are not encouraged to construct or devise their own ways of recording data. Bar graphs, ordered pairs and coordinates are valuable by-products. Fractions are reviewed extensively. Attractive illustrations and good spacing are features of the format of this series.

▶Lovell, Kenneth. "Proportionality and Probability" in Myron F. Rosskopf, Leslie P. Steffe and Stanley Taback (eds.) *Piagetian Cognitive-Development Research and Mathematical Education*. Washington: National Council of Teachers of Mathematics, 1971.

Piaget's view that the development of probability understanding to the point of quantification demands the onset of formal-operational thought is documented and compared with other recent research in this area. Preliminary aspects which can be profitably introduced in elementary school are suggested.

▶May, Lola J. "What are the Odds?" *Grade Teacher*, 88 (March 1971), 62-63.

The author assumes (a) that children have to be told that fractional notation is a form of expressing probability; and (b) that children and teachers already have an intuitive understanding of probability. However, the suggestion of recording throws of dice on a lattice is useful.

▶Nelson, L. Doyal, and W. W. Sawyer (general editors). *Mathex* (Junior Level, Grades 4-6. No. 6:2, pp.38-52). Toronto: Encyclopaedia Britannica, 1970.

This series of practical assignments develops the child's understanding of probability, including methods of recording data. However, some of the vocabulary is

likely to tax some elementary children. The games suggested provide enjoyable reinforcing activities.

▶The Nuffield Mathematics Project. *Probability and Statistics*. London: W & R Chambers and John Murray, 1969.

This book of only 55 pages follows the Nuffield Project pattern of gradually developing the child's understanding of and facility with the mathematical concept under discussion. The table of contents summarizes this approach as it applies to Probability:

Early Uses of Pictorial Representation
Games Leading to Ideas of Probability
Sampling
Recording
Simple Averages
Measuring Probability.

▶Ojemann, Ralph H., James E. Maxey,and Bill C. Snider. "The Effect of a Program of Guided Learning Experiences in Developing Probability Concepts at the Third-Grade Level", *Journal of Experimental Education*, 33 (Summer 1965), 321-330.

The learning program devised by the researchers contains many practical suggestions for the classroom teacher. The application of probability to everyday living is discussed without undue jargon and symbolization.

▶Razzell, Arthur, and K.G.O. Watts. *Probability* (Mathematical Topics 4). London: Rupert Hart-Davis, 1968.

This cheerful-looking book of 32 pages gives interesting historical background and shows how probability is related to everyday life. Although some activities are suggested, the contents could be used as informative reading. Terms such as random sampling are explained within appropriate situations and in simple language.

▶Skukyn, Murray J. "Probability", *Mathex*, Level 5, Pupil Bulletin 3. Toronto: Encyclopaedia Britannica, 1968.

A wide variety of open-ended activities using simple materials provide open-ended situations which children enjoy investigating.

▶Yee, Albert H. "Mathematics Probability and Decision-Making", *The Arithmetic Teacher*, 13 (May 1966), 385-387.

The author establishes a case for the inclusion of the study of probability in elementary mathematics on the grounds of the need to train children in decision-making skills. He rejects technical jargon in favor of relating probability to the everyday experiences of the young child.

A Collection of Ideas for Secondary School Mathematics Teachers

Contributors: *Miss S. Fraser, Mrs. T. Koinzan, Mr. D. Lawson, Mr. D. McIntyre, Mrs. L. Rule, Mr. T. Sorenson, Mr. D. Treslan, Mr. D. Vosburgh, Mr. J. Woloshchuk.*

These contributors are members of the Mathematics Department at Dr. E. P. Scarlett High School in Calgary.

As a department, we wish to share our ideas with other teachers involved in teaching mathematics in secondary schools. In particular, we hope that our collection will assist beginning teachers and rural teachers. Hopefully, other people of our profession will also benefit from our practical collection.

1. Stress and mark important points in your classroom teaching.
2. Vary the beginning of a mathematics class. Some suggestions for variation:
   (a) Practice mental mathematics.
   (b) Demonstrate the application of mathematics - charts, newspapers, letters from federal and provincial governments, and so on.
   (c) Use brain teasers - puzzles.
   (d) Discuss good study techniques - provide handouts.
   (e) Review key ideas of last day's work.
   (f) Short quiz - oral or written - could be an arithmetic quiz.
   (g) Tell a joke.
   (h) Demonstrate mathematical models.
   (i) Clearly indicate objective or objectives of class period. (This should be done in every class period.)
   (j) Discuss some aspect of history of mathematics.
3. There are various ways of providing for individual differences:
   (a) Independent study. Careful selection of students is necessary. A good deal of helpful information is available to select students for independent study.
   (b) Varying daily assignments.
   (c) Remedial classes.
   (d) Enrichment for the gifted.
   (e) Discovery lessons.
   (f) Mini-pak approach.
   (g) Circulate in class to provide for individual students.
   (h) Tutorials by teachers.
   (i) Allow better students to tutor weaker students in small groups.
   (j) Vocational and consumer mathematics courses.
   (k) Supervised study periods.
   (l) Student involvement in classwork.
   (m) Whenever possible, begin at the concrete level.

(n) Audio-visual presentations.
(o) Provide an adequate number of examples.
Note: The above is not an all-inclusive list. It does indicate that no one method can be used to provide for individual differences successfully.

4. Attempt to present lessons in unique ways in order to draw students' attention.
5. Make a sincere effort to demonstrate the applicability of mathematics.
6. Summarize your ideas or encourage students to summarize them.
7. Involve people in speakers' bureaus in your mathematics education.
8. Organize a class into groups of two, select a problem, let students do it, check, answer and discuss. This is a form of a "buddy" system.
9. Use response books to check work. Students do their own checking while teacher circulates to assist them individually.
10. Let students make up problems related to classwork you are doing. Exchange these problems with other students. Let the other students do them.
11. Allow students to make up examples as you teach.
12. Occasionally, spend some time discussing student concerns.
13. Discuss news items that may or may not pertain to mathematics.
14. Allow students an opportunity to talk about their life history. You may wish to tell them something about yourself.
15. Introduce a new topic with a problem.
16. Examine a new topic from a historical viewpoint.
17. Wherever possible, use an active approach to get the students to learn by doing. You may have to motivate students, because certainly not all students are self-motivated to learn mathematics.
18. Challenge better students. Expect intellectual effort from your students.
19. Sometimes, the lecture approach is the best to use with very able and mature students.
20. Have certain students' names on blackboard prior to beginning of class. These students would be required to do certain problems on the board.
Example: #1) Sally #2) Susan #3) Lynn
#4) Dennis #5) Shirley
21. Discuss literature, announcements or events of a particular interest to that grade level.
22. Let students do a problem related to previous day's work. Row competition could be used to see which row does best.
23. Give any interesting information regarding a student(s) involvement in any extra-curricular activity.
24. Give student *positive* comments when due.
25. At times, one could let students stand, stretch and move. This could be a good idea for a class break (Monday morning or any other time as seen fit).
26. Have students close their books and place pencils or pens down before you start teaching.
27. Attempt to treat students as individuals, but demand a degree of rigor from them.

28. At the beginning of a school term, set up a schedule of course events for the students. Though this schedule may be tentative, it is comforting for students.

29. Use chapter tests in texts as regular or review assignments.

30. Provide a variety of types of questions on examinations.

31. Admit you are incorrect when you make an error in class.

32. Always prepare adequately before you teach a lesson.

33. Discuss openly topics of mutual interest to the students and yourself. Healthy digressions never hurt anyone.

34. Be prepared to explain relationships between the topic under discussion and the real outside world - that is, point out the application of theory.

35. Recognize minority or individual views, but stress the democratic ideal of majority rule - for example, arriving at term marks.

36. Command respect, and you will get it. There is a fine line between you and the students. Make them aware of it without making them feel that you are unapproachable.

37. Take a humanistic approach toward students as much as possible.

38. Be flexible. Things will not always work out the way you planned them.

39. Try to involve the whole class. Students are quick to notice when certain people are left out.

40. Vary your approach. Even the highly academic student will get fed up with pure mathematics.

41. For slower streams, attempt to deal with practical topics. Avoid getting involved in abstract concepts all the time.

42. Take the time to meet students or groups of students outside the classroom. This is a good technique to develop student rapport.

43. Assess yourself as a teacher. Where can you improve? How?

44. Attempt to get students to feel that what they are doing in their classes is important.

45. Give special assistance in how to study, how to use the text or texts, how to locate information in order to promote individual progress.

Mathematics Council Executive – 1973-74

President
Dr. W. George Cathcart — Res 435-1949
11511 - 35A Avenue — Bus 432-5880
Edmonton

Vice-President and Annual Meeting Chairman
Denis Baudin — Res 693-2469
Box 861 — Bus 723-3815
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3716 - 107 Street — Bus 434-5472
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936 Cannell Road SW — Bus 281-3366
Calgary T2W 1T4

Treasurer
Donald H. Hinde — Res 782-3276
Box 741 — Bus 782-3812
Lacombe TOC 1SO

ATA Representative
Dr. Mel T. Sillito — Res 459-4811
Barnett House — Bus 453-2411
11010 - 142 Street
Edmonton T5N 2R1

NCTM Representative
Dr. W. George Cathcart

Faculty of Education Representative
Dr. Mary Beaton — Res 282-6825
657 Northmount Drive NW — Bus 284-5643
Calgary

Dept. of Education Representative
Wilf Lencucha — Res 328-5881
Room 236, Admin. Bldg. — Bus 328-1157
Community College
Lethbridge

Mathematics Representative
Dr. Jack W. Macki — Res 435-6897
Mathematics Department — Bus 432-5725
University of Alberta
Edmonton

Editor of *Delta-K* and Publicity Chairman
Ed Carriger — Res 843-6138
R.R. 1, Site 2, Box 3 — Bus 843-6528
Bluffton

Editor of *Monograph No. 2*
Dr. Bruce Harrison — Res 284-1595
Faculty of Education — Bus 284-5645
University of Calgary
Calgary

Directors
Ed Carriger

Brian Chapman — Res
Box 1525 — Bus
Lacombe

Dr. Joan Kirkpatrick — Res 488-2634
Dept. of Elementary Ed. — Bus 432-3662
University of Alberta
Edmonton

Stuart McCormick — Res 469-0972
6428 - 84 Street — Bus 466-2976
Edmonton

Edward Mitchell — Res 743-2052
Box 1355 — Bus
Fort McMurray

George Stepney — Res 453-1248
12414 - 125 Street — Bus 466-3161
Edmonton

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