The "Inner Stars," "-Pictures" and "-Games,"

A Contribution to Reform in Mathematics and Art Teaching - Part 1

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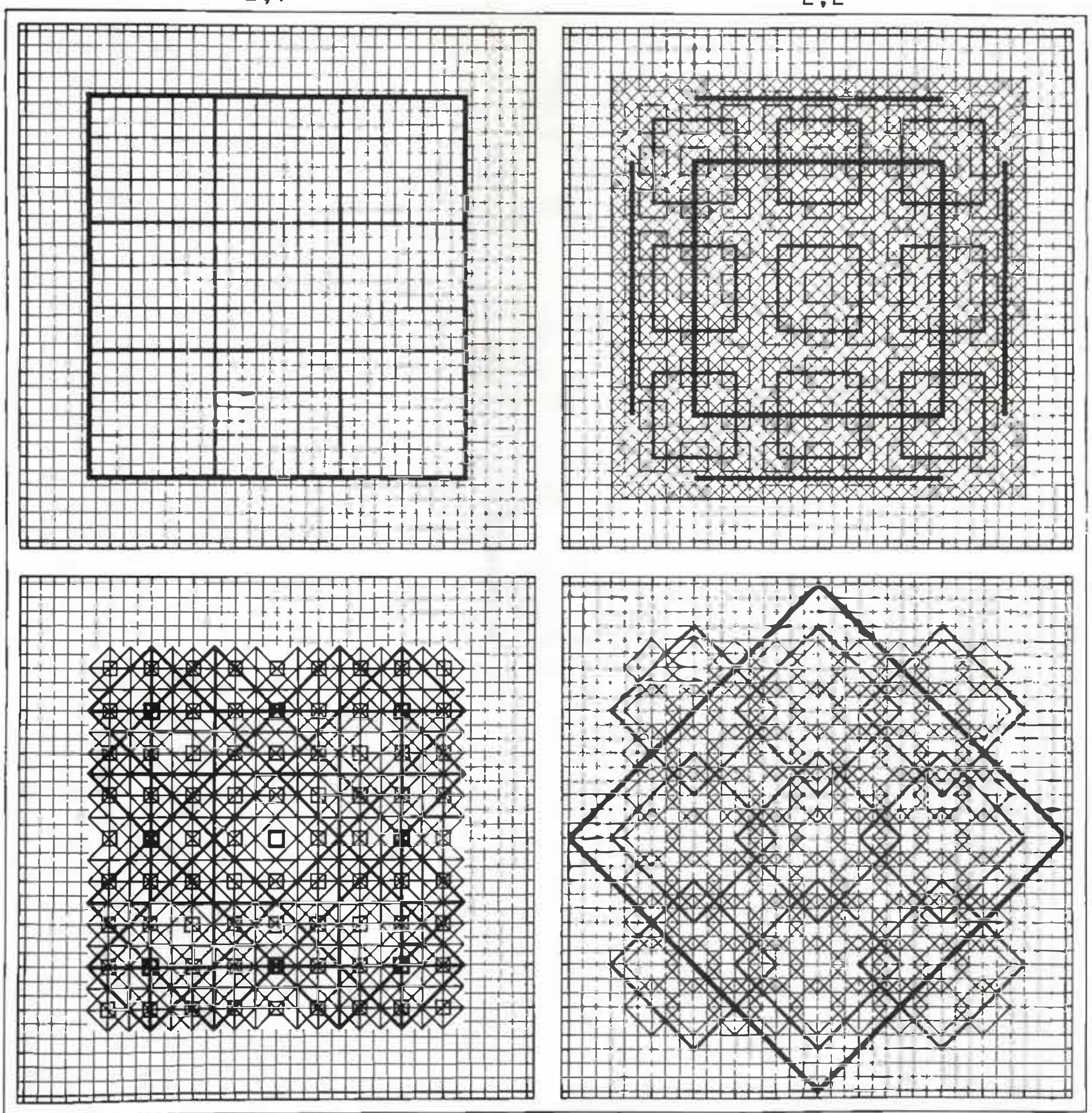
This article deals with modern and extensive geometrical figures discovered by the author. In his opinion, their appropriate treatment in lessons of geometry and/or art could contribute to an essential reform in the instruction of these subjects.

The figures are called "inner-stars," "-pictures" and "-games." The "inner-stars" are layered geometrical surface designs. They are well suited as screens for making esthetically attractive pictures, fixed or changing. These are the "inner-pictures" and the "inner-games." The simpler "inner-stars" and "-pictures" can be designed and examined with simpler means. They seem suited for improving and enlarging the introductory as well as the advanced instruction in mathematics and art.

The importance of the figures exceeds the interest of school instruction. For cogent reasons the "inner-stars," as they form scales of a future luminous music, are suited to achieve the same effects as the scales for music (including the twelve-tone scales). This visual music is not to be a tone music made visible, but an independent art, the works of which, as a rule, are films without special subjects. In the following, we examine square "inner-stars" and "-pictures" of the number of extension 3.

Picture 2;1 shows one large square (layer 0, to be conceived as an area), 9 smaller areas (layer 1), 81 still smaller areas (layer 2), and 729 smallest areas (layer 3). We introduce parallel coordinates. The center is the point (0;0), the center of the right side of the large square is (1/2;0), the center of the upper side is (0;1/2). In our thoughts, we add firstly the finer layers 4, 5 . . . and secondly we imagine the figure obtained thus continued beyond the borders of the large square over the whole plane. Thirdly, to the figure obtained in this way we add in our thoughts the coarser layers -1, -2 . . . The total figure which now exists is called the square layered "cell-star" of the number of extension 3, or C3. The squares are called its "cells."

For n = 0, ± 1, ± 2 . . . We call the centers of the "cells" of layer n the "strong-points" of layer n. Each "strong-point" of layer n is at the same time also a "strong-point" of all finer layers (n+1), (n+2). The center of the picture, and only it, is the "strong-point" of all the layers. The "strong-points" of layer 0 have coordinates which are whole numbers, those of layer 1 have coordinates with the denominator 2^1, and so on. We call a "strong-point" of layer n, which is not at the same time a "strong-point" of the next coarser layer (n-1), a "strong-point in a new place" of layer n; every other, a "strong-point in an old place." The total figure, consisting of all the "strong-points," is called the square "point-star" of the number of extension 3, or P3.



Picture 2

"M," 139 of layer 3 forming the stylized letters "LICHT-MUSIK," and 241 of layer 4 forming five rows of ornaments. The pictures are colored according to the following law: border lines are black; where "base-figures" in odd numbers cover the area the color is white; in even numbers it is black.

For n = 0, ±1, ±2 . . . A straight line passing through at least two, and therefore through infinitely numerous "strong-points" of layer n, is called a "strong line" of layer n. A "strong-line" of layer n which is not at the same time "strong-line" of the next coarser layer (n-1), is called a "strong-line in a new place" of layer n; every other, a "strong-line in an old place." Every segment of a "strong-line" is called a "strong-segment."

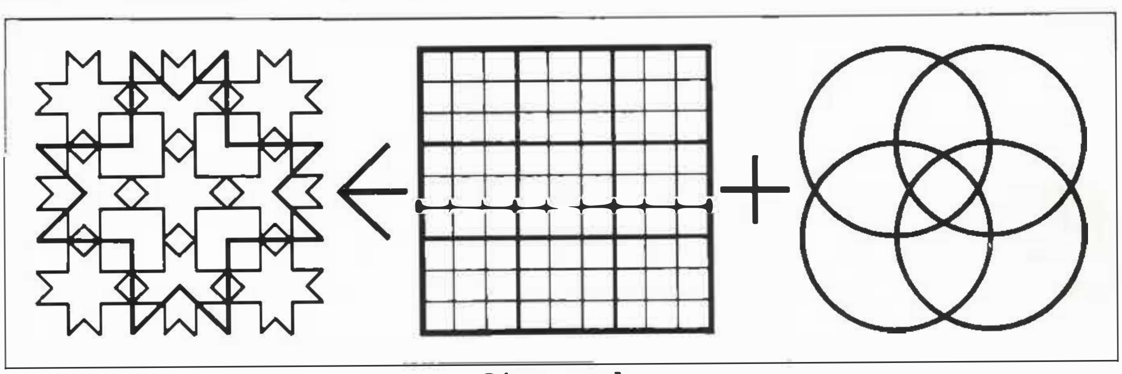
For n = 0, ±1, ±2 . . . The straight lines dividing the plane sector between two neighboring "strong-lines" of layer n into f equal parts, are called "field-lines" of layer n and of the number of division f. As such numbers of division f all natural numbers from 2 onward are possible so far as they have no common divisor with the number of extension e. In the present article e = 3. A "field-line" of layer n which is not at the same time "field-line" of the next coarser layer (n-1), is called a "field-line in a new place" of layer n; every other, a "field-line in an old place." Every segment of a "field-line" is called a "field-segment."

The whole of all "strong-lines" belonging to fixed gradients as well as of all appertaining "field-lines" to a fixed number of division is called a "general-star." The corresponding figure consisting of all lines "in a new place" is called a "complete-star." The "complete-stars" serve for the construction of the "inner-stars."

Picture 2;2 shows a modification of the "cell-star" C3. All "cells" have been reduced in the scale 1:(2/3). The new total figure is called the "screen-star" or S3, and the new squares its "base-figures." Let us compare C3 and S3. In C3 the boundary lines of "cells" of different layers are heaped one upon another in large numbers. In S3 boundary lines of "base-figures" of different layers have, at best, isolated points in common but never whole segments of straight lines. Therefore, we call the S3 "transparent."

For the central "cell" of layer 0 and for the gradients 0, ∞, 1, -1, picture 2;3 shows the "strong-lines in a new place" and the "field-lines in a new place" of layers 1 and 2, and that, as far as the "field-lines" are concerned, to the number of division f = 2. It therefore shows a segment of a "complete-star" adjoined to C3 and P3. This "complete-star" contains also the "inner-stars" of pictures 1;3 and 2;4.

Picture 2;4 shows a further modification of the "cell-star" C3. The new "base-figures" are rhombuses (to be conceived as areas). The new total figure is an "inner-star." For this article we call it I(2;4). Let us compare S3 and I(2;4). The I(2;4) is also "transparent." In contrast to S3, neighboring "base-figures" (in the directions up and down, and from right to left) have some points in common. Therefore we call the I(2;4) "coherent."



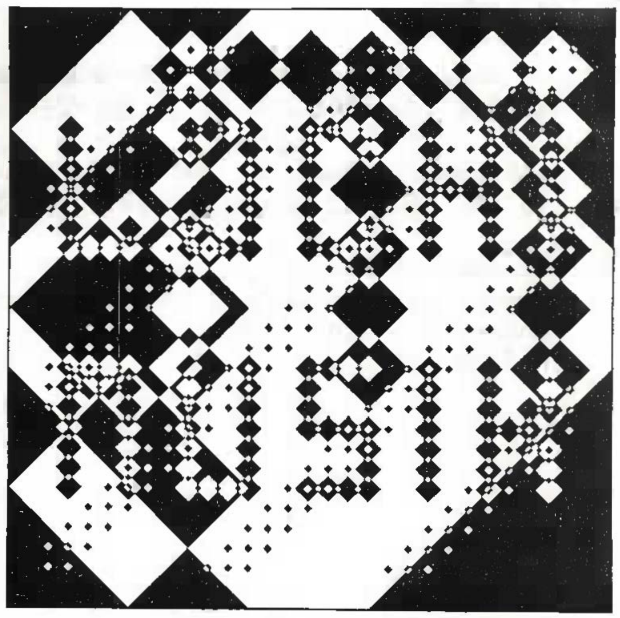
Picture 1

Picture 1 represents the basic idea of the "inner stars." It shows three partial pictures. The left figure represents the intersection law of seeing. Two or more areas of good form (here, circular areas) intersecting each other can, at the same time, be seen as intact forms. The middle figure shows layers 0, 1 and 2 of the "cell-star" C3 of the number of extension 3. The figure on the right shows layers 0 and 1 of an "inner-star" of the number of extension 3. We call this "inner-star" the I(1;3).

For every "inner-star" we call the whole figure consisting of the border-lines of all "base figures," the pertinent "line-inner-star." To every "inner-star" exactly one "line-inner-star" is coordinated. To every "line-inner-star" several "inner-stars" are coordinated, exactly even. The "inner-stars" are not an end in themselves. They serve as screens for the production of "inner-pictures" and "-games."

An "inner-picture" develops from an "inner-star" in the following way. Among the "base-figures" of the "inner-star" a limited number are chosen to appear or shine. The others are to remain invisible. The appearing "base-figures" are covered with colors.

Picture 3



Picture 4

Pictures 3 and 4 are 5-layered "inner-pictures" of the "inner-stars" I(1;3) and I(2;4). They show 1 "base-figure" of layer 0, 3 "base-figures" of layer 1 at the left side of the picture, 22 "base-figures" of layer 2 forming an "M", 139 of layer 3 forming the stylized letters "LICHT-MUSIK", and 241 of layer 4 forming five rows of ornaments. The pictures are colored according to the following law: boarder lines are black. Where "base-figures" in odd numbers cover the area, the color is white; in even numbers, it is black.

An "inner-game" is a changing "inner-picture." In it, "base-figures" can appear (rhythmically), change color and be extinguished. On a later stage of development they can also move (subject to certain laws) and change size and form. "Inner-games" are produced with the help of "luminous organs." These are electronic devices with a veiwing screen.

Artistically formed "inner-pictures" and "-games" are to be works of a new art, a static and dynamic visual music. Three-dimensional "inner-pictures" can be produced physically, but cannot be "looked-into."

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