Formulae Chart and Summary of Main Points

Text: Modern Intermediate Algebra - Nichols et al

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CHAPTER 6: <u>Relations and Functions</u>

<u>Relation:</u> any set of ordered pairs.

Domain: the set of all first elements of the ordered pairs of a relation.

Range: the set of all second elements of the ordered pairs of a relation.

<u>Methods of stating relations:</u>

a) List or roster method: $R = \{1, 4, 9, \ldots\}$

b) Rule method: $R = \{$the squares of the natural numbers$\}$

c) Graph method: 

Inverse of a Relation (i) obtained by interchanging the elements in each of the ordered pairs of the original relation.

(ii) obtained by interchanging the variables in the open sentence defining the relation.

Note: The graph of a relation and its inverse are "mirror reflections" through the line defined by $y = x$

- If R is the relation, the R' is the inverse of that relation.

<u>Function:</u> a unique relation in which each domain element is mapped onto one and only one range element.

Vertical line test: A relation is a function if and only if no vertical line intersects the graph of the relation in more than one point.

<u>Types of Functional Notation:</u>

a) Set builder: $g = \{(x,y) \mid y = 2x + 1,\ x,y \in R\}$

b) Mapping: $g : x \longrightarrow 2x + 1,\ x \in R$

c) Image: $g(x) = 2x+1,\ x \in R$

In general, the function f pairs (c) from its domain and (f(c)) from its range to form (c,f(c)), an element of f.

<u>Special Functions:</u>

a) Constant function: eg. $y=3$.

- the range of f contains exactly one element.

b) Identity function: eg. $y=x$.

- each element of the function is of the form (x,x).
- (note that this is a special one - to - one function)

c) One - to - one function: eg. $y=x+3$

- each element in the domain is paired with a unique range element.

d) Many - to - one function: eg. $Y = |x|$

-each range element is associated with two or more domain elements.

e) Greatest Integer Function: eg $(x, [x])$

- each element of the function is of the form $(x, [x])$, where $[x]$ is the greatest integer not greater than x.
- (Sometimes referred to as the step-function)

f) Linear function: eg $y = 2x + 1$

- all functions of this type must be written in the form $y=mx+b$
- note that $m \neq 0$

<u>Composition of Functions:</u>

- a function may be defined in terms of other functions.

<u>Def'n</u>: Given a pair of functions f and g so that the range of f is the domain of g, the function j is the function composed of g with f if and only if $j(x) = g(f(x))$ for each x in the domain of f.

<u>Inverse Functions:</u> If f is a given function, then f^{-1} is the notation used for the inverse of that function.

NOTE that the methods used to obtain <u>inverse functions</u> are the same as those used to form <u>inverse relations</u>.

<u>Direct Proportion:</u>

Def'n: For every real number $c \neq 0$, $\{(x,y) \mid y = cx \text{ and } x \in D\}$ is a direct proportion function, where D is the domain of the function. (c) is called the constant of proportionality or constant of variation.

<u>Inverse Proportion:</u>

Def'n: For every real number $C \neq 0$, $\{(x,y) \mid y = \frac{c}{x} \text{ and } x \in D\}$ is an inverse proportion function, where D is the domain of the function. (c) is called the constant of proportionality.

CHAPTER 7: Quadratic Functions

Def'n: A function Q is a quadratic function iff $Q = \{(x, ax^2 + bx + c)\}$ where a, b, and c are real numbers, $a \neq 0$

The graph of a quadratic function is called a parabola.

Axis of Symmetry: the line about which the curve opens and which divides the graph into two equal parts.

Vertex: the point of intersection between the curve and the axis of symmetry.

Role of (a) in $y = ax^2 + bx + c$

1. The sign of (a) determines the direction of the curve (up of down)
2. The magnitude of (a) determines the size of the curve.

Role of (c) in $y = ax^2 + bx + c$

1. The y-intercept of the curve is (c)

Role of (b) in $y = ax^2 + bx + c$

1. (b) involves a horizontal shift of the curve from standard position.

Types of Quadratic Functions:

A. Form $y = ax^2 + p$

1) The graph formed will always be the graph of $y = ax^2$ shifted $|p|$ units vertically.
2) If $p > 0$, the parabola is shifted upward from standard position. If $p < 0$, the parabola is shifted downward from standard position.

B. Form $y = a(x-k)^2$

1) The graph of $y = a(x-k)^2$ is the graph of $y = ax^2$ shifted $|k|$ units to the right if $k > 0$, and to the left if $k < 0$.
2) The vertex of the graph of $y = a(x-k)^2$ has coordinates (K,0)
3) The equation of the axis of symmetry is $x = k$.

C. Form $y = a(x-k)^2 + p$ (The general Case)

1) The graph of $y = a(x-k)^2 + p$ is the graph of $y = a(x-k)^2$ shifted $|p|$ vertically; that is, if $p > 0$ or $p < 0$.
2) The graph is a parabola.
3) The coordinates of the vertex are (k,p)
4) The equation of the axis of symmetry is $x = k$.

Techniques for determining the graph of any quadratic function:

1) Completing the square.
2) Solving the general quadratic equations $y = ax^2 + bx + c$ to obtain:

Axis of Symmetry: $x = \frac{-b}{2a}$

Range:
- Maximum: $y \leq \frac{4ac - b^2}{4a}$, iff $a < 0$.
- Minimum: $y \geq \frac{4ac - b^2}{4a}$, iff $a > 0$.

Vertex: $\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$

Note applications of quadratic theory in problem solving.

CHAPTER 8: Quadratic Equations and Inequalities

Def'n: A quadratic equation is an equation of the form $ax^2 + bx + c = 0$ where a, b and c are real numbers and $a \neq 0$.

Three cases are considered:

1) $b = 0$ ....$ax^2 + c = 0$
2) $c = 0$ ....$ax^2 + bx = 0$
3) $b \neq 0$, $c \neq 0$ ....$ax^2 + bx + c = 0$ (General Case)

Theorem 1: $x^2 \geq 0$ Any real number multiplied by itself yields a product which is a non-negative real number.

Theorem 2: $x^2 = k$ iff $x = \sqrt{k}$ or $x = -\sqrt{k}$, for each $k \geq 0$

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Characteristics:

1. Roots are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$
2. Axis of Symmetry: $x = \frac{-b}{2a}$
3. Sum of the roots: $\frac{-b}{a}$
4. Product of the roots: c/a
5. Discriminant test (D) : note that $D = b^2 - 4ac = \Delta$
   a) If $D > 0$, then there are two distenct roots.
   b) If $D = 0$, then there are two coincident roots.
   c) If $D < 0$, then there are no real roots.

NOTE that the roots are to the quadratic equation as the x-intercepts are to the quadratic function.

Equations with the Quadratic Pattern:

1. Fractional equations
2. Radical equations - note technique for solving equations containing one or more radicals.

Quadratic Inequalities:

Def'n: A quadratic inequality is an inequality of the form $ax^2 + bx + c > 0$ or $ax^2 + bx + c < 0$ where $a \neq 0$.

Theorem 1: $ab>0 \Longleftrightarrow [a>0 \text{ and } b>0]$ or $[a<0 \text{ and } b<0]$

Theorem 2: $ab<0 \Longleftrightarrow [a>0 \text{ and } b<0]$ or $[a<0 \text{ and } b>0]$

NOTE examples on pages 284 - 286 in the text.

CHAPTER 9: Complex Number System

Need for Complex Numbers:

1. Studies in electricity - alternating current theory.
2. Studies in magnetism - Oersted.
3. Problems in heat conduction and electrostatics.
4. Problems in physics; hydrodynamics; etc.

Def'n: Each ordered pair of real numbers (a,b) is a complex number, (Z). Note the geometric derivation there of.

Properties of Complex Numbers:

[NOTE that $Z_1 = (a,b)$ and $Z_2 = (c,d)$ and $Z_3 = (e,f)$]

1. Property of Equality: $Z_1 = Z_2$ iff $a = c$ and $b = d$.
2. Addition of complex numbers: $Z_1 \oplus Z_2 = (a+c,\ b+d)$
   a) Closure Property: $Z_1 \oplus Z_2$ is a unique complex number, for all complex numbers Z.
   b) Commutative Property: $Z_1 \oplus Z_2 = Z_2 \oplus Z_1$
   c) Associative Property: $Z_1 \oplus (Z_2 \oplus Z_3) = (Z_1 \oplus Z_2) \oplus Z_3$
   d) Additive Identity Element: (0,0)
   e) Additive Inverse Element (-a, -b)
3. Subtraction of Complex numbers: $Z_1 \ominus Z_2 = (a-c,\ b-d)$
4. Multiplication of complex numbers: $Z_1 \odot Z_2 = (ac - bd,\ ad + bc)$
   a) Closure Property: $Z_1 \odot Z_2$ is a unique complex number, for all complex numbers Z.
   b) Commutative Property: $Z_1 \odot Z_2 = Z_2 \odot Z_1$
   c) Associative Property: $Z_1 \odot (Z_2 \odot Z_3) = (Z_1 \odot Z_2) \odot Z_3$
   d) Multiplicative Identity Element: (1,0)
   e) Multiplicative Inverse Element: $\frac{1}{Z} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$
   f) Distributive Property: $Z_1 \odot (Z_2 \oplus Z_3) = (Z_1 \odot Z_2) \oplus (Z_1 \odot Z_3)$
5. Division of Complex Numbers: $Z_1 \oplus Z_2 = Z_1 \odot \frac{1}{Z_2}$

   NOTE that $Z_2 \neq (0,0)$

Any number system composed of a set (T) of elements and two operations (+) and (X) for these elements is called a **field**. The complex number system can be considered a field, and it contains all the number properties of a field.

Some complex numbers behave like real numbers. For example, $(2,0) \leftrightarrow 2$. In general, the complex number (a,0) will always behave like the real number (a).

Standard Form: an alternate format for writing and working with complex numbers.

$Z = (a,b) = a + bi$ Standard form

Note that all previous statements concerning properties of complex numbers can now be converted to standard format.

Absolute Value of a Complex Number:

$|Z| = \sqrt{a^2 + b^2}$

NOTE the geometric derivation thereof.

The Conjugate of a Complex Number:

For each complex number Z, if $Z = (x,y)$ or $x + yi$, then the conjugate of Z, denoted by $\bar{Z}$ is $(x, -y)$ or $x - yi$.

The product of a complex number and its conjugate is a real number.

NOTE the geometric interpretation of addition, subtraction and absolute value re: Complex numbers, Pages 316 - 318 in the text.

Square Roots which are Complex Numbers:

For each real number $a<0$, $i\sqrt{-a}$ is a square root of (a), also $-i\sqrt{-a}$ is a square root of (a)

- Agreement: For each $x>0$, $\sqrt{-x} = i\sqrt{x}$ and $-\sqrt{-x} = -i\sqrt{x}$

Quadratic Equations with Complex Solutions:

Recall the Discriminant test and the Quadratic Formula previously studied. Use these two facts in solving quadratic equations with complex roots.

NOTE that in further studies in mathematics you will find that each number (real or complex) has n different n^{th} roots among the complex numbers (for each natural number $n \geq 2$)

CHAPTER 10: Solution Sets of Systems

In this chapter we are interested, primarily, in the general linear equation $Ax+By+C=0$

Types of Equation Systems:

1) Independent System:

-here there is exactly one ordered pair in the solution set.

Ex. $\{x - y - 1 = 0 \cap 2x + y + 4 = 0\} = \{(-1,-2)\}$

2) Inconsistent System:

-here the empty set comprises the solution set.

Ex. $\{y = x - 1 \cap y = x + 2\} = \emptyset$

-note the identical slopes. Hence parallel lines.

3) Dependent Systems:

- here all real number ordered pairs which satisfy one equation will satisfy the second equation. The two equations are equivalent. (i.e. the same graph)

Ex. $\{x - y = -1 \cap 2x - 2y = -2\} = \{x - y = -1\}$

Def'n: Two systems of open sentences are equivalent iff they have the same solution set. (Equivalent Systems)

• **Methods of Solving Systems of Equations:**

1) Graphic method
2) Comparison method
3) Substitution method
4) Addition - Subtraction method

NOTE examples of above techniques on pages 333 - 342 in the text.

The Solution set of a General System:

Given: $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$

Therefore: $x = \dfrac{b_1c_2 - b_2c_1}{a_2b_1 - a_1b_2}$ $\qquad y = \dfrac{a_2c_1 - a_1c_2}{a_2b_1 - a_1b_2}$

Determinants:

Def'n: For all numbers a,b,c and d, the determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$

Determinants may be used to solve systems of equations as follows:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

We have considered only systems comprised of two linear equations so far. In advanced courses you will study systems comprised of more than two linear equations.

Systems of equations may also be comprised of:

a) One linear and one second- degree equation
  - See pages 349 - 354 in the text

b) Two second- degree equations
  - See pages 354 - 360 in the text

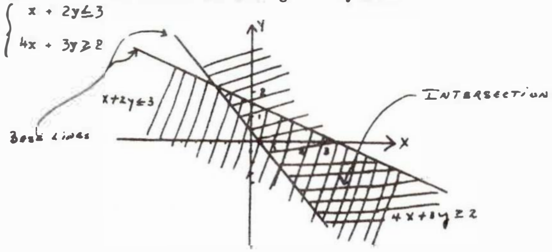
Also note techniques for solving such systems.

• Def'n: A conic is the intersection of a plane with a right circular conical surface or a right circular cylindrical surface.

Systems of Inequalities:

The graphing technique is the most useful method for solving such systems.

Consider the following system: $\begin{cases} x + 2y \le 3 \\ 4x + 3y \ge 2 \end{cases}$



NOTE the importance of "dashed" base lines for $<$ or $>$ situations.

Study examples on pages 362 - 365 in the text.

Systems involving Absolute Value:

In graphing inequalities involving absolute value, the basic property used is:

$\begin{cases} \text{For every } a>0, \text{ if } |x|<a, \text{ then } -a<x<a \\ \text{For every } a<0, \text{ if } |x|>a, \text{ then } x<-a \text{ or } x>a \end{cases}$

NOTE examples on pages 366 - 368 in the text.

CHAPTER 11: Logarithmic Functions

Def'n: $\log_{10}(x) = y$ iff $10^y = x$, $[x>0]$

The domain of the logarithmic function is the set of positive real numbers; the range of the logarithmic function is the set of all real numbers.

NOTE: In order to understand the association between logarithmic and exponential format, memorize the following:

$10^2 = 100 \iff \log_{10} 100 = 2$

exponential format — logarithmic format

A logarithm is merely an exponent for a power with base ten. Therefore, logarithms obey the laws of exponents.

A logarithm consists of two parts: characteristic and mantissa.

Eg. $28 = 10^{1.4472}$ or $\log_{10} 28 = 1.4472$

characteristic — mantissa

Recall that one may use scientific notation to obtain the charateristic of a logarithm.

•Theorems about Logarithms:

1. Logarithm of a Product: $\log_a MN = \log_a M + \log_a N$
2. Logarithm of a Quotient: $\log_a \frac{M}{N} = \log_a M - \log_a N$
3. Logarithm of a Power: $\log_a M^n = n \log_a M$
4. Logarithm of a Root: $\log \sqrt[n]{M} = \frac{1}{n} \log_a M$

The above theorems are used in solutions to questions inv[illegible] combinations of products, quotients and powers.

Antilogarithm:

The procedure of calculating an antilogarithm is the reverse of the procedure used for finding logarithms.

Eg. Suppose $\log_{10} 28 = 1.4472$

Then Antilogarithm $1.4472 = 28$

The antilogarithm gives us the number whose logarithm we have just calculated.

Other logarithmic functions:

Any positive real number (except the number one) may be used as the base of a logarithmic function. In general, for each positive real number (a) [except 1], the base (a) logarithmic function is $\{(x,y) \quad x = a^y\}$ The previous theorem for logarithms still apply.

Change of Base

Note carefully the examples on page 394 in the text.

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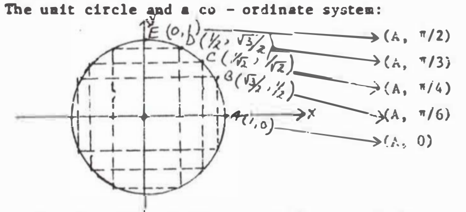
Chapter 12: Trigonometric functions:

1) Unit circle: $x^2 + y^2 = r^2 = 1$
   - Clockwise movement: Negative direction ↓
   - Counter - clockwise movement: Positive direction ↑
2) The unit circle and a co - ordinate system:



   - note the geometric figures within this circle in order to memorize the co - ordin of points in the circumference
3) Wrapping function: $W(\theta) = (x,y)$
4) Periodic functions: $f(x) = f(x + p)$, $p \neq 0$. This definition must hold for every x in the domain. The number (p) is called the period of f.
5) Cosine Function: $\{(\theta, x)\}$ for which $W(\theta) = (x,y)$.
6) Sine Function: $\{(\theta, y)\}$ for which $W(\theta) = (x,y)$.
7) Tangent Function: $\{(\theta, y/x)\}$ for which $W(\theta) = (x,y)$. $[x \neq 0]$
8) Basic formula: $\cos(\theta_1 - \theta_2) = \cos\theta_1 \times \cos\theta_2 + \sin\theta_1 \times \sin\theta_2$
9) Related formulae: $\cos(\theta_1 + \theta_2) = \cos\theta_1 \times \cos\theta_2 - \sin\theta_1 \times \sin\theta_2$

   $\cos^2\theta + \sin^2\theta = 1$

   $\tan\theta = \dfrac{\sin\theta_1}{\cos\theta_1}$, $[\cos\theta_1 \neq 0]$

   $\sin(\theta_1 - \theta_2) = \sin\theta_1 \times \cos\theta_2 - \sin\theta_2 \times \cos\theta_1$

   $\sin(\theta_1 + \theta_2) = \sin\theta_1 \times \cos\theta_2 + \sin\theta_2 \times \cos\theta_1$

   $\cos(-\theta_1) = \cos(\theta_1)$

   $\cos(\pi/2 - \theta_1) = \sin(\theta_1)$

   $\sin(-\theta_1) = -\sin(\theta_1)$

   $\sin(\pi/2 - \theta_1) = \cos(\theta_1)$
10) Characteristics of the sine function:
    1) Periodicity is 2π.
    2) Domain: $\{x/x \in R\}$
    3) Range: $\{y/-1 \leq y \leq 1, y \in R\}$
11) Characteristics of the cosine function:
    1) Periodicity is 2π.
    2) Domain: $\{\theta/\theta \in R\}$
    3) Range: $\{x/-1 \leq x \leq 1, x \in R\}$
12) Characteristics of the tangent function:
    1) Periodicity is π.
    2) Domain: $\{x/x \in R, x \neq 0\}$
    3) Range: $\{y/y \in R\}$

Chapter 13: Applications of trigonometric functions.

1) For each path (A, θ) with terminal point P, a degree - measure of $\angle AOP$ is $\frac{180}{\pi} \times \theta$
2) For all real numbers c and θ, if $c = \frac{180}{\pi} \times \theta$, then:

   $\cos(c^\circ) = \cos(\theta)$

   $\sin(c^\circ) = \sin(\theta)$

   $\tan(c^\circ) = \tan(\theta)$ $[c \neq 90 \pm 180n]$

Note that the Basic and related formulae, previously stated, can now be converted from radian to degree measurement.

3) Similar triangles: - corresponding angles are congruent.
   - corresponding sides are proportional.

   $\sin(\theta) = \dfrac{\text{opposite}}{\text{hypotenuse}}$

   $\cos(\theta) = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

   $\tan(\theta) = \dfrac{\text{opposite}}{\text{adjacent}}$

   -For trigonometric ratios of all angles, $0 \leq \theta \leq 90$, Knott's mathematical tables may be used. Also page 601 of this text.
4) Characteristics of the $30^\circ - 60^\circ - 90^\circ$ triangle:
   1) the shorter leg is one - half the length of the hypotenuse.
   2) the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.
5) Characteristics of the $45^\circ - 45^\circ - 90^\circ$ triangle.
   1) both legs have the same length.
   2) the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.
6) Law of cosines:

   $a^2 = b^2 + c^2 - 2bc \cos\alpha$ -note applicability to triangles where α is acute, obtuse, or right.
7) Law of sines:

   $\dfrac{a}{\sin(\alpha)} = \dfrac{b}{\sin(\beta)} = \dfrac{c}{\sin(\gamma)}$ -note applicability to triangles where α, β, γ are acute, obtuse, or right.

Chapter 14: Sequences, Series and Limits.

1) Sequence - any arrangement of numbers in order. - finite : $\{a_1, a_2, a_3, \ldots a_n\}$
   - infinite: $\{a_1, a_2, \ldots, a_n, \ldots\}$
2) Term - each element of the sequence.
3) Series - the indicated sum of the terms of a sequence
   - finite: $\{a_1 + a_2 + a_3 + \ldots + a_n\}$
   - infinite: $\{a_1 + a_2 + \ldots + a_n + \ldots\}$
4) Arithmetic progression: a sequence exhibiting a constant difference between successive terms.

   $a_1, a_1 + d, a_1 + 2d, \ldots, a_1 + (n-1)d.$

   General term: $a_n = a_1 + (n-1)d.$
5) Arithmetic series: $S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + a_n$

   $S_n = \dfrac{n(a_1 + a_n)}{2}$ Also $S_n = \dfrac{n}{2}[2a_1 + (n-1)d]$

6) Geometric Progression: a sequence exhibiting a constant ratio between successive terms: $a_1, a_1r, a_1r^2, \ldots a_1r^{n-1}$

7) Geometric Series: $S_n = a_1r + a_1r^2 + a_1r^3 + \ldots + a_1r^n$

$S_n = a_1\frac{(r^n - 1)}{r - 1}$ Also: $S_n = \frac{a_n r - a_1}{r - 1}$

8) Summation Notation: $\text{(stop)} \sum_{k=1\,\text{(start)}}^{n} a_k$ (substitute) $= a_1 + a_2 + a_3 + \ldots + a_n$.

9) Infinite Sequences and Limits:

- convergent sequence: tends towards a limit.
- divergent sequence: does not tend towards a limit.

10) Limit Properties:

1. $\lim_{n \to \infty} (KA_n) = K \times \lim_{n \to \infty} A_n$
2. $\lim_{n \to \infty} (a_n + b_n) = \lim_{n \to \infty} a_n + \lim_{n \to \infty} b_n$
3. $\lim_{n \to \infty} (a_n - b_n) = \lim_{n \to \infty} A_n - \lim_{n \to \infty} b_n$
4. $\lim_{n \to \infty} (A_n \times b_n) = \lim_{n \to \infty} A_n \times \lim_{n \to \infty} b_n$
5. $\lim_{n \to \infty} (A_n / b_n) = \frac{\lim_{n \to \infty} A_n}{\lim_{n \to \infty} b_n}$ $\left[\lim_{n \to \infty} b_n \neq 0\right]$
6. $\lim_{n \to \infty} c = c$
7. $\lim_{n \to \infty} 1/n = 0$ Note: Nos. 6 and 7 are very important.

11. Sum of an infinite geometric series:
- defined to be the limit of the sequence of its partial sums.

$S = \frac{a_1}{1 - r}$ Note: $\lim_{n \to \infty} r^n$ exists only for $-1 < r \leq 1$

$\lim_{n \to \infty} r^n = 0$ only for $|r| < 1$

Chapter 15: Permutations, Combinations and the Binomial Theorem.

1) Permutation: an arrangement, or ordering, of the elements of a set.
Types: linear and circular.

2) Fundamental Counting Principle: If an operation can be performed in K_1 ways, and after it is performed, a second operation can be performed in K_2 ways, etc., then collectively the n operations can be performed in $(K_1 \times K_2 \times K_3 \times \ldots\ldots K_n)$ ways.

3) Factorial: $n! = n(n - 1)(n - 2) \ldots\ldots 3 \times 2 \times 1$

4) Permutation formula:

$_nP_k = \frac{n!}{(n - k)!}$ Note: $0! = 1;\ 1! = 1$

-Distinguishable linear permutations with "like" elements: $P = \frac{n!}{k_1! \times k_2! \ldots k_r!}$

- Distinguishable circular permutations: $P = (n - 1)!$

Note: For "keychain" situations, $P = \frac{(n - 1)!}{2}$

5) Pascal's Triangle: - displays the coefficients of terms in a binomial expansion.
- displays the symmetry present in a binomial expansion,

here $\binom{n}{k} = \binom{n}{n - k}$

$(a + b)^0 \longrightarrow$ ①

$(a + b)^1 \longrightarrow$ ①a + ①b

$(a + b)^2 \longrightarrow$ ①a^2 + ②ab + ①b^2

$(a + b)^3 \longrightarrow$ ①a^3 + ③a^2b + ③ab^2 + ①b^3

6) Combination formula: $\binom{n}{k} = \frac{n!}{k! \times (n - k)!}$

-"order" is not important here.

7) The Binomial Theorem:

$(x + y)^n = \binom{n}{n,0} a^n + \binom{n}{n - 1,\ 1} a^{n-1}b + \ldots + \binom{n}{n - r,\ r} a^{n-r}b^r + \ldots + \binom{n}{0,\ n} b^n$

or $(x + y)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1}b + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + \binom{n}{n} b^n$

Note: $T_{r+1} = \binom{n}{r} a^{n-r}b^r$

Chapter 16: The Probability Function:

1) Sample space: the set of all possible outcomes of an experiment.

2) Event: any subset of a sample space of an experiment.

3) Definition of classical probability: $P(E) = \frac{n(E)}{n(S)} = \frac{\text{cases favorable}}{\text{cases possible}}$

Note: $P(E) = 0$ iff $E = \emptyset$ --- impossible event.
$P(E) = 1$ iff E is the event certain.

4) The Addition Theorem:

Case 1: $P(E \cup F) = P(E) + P(F)$ iff $E \cap F = \emptyset$ (mutually exclusive events)

Case 2: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ iff $E \cap F \neq \emptyset$.

5) The Multiplication Theorem:

Case 1: $P(E \cap F) = P(E) \times P(F/E)$ .... here E and F are dependent events.

Case 2: $P(E \cap F) = P(E) \times P(F)$ .... here E and F are independent events.

Case 3: $P(E \cap F) = 0$ .... here E and F are disjoint events.

Chapter 17: The Polynomial Function.

1) Definition: A polynomial function is a set of ordered pairs (x, f(x)), where $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$

2) Divisibility: The polynomial P(x) is divisible by the polynomial $D(x) \neq 0$ iff $P(x) \div D(x)$ is a polynomial.

3) The Division Algorithm: $P(x) = Q(x) \times D(x) + R(x)$.

a) If P(x) is divisible by D(x), Then $R(x) = 0$.

b) P(x) is not divisible by D(x) if $R(x) \neq 0$.

Note: the division process is completed when the degree of R(x) becomes less than the degree of D(x) or when $R(x) = 0$.

4) Remainder Theorem: Given the polynomials P(x), Q(x), and (x - a), if $P(x) = Q(x) \cdot (x - a) + R$ for some number R, then $R = P(a)$.

5) Synthetic Division: note the process and characteristics thereof.
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6) Zero of a polynomial: The number r is called a zero of a polynomial P(x) iff $P(r) = 0$.

7) Factor Theorem: The binomial $x - r$ is a factor of P(x) iff $P(r) = 0$.

8) **Zeros of Integral Polynomials:** An integral polynomial may have the following:

a) a complex number for a zero. eg. $x^2 + 1$

b) an irrational number for a zero: eg. $x^2 - 2$

c) a non - integral rational number for a zero. eg. $2x - 1$

9) <u>Integral Zero Theorem:</u> If an integer r is a zero of an integral polynomial P(x), then r is a factor of the constant term of P(x).

10) <u>Rational Zero theorem:</u> If a/b $\left[b \neq 0\right.$, (a) and (b) relatively prime integers$\left.\right]$ is a zero of the integral polynomial, P(x), Then (a) is a divisor of A_n and (b) is a divisor of A_0.

11) <u>Fundamental Theorem of Algebra:</u> If P(x) is a polynomial of degree greater than 0 over the complex numbers, than there is a complex number (r) for which P(r) = 0.

12) <u>Unique Factorization Theorem:</u> Every polynomial P(x) of degree $n \geq 1$ over the complex numbers can be factored uniquely into n first - degree factors, not all of which are necessarily distinct, and a constant factor which is the coefficient of the highest degree term of P(x).

- note the phrase <u>"Sum of multiplicities."</u>

13) Graphing Polynomial Functions:

- note this general review of graphs. Observe how the theory of <u>polynomial zeros</u> can assist here.

14) Complex Zeros of Real Polynomials: (Conjugate Zero Theorem)

If a + bi is a zero of a real polynomial P(x), then a - bi is also a zero of P(X).

15) Descartes' Rule of Signs: The sum of multiplicities of <u>positive real</u> zeros of a real polynomial P(x) is at most equal to the number of changes in sign in P(x). If this sum of multiplicities is less than the number of changes in sign, then it differs from it by an even number.

The sum of multiplicities of negative real zeros of P(x) is at most equal to the number of changes in sign in P(-x). Again, if this sum of multiplicities is less than the number of changes in sign, then it differs from it by an even number.

<u>Series and Mathematical Induction ..... Vance booklet</u>

1) Mathematical Induction: Part (a): Verification for a specific value.

Part (b): Induction property: If the statement is true for n = k, then we wish to prove it true for the next larger value of n, say k + 1.

2) The Circle: Standard Equation: $(x - h)^2 + (y - k)^2 = r^2$, where (h,k) is the center, and r is the radius.

General Equation: $x^2 + y^2 + Dx + Ey + F = 0$ where $\left(-\frac{D}{2}, -\frac{E}{2}\right)$ is the center and $\frac{1}{2}\sqrt{D^2+E^2-4F}$ is the radius.

3) <u>The Parabola:</u>

Equations:

$y^2 = 4px$, where the focus (p,o) is a point in the x-axis; directrix x = -p

$x^2 = 4py$, where the focus (o,p) is a point on the y-axis; directrix y = -p

Note: The graphs of both equations above have <u>vertex</u> at the origin,

Latus rectum = $|4p|$

Eccentricity (e) = 1

4) <u>The Ellipse:</u> Basic Equation is:

$$\frac{x^2}{(\text{length of semi - axis on x - axis})^2} + \frac{y^2}{(\text{length of semi - axis on y - axis})^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

(a) - length of the semi - major axis

(b) - length of the semi - minor axis

a > b - by definition

Latus rectum: $\frac{2b^2}{a}$

(c) - distance of each focus from the origin

<u>Note that:</u> $a^2 = b^2 + c^2$

Eccentricity (e) = c/a , (0 < e < 1)

5) <u>The Hyperbola:</u> Basic Equation

$$\frac{x^2}{(\text{length of semi - axis on x - axis})^2} - \frac{y^2}{(\text{length of semi - axis on y - axis})^2} = \pm 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$$

(a) : length of semi - transverse axis.

(b) : length of semi - conjugate axis.

(c) : distance of each focus from the origin.

Latus rectum: $\frac{2b^2}{a}$

Note that: $c^2 = a^2 + b^2$

Eccentricity (e) = $\frac{c}{a}$, (e>1)

Asymptote equation: $y = \pm\left(\frac{b}{a}\right)x$

<u>Chapter 18: Introduction to Vectors</u>

1) Magnitude: The magnitude of $\overrightarrow{XY} = |\overrightarrow{XY}|$ For any vector $\overrightarrow{AB}$ ← Terminal point, ← Initial point

2) Equivalent vectors: $\vec{A} \cong \vec{B}$ iff $|\vec{A}| = |\vec{B}|$ and $\vec{A}$ and $\vec{B}$ have the same direction.

3) Standard position: a vector whose initial point is the <u>origin.</u>

4) Rectangular form of a vector $\overrightarrow{AB}$ $[x_2 - x_1, \; y_2 - y_1]$ (x component, y component)

5) Zero Vector: Any vector whose initial point is the same as its terminal point.

6) Polar form of a vector $\overrightarrow{AB}$: $[r, u^\circ]$

7) Addition of vectors: $\vec{X} + \vec{Y} = [a + c, b + d]$, $\vec{X} = [a,b]$ and $\vec{Y} = [c,d]$

8) Multiplication of a scalar by a vector: $(s)[a,b] = [s(a), s(b)]$

9) Inner Product of Vectors: $|\vec{X}| \times |\vec{Y}| \times \cos(u^\circ)$. $= \vec{X} \cdot \vec{Y}$

10) Perpendicularity of vectors: $\vec{X} \perp \vec{Y}$ iff $\vec{X} \cdot \vec{Y} = 0$

Mr. Treslan wishes teachers to use the Formulae Chart as they see fit and he would welcome any criticism concerning deletions and/or additions.