THE ALBERTA
TEACHERS'
ASSOCIATION
MATHEMATICS
COUNCIL

delta-k

Volume XIV, Number 4, May 1975

Letters to the Editor

Dear Sir,

I am a new member of your council and I enjoyed the annual conference very much, but I would suggest that there could have been a "hot seat" type of approach to Dr. Eicholz. Both Edmonton school boards seem to be going away from his texts, and it would have been interesting to have heard Dr. Eicholz's reaction. Dr. Eicholz is very articulate, and would probably have welcomed the opportunity to explain his text in the present form.

The November issue of *Delta-K* gave a great deal of food for thought.

The guide for evaluation of texts is interesting, but I wonder if it is not rather dated. I have taught mathematics at most grade levels and feel that our present emphasis is on the elite who wish to major in mathematics at universtity.

I have seen all too many students who have an excellent grasp of set theory, yet who do not have the computational skills necessary for the needs of everyday life. Knowing that 7 x 8 is seven sets of eight is useless if the students believe that the product is 53.

We need priorities, but this evaluation does not even begin to address itself to the problems facing the discipline of mathematics.

First, texts must be written at the students' reading level. It also seems logical that very heavy emphasis be placed on whole number computation. A knowledge of measurement is also vital.

We must have a system of individualizing mathematics so that aspects such as problem solving are given to the student who likes, or needs, this kind of mental exercise. The original purpose of problem solving was to give a practical application to drill, but, like Frankenstein's monster, it seems to have developed a life independent of reason. You solve problems because they are written in the text; the problems are written in the text because we expect to have problems written in the text. Wow!

IN THIS ISSUE

It would be equally fruitful to critically examine the study of bases, sets, "logical thinking," irrationals, and such.

What is the answer? Modern thought seems to indicate that individualization is the way to go. I think the readership of *Delta-K* would welcome a column in each issue devoted to the practical individualization of mathematics.

Ronald A. MacGregor
Teacher, St. Philip School, Edmonton

Dear Sir,

I am writing to comment on the latest issue of *Delta-K* (Volume XIV, Number 3, February 1975).

I enjoyed this issue of *Delta-K*, as I have the ones in the past, and was very interested in the Metric Articles and Information Sheet that appeared. However, I was concerned about the spelling of "metre" (er instead of re). I realize there is some controversy in the United States regarding the spelling of metre and litre, but in Canada, it is very clear the "re" spelling is preferred (see National Standard of Canada CAN-3-001-01-73 or CSA Z234.2-1973). I was surprised to see the "er" spelling in an article written by Dr. S.A. Lindstedt, because of his use of the "re" spelling in his Metric Workshop.

Perhaps the "er" spelling was a typographical error and not intended to suggest a change in spelling. If this is the case, I think a statement to this effect should be made in the next edition of *Delta-K* so that confusion does not arise.

The Information Sheet was very well done and I'm sure will be used by teachers. It is unfortunate the error occurred on such a valuable article.

A second point, which is not as serious as the first but probably bears mentioning, relates to the use of the script "ℓ" for litre. The "ℓ" should be used when litre is the unit (i.e. 10 ℓ) but is not necessary if the unit is a multiple or sub-multiple of litre (i.e. 10 ml or 5 kl). Because the script "ℓ" cannot easily be typed or printed, its use should be restricted only to instances where confusion might result.

In conclusion, I hope you will accept these as positive comments because I did enjoy both the article and information sheet, and feel they will be appreciated by professional teaching personnel.

Leonard J. Hall
Metric Coordinator, Calgary Board of Education

EDITORIAL NOTE: *Over the years it has been the practice of the publications department, ATA, to use the Webster dictionary. Since the issue of* Delta-K *which Mr. Hall mentions, we have received an official paper from the Metric Commission of Canada in which we have noted its preference; henceforth, in our publications, "er" appearing in things metric will, indeed, be a typographical error. (The information sheet has been corrected and reprinted and is attached to this publication.)*

1975 Alberta High School Prize Exams - Winners and Solutions

Following is a list of the winners and their prizes in the 1975 Alberta High School Prize Examination in Mathematics. Congratulations to the finalists and our commendation to all who participated.

PROVINCIAL PRIZES

Student	Amount	School
[1]Terry Wu	$400	Lindsay Thurber High School, Red Deer
[2]Norman C. Hutchinson	$400	Brooks High School, Brooks
[3]Donald J. Reble	$200	Concordia College, Edmonton
[4]William R. Graham	$200	Harry Ainlay High School, Edmonton

SPECIAL PRIZES

[5]Malcolm W. Kern	$100	W.R. Myers High School, Taber
[6]Keith Fenske	$100	Harry Ainlay High School, Edmonton

DISTRICT PRIZES ($50 EACH)

District Number	Student/School
1	(none awarded)
2	Shawn R. Golby, Lorne Jenken High School, Barrhead
3	Larry I. Tennis, Wetaskiwin High School, Wetaskiwin
4	Leo B. Hartman, Camrose Composite High School, Camrose
5	Daniel A. Boulet, Olds High School, Olds
6	Emil L. Hallin, Crescent Heights High School, Medicine Hat
7 (1)	Raymond Kwan, Bonnie Doon High School, Edmonton
(2)	Masao Fujinaga, Harry Ainlay High School, Edmonton
8 (1)	Herman J. Ruitenbeek, Lord Beaverbrook High School, Calgary
(2)	John Soong, St. Mary's Community High School, Calgary

[1]Canadian Mathematical Congress Scholarship

[2]The Nickle Foundation Scholarship

[3]Third place winner

[4]Fourth place winner

[5]Grade 12 student placing highest (below first 4)

[6]Grade 10/11 student placing highest (below first 4)

NATIONAL OLYMPIAD NOMINEES

	Name (grade)	School
1.	WU, Terry C.-Y. (12)	Lindsay Thurber High School, Red Deer
2.	REBLE, Donald J. (12)	Concordia College, Edmonton
3.	GRAHAM, William R. (12)	Harry Ainlay High School, Edmonton
4.	HUTCHINSON, Norman C. (12)	Brooks Composite High School, Brooks
5.	KERN, Malcolm W. (12)	W.R. Myers High School, Taber
6.	HARTMAN, Leo R. (12)	Camrose Composite High School, Camrose
7.	FENSKE, Keith (10)	Harry Ainlay High School, Edmonton
8.	KWAN, Raymond R.K. (12)	Bonnie Doon High School, Edmonton
9.	RUITENBEEK, Herman J. (12)	Lord Beaverbrook High School, Calgary
10.	GOLBY, Shawn R. (12)	Lorne Jenken High School, Barrhead
11.	SOONG, John H.-C. (12)	St. Mary's Community High, Calgary
12.	FUJINAGA, Masao (11)	Harry Ainlay High School, Edmonton
13.	LAKE, Robert M. (12)	Ross Sheppard High School, Edmonton
14.	CAMPBELL, Murray S. (12)	Ross Sheppard High School, Edmonton
15.	ROGERSON, Richard D. (12)	McNally Composite High School, Edmonton
16.	HOUSTON, Gary M. (12)	Lindsay Thurber High School, Red Deer

CANADIAN MATHEMATICAL CONGRESS

1975 Alberta High School Examination

PART I - *Answer Sheet*

To be filled in by the candidate.

PRINT: ____

Last Name First Name Initial

____

Candidate's Address Town/City

____

Name of School

ANSWERS:

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

To be completed by the Deparmtent of Mathematics, University of Alberta.

Points	Points Correct	Number Wrong
1 - 20 5	5 x =	1 x =
Totals	C =	W =

SCORE = C - W =

(DO ALL PROBLEMS. EACH PROBLEM IS WORTH FIVE POINTS.) *TIME: 60 Minutes*

1. The sum of three consecutive positive integers is always

   (a) odd (b) even (c) a perfect square

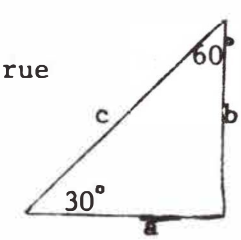
   (d) divisible by 3 (e) none of these

2. Which of the following holds true?

   (a) $\log_3 2 < \log_2 3$ (b) $\log_3 2 = \log_2 3$ (c) $\log_3 2 > \log_2 3$

   (d) $\log_3 2 = 1$ (e) $\log_2 3 = 1$

3. For the triangle as shown, which of the following is true



(a) $a = b$ (b) $b = 2a$ (c) $c = 2a$

(d) $c = 2b$ (e) none of the previous are true

4. "The operation $\circ$ is commutative" means

(a) $x \circ 1 = 1$ (b) $x \circ x = x$ (c) $x \circ y = y \circ x$

(d) $x \circ (y \circ z) = (x \circ y) \circ z$ (e) none of the previous

5. If $x = .1102$ (base 3), then x^2 is (base 3)

(a) .11021102 (b) .10102 (c) .01222111

(d) .010211 (e) none of the previous

6. Given a square inscribed in a circle inscribed in an equilateral triangle, if each side of the triangle has length 6, what is the length of each side of the square?

(a) $\frac{1}{2}\sqrt{6}$ (b) $\sqrt{3}$ (c) $\sqrt{6}$

(d) $2\sqrt{3}$ (e) none of these

7. Which is larger, the volume of a sphere of radius 1 or the volume of a right circular cone of height 1 and base radius 2?

(a) these volumes do not exist (b) they are equal

(c) the sphere (d) the cone

(e) none of the above are true

8. $\dfrac{a^4 + a^2b^2 + b^4}{a^2 + ab + b^2} =$

(a) $a^2 + ab + b^2$ (b) $a^2 + ab - b^2$, (c) $a^2 - ab - b^2$

(d) $a^2 - ab + b^2$ (e) none of the previous

9. Five years from now Bill will be twice as old as he was two years after he was half as old as he will be in one year from now. His age is

(a) 16 (b) 13 (c) 8

(d) 41 (e) cannot be determined

10. The number $1.1313\overset{\frown}{13}$ .... (the pair 13 is repeated ad infinitum) is the same as

(a) $\frac{112}{99}$ (b) $\frac{113}{99}$ (c) $\frac{100}{99}$

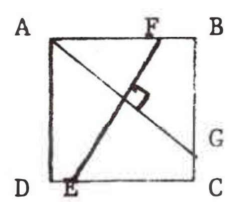
(d) $\frac{1131313}{1000000}$ (e) none of these

11. A jar contains 15 balls, of which 10 are red and 5 are black. If 3 balls are chosen at random the probability that all three will be red is

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{4}{9}$

(d) $\frac{8}{27}$ (e) none of these

12. The square ABCD has side length 1. Given that $AG \perp EF$ and that $\overline{EF} = \sqrt{\frac{7}{6}}$ $\overline{BF} = \sqrt{\frac{1}{7}}$ then $\overline{AG}$ is



(a) $\sqrt{\frac{7}{6}}$ (b) $\sqrt{7}$ (c) $\sqrt{\frac{6}{7}}$

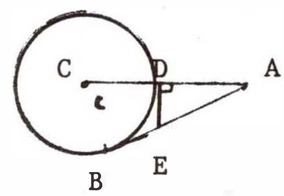
(d) $\sqrt{\frac{8}{7}}$ (e) none of these.

13. Which is the largest of 2^{4^3}, 2^{3^4}, 4^{2^3}, 3^{2^4}, 3^{4^2}?

(a) 2^{4^3} (b) 2^{3^4} (c) 4^{2^3}

(d) 3^{2^4} (e) 3^{4^2}

14. A circle of radius 3 has centre C. Let A be at a distance 5 from C and AB be a tangent to the circle. Let AC meet the circle at D and let E lie on AB with ED $\perp$ AC. Then the length $\overline{ED}$ is



(a) 1 (b) 2

(c) $\sqrt{2}$ (d) $\sqrt{3}$

(e) none of these

15. The system of equations $2x - 3y = 4$, $2y - 4x = 8$ has

(a) ten solutions (b) two solutions, (c) one solution

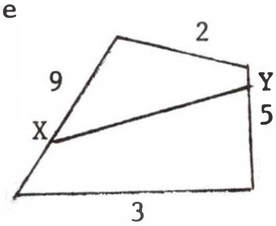
(d) no solutions (e) none of the previous

16. Suppose that a_1 is an integer not divisible by 3 and that $a_1^2 + a_2^2 + \ldots + a_n^2$ is divisible by 3, where $a_2, \ldots, a_n$ are integers. Then n is

(a) arbitrary (b) at least 3 (c) at most 2

(d) Always odd (e) none of these

17. According to the diagram, $\overline{XY}$ cannot equal



(a) 2 (b) 4

(c) 7 (d) 10 (e) any of these

18. Assume the earth is a perfect sphere and a wire is stretched tightly around the equator. The wire is lengthened one meter and then expanded uniformly so as to form a somewhat larger circle. The new radius will be approximately how many meters larger than the old one?

(a) .016 (b) .032 (c) .16

(d) .32 (e) 1

19. School X has 100 students and school Y has 50 students. These schools are to be replaced by a single school Z. If the students live in the immediate vicinities of their respective schools (X or Y), where should Z be placed so as to minimize the total distance travelled by all the students.

(a) at X (b) at Y (c) half way in between

(d) one third of the way from X to Y (e) at none of these

20. An urn contains 100 balls of different colours, 40 red, 27 green, 26 blue, and 7 white. What is the smallest number of balls that must be drawn without looking to guarantee that at least 15 balls have the same colour?

(a) 86 (b) 50 (c) 43

(d) 39 (e) none of these

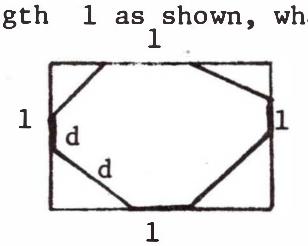
PART II

TIME: 110 Minutes

Instructions to Candidates

Attempt the problems in any order you wish. Partial credit is given for significant progress. Use any methods you like. Each problem is worth 20 points.

1. Given a regular octagon of side length d inscribed in a square of side length 1 as shown, what is d?



2. A prime number is a positive integer bigger than one, which is evenly divisible only by one and itself. Show that if a prime number is divided by 30, the remainder is prime.

3. Find all integers k so that

$$x^2 + k(x+1) + 7 = 0$$

has only integer solutions.

4. Given triangle A B C with M the midpoint of B C. Prove that $\overline{AB} + \overline{AC} > 2\,\overline{AM}$.

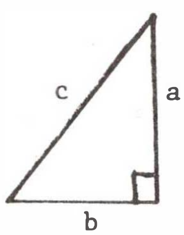
5. Find a positive number n, which is smaller than 25 such that the expression

$$(n - 1)^3 + n^3 + (n + 1)^3$$

is evenly divisible by 102.

6. Is the inequality $(99)^n + (100)^n > (101)^n$ always true, where n is a positive integer? Justify your answer.

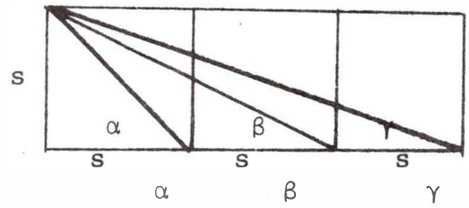
7. Given the right triangle as shown; and that a, b, c is an arithmetic progression. Show that a : b : c = 3 : 4 : 5.



8. Let a, b, c, d be four consecutive integers. Prove that $a \cdot b \cdot c \cdot d + 1$ is a perfect square.

9. A group of students write a set of k exams. Suppose that a_1 of the students failed at least 1 exam, a_2 failed at least 2 exams, $\cdot\cdot\cdot$, a_k failed exactly k exams. What was the total number of failed exams?

10. Given the three squares as shown, along with the angles α, β, γ. Show that $\alpha + \beta + \gamma = 90°$.



SOLUTIONS

1975 ALBERTA HIGH SCHOOL

PRIZE EXAMINATION IN MATHEMATICS

PART I: - KEY

1	2	3	4	5	6	7	8	9	10
D	A	D	C	C	C	B	D	E	A

11	12	13	14	15	16	17	18	19	20
E	A	B	E	D	B	D	C	A	B

PART II: - ANSWERS

1. By symmetry, the octagon divides the square into the octagon itself and four right triangles, each of side lengths x and hypoteneuse length d, such that $2x + d = 1$. By the Pythagorean theorem, $2x^2 = d^2$, or $x = \frac{\sqrt{2}}{2} d$, so that $\sqrt{2}d + d = 1$ or $d = \frac{1}{\sqrt{2} + 1} = \sqrt{2} - 1$.

2. Let p denote the prime and r the remainder. Then for some integer d we can write $p = 30d + r$. Note that $1 \leq r \leq 29$. All non-prime integers in this range are divisible by either 2 , 3 , or 5 . Thus, if r is not prime, then r is divisible by 2 , 3 or 5 (which also divides 30). The same number would then have to divide p . Since p is a prime, this cannot occur. Therefore, r is prime.

3. If r , s are the roots, then $r \cdot s = 7 + k$, $r + s = -k$. From this second equation it is clear that if k and r are integers, so is s. If r is a root of $r^2 + k(r+1) + 7 = 0$, then

$$k = -\frac{r^2 + 7}{r^2 + 1} = -(r-1) - \frac{8}{r+1} .$$

Therefore <u>both</u> k and r are integers if and only if $8 \div (r+1)$ is an integer,

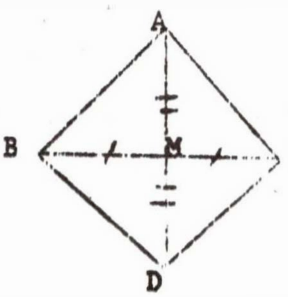
$$\frac{8}{r+1} = j , \quad \text{or} \quad r = -1 + \frac{8}{j} \quad \text{for}$$

some integer j . For r to be an integer, $j = \pm 1, \pm 2, \pm 4, \pm 8$. We can construct the following table.

j	1	-1	2	-2	4	-4	8	-8
$r = -1 + \frac{8}{j}$	7	-9	3	-5	1	-3	0	-2
$k = 1-r-j$	-7	11	-4	8	-4	8	-7	11

Therefore $k = -7, 11, -4, 8$ with corresponding root pairs (7,0), (-9,-2), (3,1), (-5,-3).

4.



In ΔABC , produce AM to D so that $\overline{AM} = \overline{MD}$. Then since $\overline{BM} = \overline{MC}$ and $\angle AMC = \angle BMD$, it must be that $\overline{AC} = \overline{BD}$.

Hence, $\overline{AB} + \overline{AC} = \overline{AB} + \overline{BD} > \overline{AM} + \overline{MD} = 2\overline{AM}$, since the shortest distance between two points is a straight line.

5. $n^3 - 3n^2 + 3n - 1 + n^3 + 3n^2 + 1 = 3n^3 + 6n = 3n(n^2 + 2)$

We require $3n(n^2 + 2) = 102j$ for some integer j, i.e.

$$\frac{n(n^2 + 2)}{2.17} = j$$

If n were odd, so would be $n^2 + 2$, and then so would be $n(n^2 + 2)$. Therefore n is <u>even</u>.

Also, since $0 < n < 25$, n cannot be an even multiple of 17, therefore $n^2 + 2$ is a multiple of 17, $n^2 + 2 = 17k$, or $n = \sqrt{17k-2}$.

We have:

k	1	2	3	4	5	6
n	$\sqrt{15}$	$\sqrt{32}$	$\sqrt{49}=7$	$\sqrt{66}$	$\sqrt{83}$	$\sqrt{100}=10$

So $n = 10$ will do ($n = 7$ is not even). If one checks all the cases $k = 1, 2, .., 37$ one finds that this is the only solution with $n \leq 25$. (The checking was done with the help of a digital computer).

6. Divide the inequality on both sides by $(100)^n$ to get $(.99)^n + 1 > (1.01)^n$. The left hand side is always less than or equal to 2 for every positive n. However, the right hand side increases without bound as n gets larger, so that for large enough n, the inequality must be false. (The smallest integer for which the inequality is not true is $n = 49$)..

7. Let $a = b - d$ and $c = b + d$. Then $(b + d)^2 = b^2 + (b - d)^2$, or upon simplifying, $b = 4d$, $a = 3d$, $c = 5d$. Hence $a: b: c = 3d: 4d: 5d = 3: 4: 5$.

8. $n(n + 1)(n + 2)(n + 3) = n^4 + 6n^3 + 11n^2 + 6n + 1 = (n^2 + 3n + 1)^2$.

9. We first count how many people failed exactly z exams for $z = 1, 2, \ldots, k$:

$a_1 - a_2$	failed	exactly	1 exam
$a_2 - a_3$	"	"	2 exams
$\vdots$			
$a_{k-1} - a_k$	"	"	k - 1 exams
a_k	"	"	k exams

The total number of failed exams is then $1(a_1 - a_2) + 2(a_2 - a_3) + \cdots + (k - 1)(a_{k-1} - a_k) + ka_k = a_1 + a_2 + \cdots a_k$.

10. $\tan \alpha = s/s = 1$, $\tan \beta = s/2s = 1/2$, $\tan \gamma = s/3s = 1/3$.

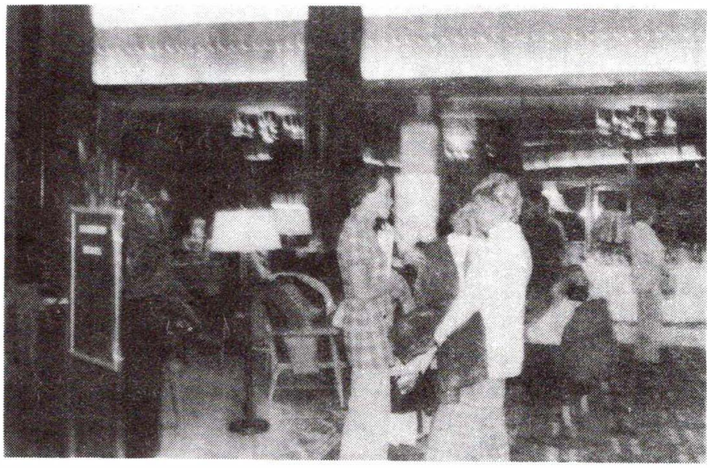
Hence $\alpha = 45^\circ$. $\tan (\beta+\gamma) = \dfrac{\tan \beta + \tan \gamma}{1 - \tan \beta \tan \gamma} = \dfrac{1/2 + 1/3}{1 - 1/6} = 1$,

giving $\beta + \gamma = 45^\circ$. Hence $\alpha + \beta + \gamma = 90^\circ$.

Once again...

At the business session of the annual meeting (October, 1974) two changes in the constitution were approved. (A copy of the constitution with these changes incorporated was mailed with the November issue of *Delta-K*.) Fees were increased to $6 (regular membership and subscription), $3 (students), effective January 1, 1975.

Also effective January 1, 1975, the costs for the Metric Missionary Workshops include expenses of participants, plus an honorarium to MCATA in an amount to be negotiated (minimum $50). The program has been extended to June 1 and could be renewed next year if there is sufficient need expressed.



MCATA annual meeting, 1974. Were you there?

The 1975 MCATA annual meeting is scheduled for Calgary - October 3/4, at the Calgary Inn.

A new series of ten 15-minute color instructional TV programs has been developed to combat math phobias. "*Math Matters*" is designed for the intermediate level, and these programs can be used for both refresher work and to introduce new mathematics topics. Lessons in this new series include fractions, metric measurement, geometry, and mathematical properties. "*Math Matters*" was produced by KLRN-TV, Southwest Texas Public Broadcasting Council in Austin, and is being distributed by the Agency for Instructional Television (AIT), Box A, Bloomington, Indiana 47401.

BOOKS REVIEWED

Activities in Mathematics First Course & Second Course
Johnson, Hansen, Peterson, Rudnick, Cleveland, Bolster
Publisher - Scott, Foresman & Co. (now Gage Publishing Limited), Price $7.90.

by *Art Jorgensen*
Principal
Jubilee Junior High School
Edson

The topics covered in these books are patterns, numbers, measurement, probability, graphs, statistics, proportions, and geometry.

The teacher's edition introduces each broad topic with a comprehensive overview. For each activity to be used in relation to the topic there is provided the objectives, an overview, a list of necessary materials, and practical procedures.

The students' books are very attractive and the vocabulary is very readable. The activities used to develop the desired understandings and concepts deal with contemporary issues of high interest to students. To avoid the monotony of drill, many activities are introduced using games. Emphasis is placed on student involvement. Among activities used to develop each topic are those that will interest and challenge students with a wide range of mathematical ability.

These books should prove of particular interest to students at the elementary and junior high school levels that have found mathematics to be difficult and uninteresting.

The textual material is supplemented with an excellent set of duplicating masters and overhead visuals.

A Symposium on the Evaluation of Modern Mathematics Curricula – A Report

D. Alexander, University of Toronto
J. Beamer, University of Saskatchewan
W. Higginson, Queen's University

At the International Congress of Mathematicians, Vancouver, August 21-29, 1974, a three-day symposium was organized by the International Commission on Mathematical Instruction (I.C.M.I.) to discuss evaluation of Modern Mathematics Curricula. Reports were presented by representatives of the United Kingdom, Russia, Poland, Brazil, Japan, India, U.S.A., Canada, Germany, Denmark, and Hungary with a general discussion on the topic on the third day. An official report will be sent in due course to all affiliated organizations (in Canada, the Canadian Mathematical Congress). The following reflects the impressions of the Canadian representatives.

There appeared to be general agreement that initial evaluations conducted in the experimental phases of the new curricula had been favorable while the evaluations became less significant as the curricula were expanded to encompass more schools. Reference was made to three factors which could explain this: 1) the reduction in the selectivity of the secondary and tertiary student bodies which coincided with the introduction of the new programs in most countries; 2) the lack of adequate teacher training for implementing the new programs; 3) the inability to define "modern mathematics curricula" (is it primarily content or method; what content, what method?).

The only large-scale study reported was the National Longitudinal Study of the U.S.A. where the most important factor determined was the teacher. No characteristics were identified for the effective teacher and the effectiveness of the teacher was not consistent in successive years. It is expected that the National Assessment of Educational Progress will contain some evaluation of mathematics curricula in the U.S.

In the U.K. a study is just being undertaken which will be an observational type rather than statistical. There is a definite feeling that large-scale statistical studies are doomed to failure because of the many uncontrollable variables involved.

Dr. Christiansen (Denmark and UNESCO) emphasized the need for evaluation of curricula on the basis of well-established goals (i.e. the goals of a developing country like Brazil, where illiteracy is a major problem, are entirely different from those of the U.S.A.). He also referred to his experience with UNESCO as impressing him with the impossibility of successfully 'transplanting' curricula. "Each country must find its own salvation."

The Polish answer to teacher training was enlightening. At 4 p.m. on a given day each week, every elementary teacher will be required to watch an in-service T.V. program and assignments will be marked weekly by the local superintendent. This is one way of solving the teacher training problem. (It was not clear what happens to delinquents.)

Although no conclusions were reached on the evaluation of the modern mathematics curricula or on how such evaluation should take place -- or indeed even whether it should take place -- the symposium did provide a forum for an exchange of information and opinions on an international level and was valuable for this alone.

The ICMI is a commission of the International Mathematical Union. Canadian representation is through the Canadian Mathematical Congress. Dr. A.J. Coleman, President of the CMC, hopes to institute a mechanism for making this representation more effective in the future through the formation of a national committee or commission for mathematics education. It is hoped that he will find the support, political and financial, for such a committee.

A new booklet, *The Overhead Projector in the Mathematics Classroom*, by George Lenchner, describes how to get the most out of this ubiquitous but under-used aid. Describing precisely what materials to use, how to design and make them, and how to present them effectively, this booklet helps the classroom teacher make optimum use of his overhead projector. With many detailed diagrams and two-color illustrations, plus an extensive list of suppliers, this is a practical as well as an imaginative guide. The handy 32-page booklet is available from NCTM for $1.10.

Some Comments on Methodology

1. Think Metric: 100% Immersion

There is consensus that the best approach to teaching and learning the metric system is by 100% conversion. It is unnecessary and unsatisfactory to teach a dual system. In particular, conversion from metric to Imperial units, and vice versa, is awkward, memory-taxing, and conceptually disruptive.

If measurement is being introduced to a Grade 1 class, only metric units should be used; if the advanced grades have been using Imperial units, they should be given a "crash" transitional programme to develop a background in metric units and then carry on exclusively with the metric system. (Adults will probably want to make comparisons to the Imperial system because it is an engrained part of their thinking.)

2. Learn Measurement by Measuring

The best cardinal capsule for the method of teaching and learning measurement is the Dewey principle, "Learn by doing." Measure, measure, measure.

3. Conceptualization of Measurement

How does a pupil develop the concept of measurement? What is the learning pattern? Consider the following four steps:

A. The identification and understanding of what is being measured. We measure different attributes, each distinct and discrete, e.g., length (1-dimensional), area (2-dimensional), volume (3-dimensional) time, speed (length/time), acceleration, force, work, power, etc. Confusion as to what is being measured leads to very muddled thinking. We teach this phase in elementary grades by using examples: length refers to our height, the width of the room, the depth of the sea, how far we can step, how high we can jump, the distance to the zoo, or to the movie, how long the worm is, the thickness of a book, the distance around the yard. We sharpen our understanding of the attribute being measured by comparison: John is taller than Jim, the door is twice as high as it is wide, it is further to Edmonton than to Calgary, etc. Further understanding is developed by ordering various objects according to some measure, e.g., arranging rods in order of length, arranging cups in order of capacity, arranging stones in order of mass.

B. Identification and selection of a unit. An appropriate unit must possess the same attributes as the one being measured. You use a toothpick, or a bobby-pin or a piece of string to measure length because they have the common attribute of length. Appropriate units may be non-standard and we usually (but not always) begin with non-standard units to accentuate the understanding of what we are measuring. However, we soon discover the inadequacies of non-standard units and introduce standard units. Metric, of course.

C. Measuring. The actual activity of measuring involves counting the units. In every case, we are faced with the problem of counting part of a unit -- part of a centimetre, or of a gram, or of a second. Counting of units is facilitated by using scales - really number lines with points matched to the number of units. The reading of scales, or rulers, or clock faces, or dials, needs to be carefully developed. On the metric system, parts of a unit are in the decimal system of notation rather than in the fractional system. We develop a thinking pattern of "two point three kilograms of meat" (2.3 kg). We subdivide in multiples of ten rather than in halves, quarters, eighths, etc.

D. Symbolization. Finally we express in some meaningful way the measurement. In the metric system. The problem of symbolization has been carefully considered for universal application and understanding. The SI conventions for symbolization should be meticulously followed.

4. The Use of Estimating

The practice of estimating a measurement is educationally very worthwhile. It promotes and augments conceptualization. It reinforces the imagery of the unit that is being used, it makes one "think" of the attribute that is being measured.

Look (at the object to be measured) -- *Visualize* (the unit to be used) -- *Estimate* (really "counting" of units in your mind) -- *Measure* (using standard units and your skill of reading scales) -- then *Compare* (your measurements to your estimate) to sharpen and develop the whole conceptualization process.

5. Utilitarian vs Structural Approach

How should we select and sequence measurement concepts? By their utilitarian value or by a rational, structural plan? For example, should we teach centimetres, metres and kilometres only, because they are the units of length that are commonly used, or should we teach all the subunits from milli- to kilo- (including decimetres, decametres, hectometres) because together they form a rational, well-structured system? The answer to this dilemma and controversy is probably some kind of synthesis. Certainly, attention must be given to the common, everyday application of units of measurement in order to have enough relevancy to maintain the familiarity and understanding of the units.

On the other hand, understanding is enhanced by having some rational structure. A good solution is probably to select one attribute--length is undoubtedly the best one--and teach all the subunits, millimetre, decimetre, metre, decametre, hectometre, kilometre, in order to have a complete example, a referent, for the decimal structure of the metric system. Even though a decametre may never be used in the big wide world of applied measurement, the unit has a place in the logical scheme of the metric system and should be taught for educational (cognitive) reasons.

Having one complete example, it is probably unnecessary to include all the subunits in the measurements of other attributes, e.g., in mass, the units of kilogram, gram, milligram and megagram (tonne) will suffice.

6. Appropriate Teaching (learning) Materials

Teachers should select materials that relate clearly and directly to the ideas that are being taught. Rulers are a good example. If the centimetre unit is being introduced, the ruler should clearly and directly show the centimetre scale; it should not be obscured with millimetre, decimetre or metre markings. If measurement using decimetres to one place of decimal is being taught, the ruler should show decimetre markings with each decimetre divided into decimal parts (called centimetres!).

7. Decimal Notation vs Fractional Notation

Because the metric system is based on powers of ten, we should capitalize and exploit this principle. Parts of a unit are expressed by decimals. In most cases each place of decimal can be associated with a subunit, e.g., 6.75 m means 6 metres, 7 decimetres and 5 centimetres. The fractional notation 6 3/4 m does not have this added association to subunits.

Fractional notation for parts of a unit should be avoided with the exception perhaps of ½, ¼ and maybe other unit fractions. These examples have a "visual imagery" impact and are therefore meaningful. But decimal notations will have greater dividends and lead naturally to other important ideas such as precision.

Decimal notation should be introduced in specific context. For example, we have always introduced two places of decimals in the context of writing dollars and cents. "Two dollars and sixty-five cents" is written as "$2.65." In the same way "Two metres and sixty-five centimetres" can be written as "2.65 m."

8. Precision and Approximations

All measurements made by you and me and our pupils are approximations. (The only exact measurements are those that are defined, e.g., the length of 1 650 763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels 2_{P10} and $5d_5$ of the krypton-86 atom is exactly 1 metre). We should avoid saying "Johnny is exactly 135 centimetres tall" (or even 1 352 millimetres!).

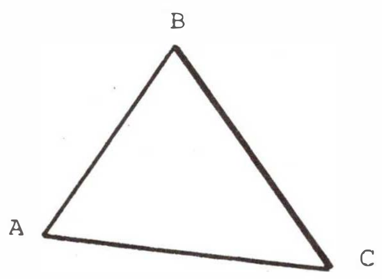
The idea of approximate measurements leads us to a consideration of precision, or "tolerances" as the tradesmen and engineers would say. This is an important educational idea - one that we have practically ignored in elementary grades.

Any notation of a metric unit gives the precision of the measurement, e.g., 1.925 m (my height) is a measurement precise to the nearer millimetre.

This attention to precision will lead in higher grades to the idea of scientific notation and significant digits.

9. "Ragged" Decimals

Before measures can be added or subtracted, they must have the same degree of precision. For example, consider the following problem:



If Amy measured the side AB as 2.4 m and Betty measured the side BC as 2.78 m and Christine measured the side AC as 2.923 m, what is the perimeter of the triangle?

```
  2.4   m
  2.78  m
  2.923 m
  -------
     ?
```

Do we merely fill in the gaps in the "ragged" decimals with zeros? This ignores the precision of each measurement. Amy was content to measure to the nearer decimetre, Betty to the nearer centimetre, Christine to the nearer millimetre. In order to find the perimeter to a known degree of precision, either Amy and Betty should remeasure and use the same precision as Christine did, or we must reduce the precision of the last two measures to Amy's standard, i.e.,

2.4 m
2.8 m
2.9 m

10. Problem Solving

Most of the mathematical problems in elementary textbooks (and in real life) involve measurements. As we look at methodology for teaching measurement, it is therefore relevant to look at the methods used for problem solving.

The usual initial step in solving a math problem is to formulate the problem situation, that is, to determine the essential components of the problem. When measurement is involved, it is useful at this stage to identify, select and visualize the units of measurements. Very often a diagram is a possibility.

The next step is usually to translate the problem situation into a mathematical sentence. Here, a clear understanding of the units used is important. Precision of units should be consistent.

The next step is usually computational. The comments made above in sections related to decimal notation, precision and ragged decimals are relevant.

Finally, in the interpretation of the solution to the problem, the complete understanding of units of measurement play an important part.

The work in problem solving will mean that decimal computation will be emphasized with a corresponding de-emphasis for fractional computation.

11. Concomitant Referents: All Subject Areas

As the change is made to the metric system, teachers should be aware of the appearance of metric referents in the environment. Metric units will appear more and more frequently in the supermarket and on packaging. Watch for them -- refer to them.

Although the math and science teachers will probably be the "experts" in metrication, all teachers should be informed of the basic principles of the metric system. Maps in social studies can easily be changed to metric by changing the scale; references to measures in language and reading should be interpreted in metric terms. Of course, many references are merely expressions and carry with them little measurement connotation, e.g., "There was a crooked man, who walked a crooked mile" or "I love you a bushel and a peck." Do not destroy the charm of these euphemisms by insisting on the metric translation "I love you 36.368 72 litres and an additional 9.092 18 litres."

From *Think Metric, a Workshop on the Metric System of Measurement*, prepared by Sidney A. Lindstedt, Consultant, Alberta Department of Education.

CANADIAN SOCIETY FOR HISTORY AND PHILOSOPHY OF MATHEMATICS
Société Canadienne d'Histoire et de Philosophie des Mathématiques

Dear Colleague,

Each year more mathematics teachers are realizing the relevance of the history or philosophy of mathematics to their own professional development and to their teaching.

In June of 1974, a new society was formed to encourage teaching, study and research in the history and philosophy of mathematics throughout Canada. In the first six months we had grown to a membership of 100 and we soon hope to publish a list of members and their areas of interest.

If you have an interest in the history or philosophy of mathematics, we invite you to join us by filling out and returning the application form below along with a cheque for $4 (the dues for 1975). A cheque for $10 will pay for your 1975 dues as well as a subscription to Vol. II (4 issues) of the Society's regular official journal, *Historia Mathematica,* an international journal, published and edited in Canada, with over 1,000 subscribers.

Join your colleagues and join now.

Yours very truly,

J. L. Berggren
Secretary-Treasurer

NAME ______________________________

MAILING ADDRESS ______________________________

____
(province) (post code)

SPECIAL AREAS OF INTEREST ______________________________

AMOUNT ENCLOSED $10 ( ) $4 ( )

On the Transformation of Rectangular Regions into Rectangular Regions of Equal Areas

by *William J. Bruce*

Consider two rectangles, ABCD with sides of lengths a and c, and EFGH with sides of lengths b and d, as shown in Figure 1, and such that $b < a < 2b$, $ac = bd$.

The problem is either to cut up the rectangular region ABCD so that the pieces will cover exactly the rectangular region EFGH, or to construct from ABDC a rectangle of area equal to that of EFGH and having the same shape.

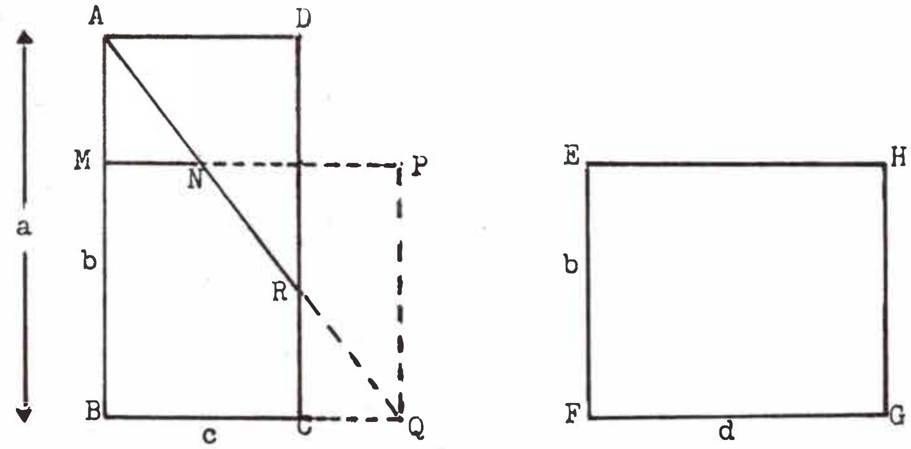


Figure 1

Construct MB = DR = EF, MN parallel to BC, and draw AR. Slide △ ADR to the position NPQ. Then Area △ RCQ = Area AMN and we have Area Rectangle ABCD = Area Rectangle MBQP = Area Rectangle EFGH. If cutting is done, the triangular pieces AMN and ADR combined with the polygonal piece MBCRN will cover exactly the rectangular region EFGH.

If $2b < a < 3b$, a modification of the procedure is necessary. In this case, it is essential first to reduce the problem to that of the first case. Division of the inequality by 2 gives $b < a/2 < 3b/2$ from which $b < a/2 < 2b$ follows, so that the first case applies. Figure 2 indicates the procedure.

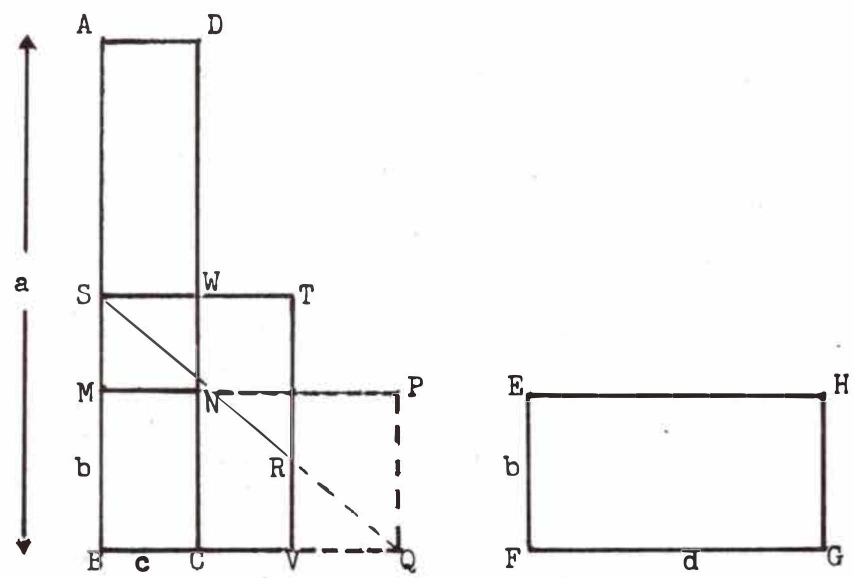


Figure 2

Divide the region ABCD into two equal parts by drawing SW. Place region ASWD in the position WCVT. Then the region SBVT satisfies the conditions of the first case and the construction proceeds as before, starting with this region, to obtain Area Rectangle ABCD = Area Rectangle SBVT = Area Rectangle MBQP = Area Rectangle EFGH.

In general, if AB is of length L, we can divide the rectangular region into n equal parts such that we always have $b < L/n < 2b$ for $L > b$ and the procedure is similar to that shown in Figure 2.

Trivial cases exist when $L = nb$ and require at most the division of the region into n equal rectangular parts and side-by-side placement of these parts as in Figure 2.

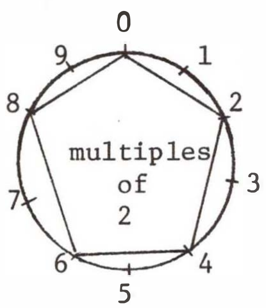
Ideas and Manipulatives you can try

Games, Games, Games

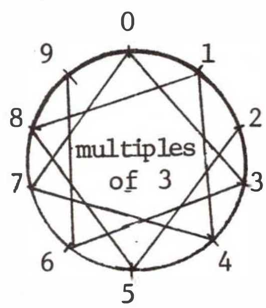
GAME I: Multiplication Fun

Directions: To graph multiples on the circle, start with 0 and connect in order the points with line segments until 0 is reached again.

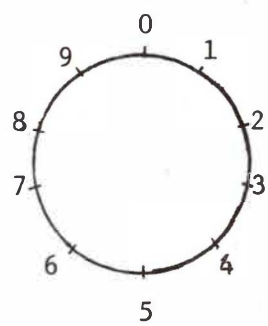
Example A



Example B



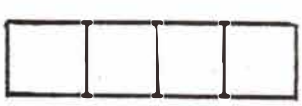
Try One

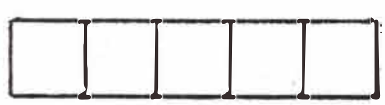


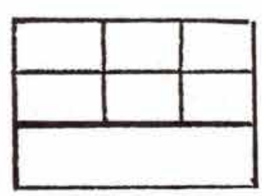
GAME II: Biggest Number

Materials: Make 3 of each of the 10 digits (0,1,2...9) on transparency squares (for class demonstration), or on construction paper squares (for math center).

Directions: Pull one square out of container at a time. Place it (or its digit) in box of your choice. - Remember it can't be moved after it is placed!! When boxes are filled, decide biggest number (or sum or difference).







GAME III: Can-u-go

1	2	3	4	5	6	7	8	9

Directions: Player covers any number of places that total value of the thrown dice. He continues until no other numbers can be used and totals those uncovered. Each player does this in turn. Winner has smallest sum.

Contributed by Shirley Frye

Hollywood Squares

. This game is one that I have found to be effective at the junior high level. It can be used to help students achieve a variety of curricular objectives, it is interesting to play, it can be learned quickly, and it adapts well to class size groups. I believe the game could be profitably used for students of other ages.

The Players:

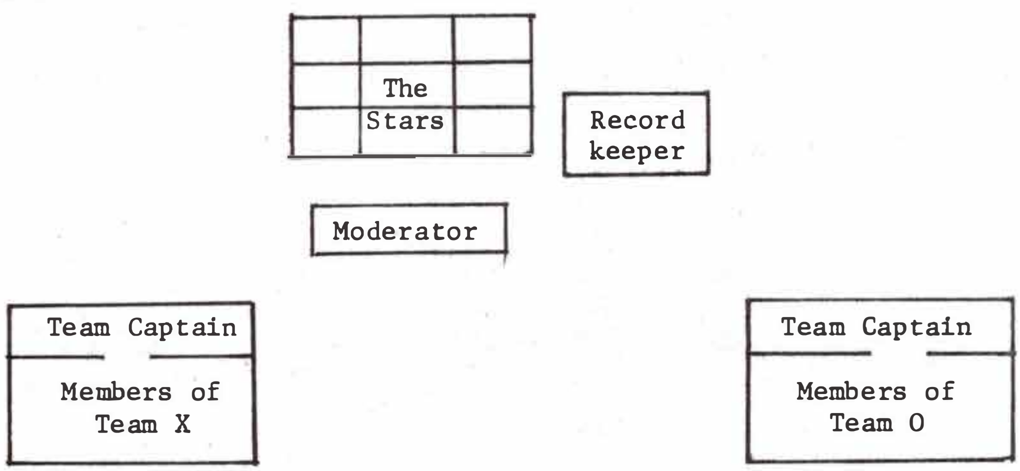
Stars - Nine persons are chosen to be the stars. They sit in desks arranged in a 3 by 3 array facing the class.

Record Keeper - One student is chosen to record results on a tic-tac-toe grid on the board.

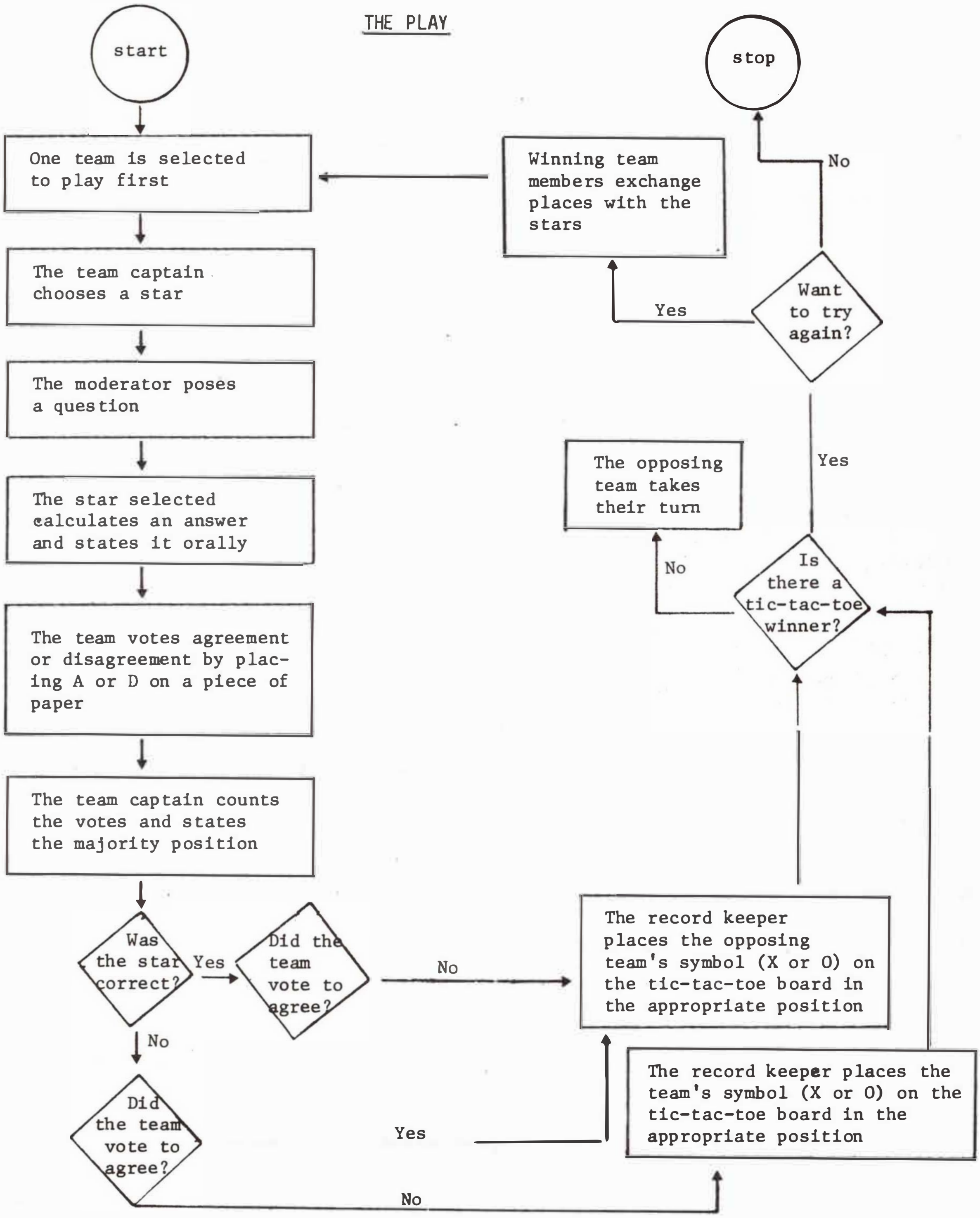
Moderator - One student acts as the moderator.

Teams - The rest of the students are divided into two groups; team X and team O.

Team Captains - Each team selects a Captain.



THE PLAY



Suggestions

- Provide the moderator with sample questions that are not too easy.
- Encourage the stars to try to deceive the teams by dramatizing responses.
- Questions answered incorrectly can be reused until the correct answer is given, thus all stars and team members should be prepared to respond to all questions.

Contributed by Francis Somerville

Formulae Chart and Summary of Main Points

Text: Modern Intermediate Algebra - Nichols et al

Designed for the Mathematics Department
Dr. E.P. Scarlett Senior High School, Calgary
by D.L. Treslan

MATHEMATICS 20 (January 1975)

CHAPTER 6: <u>Relations and Functions</u>

Relation: any set of ordered pairs.

Domain: the set of all first elements of the ordered pairs of a relation.

Range: the set of all second elements of the ordered pairs of a relation.

Methods of stating relations:

a) List or roster method: $R = \{1, 4, 9, \ldots\}$

b) Rule method: $R = \{$the squares of the natural numbers$\}$

c) Graph method:

Inverse of a Relation (i) obtained by interchanging the elements in each of the ordered pairs of the original relation.

(ii) obtained by interchanging the variables in the open sentence defining the relation.

Note: The graph of a relation and its inverse are "mirror reflections" through the line defined by $y = x$

- If R is the relation, the R' is the inverse of that relation.

Function: a unique relation in which each domain element is mapped onto one and only one range element.

Vertical line test: A relation is a function if and only if no vertical line intersects the graph of the relation in more than one point.

Types of Functional Notation:

a) Set builder: $g = \{(x,y) \mid y=2x+1,\ x,y \in R\}$

b) Mapping : $g : x \longrightarrow 2x+1,\ x \in R$

c) Image: $g(x) = 2x+1,\ x \in R$

In general, the function f pairs (c) from its domain and (f(c)) from its range to form (c,f(c)), an element of f.

Special Functions:

a) Constant function: eg. $y=3$.

- the range of f contains exactly one element.

b) Identity function: eg. $y=x$.

- each element of the function is of the form (x,x).

- (note that this is a special one - to - one function)

c) One - to - one function: eg. $y=x+3$

- each element in the domain is paired with a unique range element.

d) Many - to - one function: eg. $Y = |x|$

-each range element is associated with two or more domain elements.

e) Greatest Integer Function: eg $(x, [x])$

- each element of the function is of the form $(x, [x])$, where $[x]$ is the greatest integer not greater than x.

- (Sometimes referred to as the step-function)

f) Linear function: eg $y = 2x + 1$

- all functions of this type must be written in the form $y=mx+b$

- note that $m \neq 0$

Composition of Functions:

- a function may be defined in terms of other functions.

Def'n : Given a pair of functions f and g so that the range of f is the domain of g, the function j is the function composed of g with f if and only if $j(x) = g(f(x))$ for each x in the domain of f.

Inverse Functions: If f is a given function, then f^{-1} is the notation used for the inverse of that function.

NOTE that the methods used to obtain <u>inverse functions</u> are the same as those used to form <u>inverse relations</u>.

Direct Proportion:

Def'n: For every real number $c \neq 0$, $\{(x,y) \mid y = cx \text{ and } x \in D\}$ is a direct proportion function, where D is the domain of the function. (c) is called the constant of proportionality or constant of variation.

Inverse Proportion:

Def'n: For every real number $C \neq 0$, $\{(x,y) \mid y=\frac{c}{x} \text{ and } x \in D\}$ is an inverse proportion function, where D is the domain of the function. (c) is called the constant of proportionality.

CHAPTER 7: Quadratic Functions

Def'n: A function Q is a quadratic function iff $Q = \{(x, ax^2 + bx + c)\}$ where a, b, and c are real numbers, $a \neq 0$

The graph of a quadratic function is called a parabola.

Axis of Symmetry: the line about which the curve opens and which divides the graph into two equal parts.

Vertex: the point of intersection between the curve and the axis of symmetry.

Role of (a) in $y = ax^2 + bx + c$

1. The sign of (a) determines the direction of the curve (up of down)
2. The magnitude of (a) determines the size of the curve.

Role of (c) in $y = ax^2 + bx + c$

1. The y-intercept of the curve is (c)

Role of (b) in $y = ax^2 + bx + c$

1. (b) involves a horizontal shift of the curve from standard position.

Types of Quadratic Functions:

A. Form $y = ax^2 + p$

1) The graph formed will always be the graph of $y = ax^2$ shifted $|p|$ units vertically.
2) If $p > 0$, the parabola is shifted upward from standard position. If $p < 0$, the parabola is shifted downward from standard position.

B. Form $y = a(x-k)^2$

1) The graph of $y = a(x-k)^2$ is the graph of $y = ax^2$ shifted $|k|$ units to the right if $k > 0$, and to the left if $k < 0$.
2) The vertex of the graph of $y = a(x-k)^2$ has coordinates (K,0)
3) The equation of the axis of symmetry is $x = k$.

C. Form $y = a(x-k)^2 + p$ (The general Case)

1) The graph of $y = a(x-k)^2 + p$ is the graph of $y = a(x-k)^2$ shifted $|p|$ vertically; that is, if $p > 0$ or $p < 0$.
2) The graph is a parabola.
3) The coordinates of the vertex are (k,p)
4) The equation of the axis of symmetry is $x = k$.

Techniques for determining the graph of any quadratic function:

1) Completing the square.
2) Solving the general quadratic equations $y = ax^2 + bx + c$ to obtain:

Axis of Symmetry: $x = \frac{-b}{2a}$

Range:
- Maximum: $y \leq \frac{4ac - b^2}{4a}$, iff $a < 0$.
- Minimum: $y \geq \frac{4ac - b^2}{4a}$, iff $a > 0$.

Vertex: $\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a} \right)$

Note applications of quadratic theory in problem solving.

CHAPTER 8: Quadratic Equations and Inequalities

Def'n: A quadratic equation is an equation of the form $ax^2 + bx + c = 0$ where a, b and c are real numbers and $a \neq 0$.

Three cases are considered:

1) $b = 0$ ....$ax^2 + c = 0$
2) $c = 0$ ....$ax^2 + bx = 0$
3) $b \neq 0, c \neq 0$ ....$ax^2 + bx + c = 0$ (General Case)

Theorem 1: $x^2 \geq 0$ Any real number multiplied by itself yields a product which is a non-negative real number.

Theorem 2: $x^2 = k$ iff $x = \sqrt{k}$ or $x = -\sqrt{k}$, for each $k \geq 0$

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Characteristics:

1. Roots are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$
2. Axis of Symmetry: $x = \frac{-b}{2a}$
3. Sum of the roots: $\frac{-b}{a}$
4. Product of the roots: c/a
5. Discriminant test (D) : note that $D = b^2 - 4ac = \Delta$
   a) If $D > 0$, then there are two distenct roots.
   b) If $D = 0$, then there are two coincident roots.
   c) If $D < 0$, then there are no real roots.

NOTE that the roots are to the quadratic equation as the x-intercepts are to the quadratic function.

Equations with the Quadratic Pattern:

1. Fractional equations
2. Radical equations - note technique for solving equations containing one or more radicals.

Quadratic Inequalities:

Def'n: A quadratic inequality is an inequality of the form $ax^2 + bx + c > 0$ or $ax^2 + bx + c < 0$ where $a \neq 0$.

Theorem 1: $ab > 0 \iff [a > 0 \text{ and } b > 0]$ or $[a < 0 \text{ and } b < 0]$

Theorem 2: $ab < 0 \iff [a > 0 \text{ and } b < 0]$ or $[a < 0 \text{ and } b > 0]$

NOTE examples on pages 284 - 286 in the text.

CHAPTER 9: Complex Number System

Need for Complex Numbers:

1. Studies in electricity - alternating current theory.
2. Studies in magnetism - Oersted.
3. Problems in heat conduction and electrostatics.
4. Problems in physics; hydrodynamics; etc.

Def'n: Each ordered pair of real numbers (a,b) is a complex number , (Z). Note the geometric derivation there of.

Properties of Complex Numbers:

[NOTE that $Z_1 = (a,b)$ and $Z_2 = (c,d)$ and $Z_3 = (e,f)$]

1. Property of Equality: $Z_1 = Z_2$ iff $a = c$ and $b = d$.
2. Addition of complex numbers: $Z_1 \oplus Z_2 = (a+c,\ b+d)$
   - a) Closure Property: $Z_1 \oplus Z_2$ is a unique complex number, for all complex numbers Z.
   - b) Commutative Property: $Z_1 \oplus Z_2 = Z_2 \oplus Z_1$
   - c) Associative Property: $Z_1 \oplus (Z_2 \oplus Z_3) = (Z_1 \oplus Z_2) \oplus Z_3$
   - d) Additive Identity Element: (0,0)
   - e) Additive Inverse Element (-a, -b)
3. Subtraction of Complex numbers: $Z_1 \ominus Z_2 = (a-c,\ b-d)$
4. Multiplication of complex numbers: $Z_1 \odot Z_2 = (ac - bd,\ ad + bc)$
   - a) Closure Property: $Z_1 \odot Z_2$ is a unique complex number, for all complex numbers Z.
   - b) Commutative Property: $Z_1 \odot Z_2 = Z_2 \odot Z_1$
   - c) Associative Property: $Z_1 \odot (Z_2 \odot Z_3) = (Z_1 \odot Z_2) \odot Z_3$
   - d) Multiplicative Identity Element: (1,0)
   - e) Multiplicative Inverse Element: $\frac{1}{Z} = \left(\frac{a}{a^2 + b^2},\ \frac{-b}{a^2 + b^2}\right)$
   - f) Distributive Property: $Z_1 \odot (Z_2 \oplus Z_3) = (Z_1 \odot Z_2) \oplus (Z_1 \odot Z_3)$
5. Division of Complex Numbers: $Z_1 \oplus Z_2 = Z_1 \odot \frac{1}{Z_2}$

   NOTE that $Z_2 \neq (0,0)$

Any number system composed of a set (T) of elements and two operations (+) and (X) for these elements is called a **field**. The complex number system can be considered a field, and it contains all the number properties of a field.

Some complex numbers behave like real numbers. For example, $(2,0) \leftrightarrow 2$. In general, the complex number (a,0) will always behave like the real number (a).

Standard Form : an alternate format for writing and working with complex numbers.

$Z = (a,b) = a + bi$ Standard form

Note that all previous statements concerning properties of complex numbers can now be converted to standard format.

Absolute Value of a Complex Number:

$|Z| = \sqrt{a^2 + b^2}$

NOTE the geometric derivation thereof.

The Conjugate of a Complex Number:

For each complex number Z, if $Z = (x,y)$ or $x + yi$, then the conjugate of Z, denoted by $\overline{Z}$ is $(x, -y)$ or $x - yi$.

The product of a complex number and its conjugate is a real number.

NOTE the geometric interpretation of addition, subtraction and absolute value re: Complex numbers, Pages 316 - 318 in the text.

Square Roots which are Complex Numbers:

For each real number $a < 0$, $i\sqrt{-a}$ is a square root of (a), also $-i\sqrt{-a}$ is a square root of (a)

- Agreement: For each $x > 0$, $\sqrt{-x} = i\sqrt{x}$ and $-\sqrt{-x} = -i\sqrt{x}$

Quadratic Equations with Complex Solutions:

Recall the Discriminant test and the Quadratic Formula previously studied. Use these two facts in solving quadratic equations with complex roots.

NOTE that in further studies in mathematics you will find that each number (real or complex) has n different n^{th} roots among the complex numbers (for each natural number $n \geq 2$)

CHAPTER 10: Solution Sets of Systems

In this chapter we are interested, primarily, in the general linear equation $Ax+By+C=0$

Types of Equation Systems:

1) Independent System:

-here there is exactly one ordered pair in the solution set.

Ex. $\{x - y - 1 = 0 \cap 2x + y + 4 = 0\} = \{(-1,-2)\}$

2) Inconsistent System:

-here the empty set comprises the solution set.

Ex. $\{y = x - 1 \cap y = x + 2\} = \emptyset$

-note the identical slopes. Hence parallel lines.

3) Dependent Systems:

- here all real number ordered pairs which satisfy one equation will satisfy the second equation. The two equations are equivalent. (i.e. the same graph)

Ex. $\{x - y = -1 \cap 2x - 2y = -2\} = \{x - y = -1\}$

Def'n: Two systems of open sentences are equivalent iff they have the same solution set. (Equivalent Systems)

• **Methods of Solving Systems of Equations:**

1) Graphic method
2) Comparison method
3) Substitution method
4) Addition - Subtraction method

NOTE examples of above techniques on pages 333 - 342 in the text.

The Solution set of a General System:

Given: $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$

Therefore: $x = \dfrac{b_1c_2 - b_2c_1}{a_2b_1 - a_1b_2}$ $\qquad y = \dfrac{a_2c_1 - a_1c_2}{a_2b_1 - a_1b_2}$

Determinants:

Def'n: For all numbers a,b,c and d, the determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$

Determinants may be used to solve systems of equations as follows:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

We have considered only systems comprised of two linear equations so far. In advanced courses you will study systems comprised of more than two linear equations.

Systems of equations may also be comprised of:

a) One linear and one second- degree equation
   - See pages 349 - 354 in the text
b) Two second- degree equations
   - See pages 354 - 360 in the text

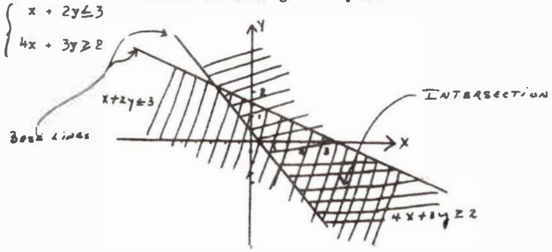
Also note techniques for solving such systems.

• **Def'n:** A conic is the intersection of a plane with a right circular conical surface or a right circular cylindrical surface.

Systems of Inequalities:

The graphing technique is the most useful method for solving such systems.

Consider the following system: $\begin{cases} x + 2y \leq 3 \\ 4x + 3y \geq 2 \end{cases}$



NOTE the importance of "dashed" base lines for < or > situations.

Study examples on pages 362 - 365 in the text.

Systems involving Absolute Value:

In graphing inequalities involving absolute value, the basic property used is:

For every $a > 0$, if $|x| < a$, then $-a < x < a$
For every $a < 0$, if $|x| > a$, then $x < -a$ or $x > a$

NOTE examples on pages 366 - 368 in the text.

CHAPTER 11: Logarithmic Functions

Def'n: $\log_{10}(x) = y$ iff $10^y = x$, $[x > 0]$

The domain of the logarithmic function is the set of positive real numbers; the range of the logarithmic function is the set of all real numbers.

NOTE: In order to understand the association between logarithmic and exponential format, memorize the following:

$10^2 = 100 \iff \log_{10} 100 = 2$

exponential format — logarithmic format

A logarithm is merely an exponent for a power with base ten. Therefore, logarithms obey the laws of exponents.

A logarithm consists of two parts: characteristic and mantissa.

Eg. $28 = 10^{1.4472}$ or $\log_{10} 28 = 1.4472$
characteristic (1) — mantissa (.4472)

Recall that one may use scientific notation to obtain the charateristic of a logarithm.

Theorems about Logarithms:

1. Logarithm of a Product: $\log_a MN = \log_a M + \log_a N$
2. Logarithm of a Quotient: $\log_a \frac{M}{N} = \log_a M - \log_a N$
3. Logarithm of a Power: $\log_a M^n = n \log_a M$
4. Logarithm of a Root: $\log \sqrt[n]{M} = \frac{1}{n} \log_a M$

The above theorems are used in solutions to questions inv[illegible]g conbinations of products, quotients and powers.

Antilogarithm:

The procedure of calculating an antilogarithm is the reverse of the procedure used for finding logarithms.

Eg. Suppose $\log_{10} 28 = 1.4472$

Then Antilogarithm $1.4472 = 28$

The antilogarithm gives us the number whose logarithm we have just calculated.

Other logarithmic functions:

Any positive real number (except the number one) may be used as the base of a logarithmic function. In general, for each positive real number (a) [except 1], the base (a) logarithmic function is $\{(x,y) \quad x = a^y\}$ The previous theorem for logarithms still apply.

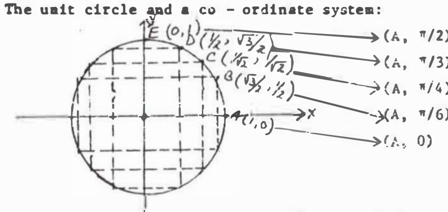
Change of Base

Note carefully the examples on page 394 in the text.

MATHEMATICS 30 (March 1974)

Chapter 12: Trigonometric functions:

1) Unit circle: $x^2 + y^2 = r^2 = 1$
   - Clockwise movement: Negative direction ↓
   - Counter - clockwise movement: Positive direction ↑
2) The unit circle and a co - ordinate system:



   - note the geometric figures within this circle in order to memorize the co - ordin of points in the circumference
3) Wrapping function: $W(\theta) = (x,y)$
4) Periodic functions: $f(x) = f(x + p)$, $p \neq 0$. This definition must hold for every x in the domain. The number (p) is called the period of f.
5) Cosine Function: $\{(\theta, x)\}$ for which $W(\theta) = (x,y)$.
6) Sine Function: $\{(\theta, y)\}$ for which $W(\theta) = (x,y)$.
7) Tangent Function: $\{(\theta, y/x)\}$ for which $W(\theta) = (x,y)$. $[x \neq 0]$
8) Basic formula: $\cos(\theta_1 - \theta_2) = \cos\theta_1 \times \cos\theta_2 + \sin\theta_1 \times \sin\theta_2$
9) Related formulae: $\cos(\theta_1 + \theta_2) = \cos\theta_1 \times \cos\theta_2 - \sin\theta_1 \times \sin\theta_2$

   $\cos^2\theta + \sin^2\theta = 1$

   $\tan\theta = \frac{\sin\theta_1}{\cos\theta_1}$, $[\cos\theta_1 \neq 0]$

   $\sin(\theta_1 - \theta_2) = \sin\theta_1 \times \cos\theta_2 - \sin\theta_2 \times \cos\theta_1$

   $\sin(\theta_1 + \theta_2) = \sin\theta_1 \times \cos\theta_2 + \sin\theta_2 \times \cos\theta_1$

   $\cos(-\theta_1) = \cos(\theta_1)$

   $\cos(\pi/2 - \theta_1) = \sin(\theta_1)$

   $\sin(-\theta_1) = -\sin(\theta_1)$

   $\sin(\pi/2 - \theta_1) = \cos(\theta_1)$
10) Characteristics of the sine function:
    1) Periodicity is 2π.
    2) Domain: $\{x/x \in R\}$
    3) Range: $\{y/-1 \leq y \leq 1, y \in R\}$
11) Characteristics of the cosine function:
    1) Periodicity is 2π.
    2) Domain: $\{\theta/\theta \in R\}$
    3) Range: $\{x/-1 \leq x \leq 1, x \in R\}$
12) Characteristics of the tangent function:
    1) Periodicity is π.
    2) Domain: $\{x/x \in R, x \neq 0\}$
    3) Range: $\{y/y \in R\}$

Chapter 13: Applications of trigonometric functions.

1) For each path (A, θ) with terminal point P, a degree - measure of $\angle AOP$ is $\frac{180}{\pi} \times \theta$
2) For all real numbers c and θ, if $c = \frac{180}{\pi} \times \theta$, then:

   $\cos(c^\circ) = \cos(\theta)$

   $\sin(c^\circ) = \sin(\theta)$

   $\tan(c^\circ) = \tan(\theta)$ $[c \neq 90 \pm 180n]$

Note that the Basic and related formulae, previously stated, can now be converted from radian to degree measurement.

3) Similar triangles: - corresponding angles are congruent.
   - corresponding sides are proportional.

   $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

   $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$

   $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$

   -For trigonometric ratios of all angles, $0 \leq \theta \leq 90$, Knott's mathematical tables may be used. Also page 601 of this text.
4) Characteristics of the $30^\circ - 60^\circ - 90^\circ$ triangle:
   1) the shorter leg is one - half the length of the hypotenuse.
   2) the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.
5) Characteristics of the $45^\circ - 45^\circ - 90^\circ$ triangle.
   1) both legs have the same length.
   2) the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.
6) Law of cosines:

   $a^2 = b^2 + c^2 - 2bc \cos\alpha$ -note applicability to triangles where α is acute, obtuse, or right.
7) Law of sines:

   $\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$ -note applicability to triangles where α, β, γ are acute, obtuse, or right.

Chapter 14: Sequences, Series and Limits.

1) Sequence - any arrangement of numbers in order.
   - finite : $\{a_1, a_2, a_3, \ldots a_n\}$
   - infinite: $\{a_1, a_2, \ldots, a_n, \ldots\}$
2) Term - each element of the sequence.
3) Series - the indicated sum of the terms of a sequence
   - finite: $\{a_1 + a_2 + a_3 + \ldots + a_n\}$
   - infinite: $\{a_1 + a_2 + \ldots + a_n + \ldots\}$
4) Arithmetic progression: a sequence exhibiting a constant difference between successive terms.

   $a_1, a_1 + d, a_1 + 2d, \ldots, a_1 + (n - 1)d$.

   General term: $a_n = a_1 + (n - 1)d$.
5) Arithmetic series: $S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + a_n$

   $S_n = \frac{n(a_1 + a_n)}{2}$ Also $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$

6) Geometric Progression: a sequence exhibiting a constant ratio between successive terms: $a_1, a_1 r, a_1 r^2, \ldots a_1 r^{n-1}$

7) Geometric Series: $S_n = a_1 r + a_1 r^2 + a_1 r^3 + \ldots + a_1 r^n$

$S_n = a_1 \frac{(r^n - 1)}{r - 1}$ Also: $S_n = \frac{a_n r - a_1}{r - 1}$

8) Summation Notation: (stop) $\sum_{k=1}^{n} a_k$ (substitute) $= a_1 + a_2 + a_3 + \ldots + a_n$. k = 1 (start)

9) Infinite Sequences and Limits:

- convergent sequence: tends towards a limit.
- divergent sequence: does not tend towards a limit.

10) Limit Properties:

1. $\lim_{n \to \infty} (KA_n) = K \times \lim_{n \to \infty} A_n$
2. $\lim_{n \to \infty} (a_n + b_n) = \lim_{n \to \infty} a_n + \lim_{n \to \infty} b_n$
3. $\lim_{n \to \infty} (a_n - b_n) = \lim_{n \to \infty} A_n - \lim_{n \to \infty} b_n$
4. $\lim_{n \to \infty} (A_n \times b_n) = \lim_{n \to \infty} A_n \times \lim_{n \to \infty} b_n$
5. $\lim_{n \to \infty} (A_n / b_n) = \frac{\lim_{n \to \infty} A_n}{\lim_{n \to \infty} b_n}$ $\left[\lim_{n \to \infty} b_n \neq 0\right]$
6. $\lim_{n \to \infty} c = c$
7. $\lim_{n \to \infty} 1/n = 0$ Note: Nos. 6 and 7 are very important.

11. Sum of an infinite geometric series:
- defined to be the limit of the sequence of its partial sums.

$S = \frac{a_1}{1 - r}$ Note: $\lim_{n \to \infty} r^n$ exists only for $-1 < r \leq 1$

$\lim_{n \to \infty} r^n = 0$ only for $|r| < 1$

Chapter 15: Permutations, Combinations and the Binomial Theorem.

1) Permutation: an arrangement, or ordering, of the elements of a set.
Types: linear and circular.

2) Fundamental Counting Principle: If an operation can be performed in K_1 ways, and after it is performed, a second operation can be performed in K_2 ways, etc., then collectively the n operations can be performed in $(K_1 \times K_2 \times K_3 \times \ldots \ldots K_n)$ ways.

3) Factorial: $n! = n(n - 1)(n - 2) \ldots\ldots 3 \times 2 \times 1$

4) Permutation formula:

$_nP_k = \frac{n!}{(n - k)!}$ Note: $0! = 1$; $1! = 1$

-Distinguishable linear permutations with "like" elements: $P = \frac{n!}{k_1! \times k_2! \ldots k_r!}$

- Distinguishable circular permutations: $P = (n - 1)!$

Note: For "keychain" situations, $P = \frac{(n - 1)!}{2}$

5) Pascal's Triangle: - displays the coefficients of terms in a binomial expansion.
- displays the symmetry present in a binomial expansion,

here $\binom{n}{k} = \binom{n}{n-k}$

$(a + b)^0 \rightarrow$ ①

$(a + b)^1 \rightarrow$ ①a + ①b

$(a + b)^2 \rightarrow$ ①a^2 + ②ab + ①b^2

$(a + b)^3 \rightarrow$ ①a^3 + ③a^2b + ③ab^2 + ①b^3

6) Combination formula: $\binom{n}{k} = \frac{n!}{k! \times (n - k)!}$

-"order" is not important here.

7) The Binomial Theorem:

$(x + y)^n = \binom{n}{n,0} a^n + \binom{n}{n-1, 1} a^{n-1}b + \ldots + \binom{n}{n-r, r} a^{n-r}b^r + \ldots + \binom{n}{0, n} b^n$

or $(x + y)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1}b + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + \binom{n}{n} b^n$

Note: $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Chapter 16: The Probability Function:

1) Sample space: the set of all possible outcomes of an experiment.

2) Event: any subset of a sample space of an experiment.

3) Definition of classical probability: $P(E) = \frac{n(E)}{n(S)} = \frac{\text{cases favorable}}{\text{cases possible}}$

Note: $P(E) = 0$ iff $E = \phi$ --- impossible event.
$P(E) = 1$ iff E is the event certain.

4) The Addition Theorem:

Case 1: $P(E \cup F) = P(E) + P(F)$ iff $E \cap F = \phi$ (mutually exclusive events)

Case 2: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ iff $E \cap F \neq \phi$.

5) The Multiplication Theorem:

Case 1: $P(E \cap F) = P(E) \times P(F/E)$ .... here E and F are dependent events.

Case 2: $P(E \cap F) = P(E) \times P(F)$ .... here E and F are independent events.

Case 3: $P(E \cap F) = 0$ .... here E and F are disjoint events.

Chapter 17: The Polynomial Function.

1) Definition: A polynomial function is a set of ordered pairs (x, f(x)), where $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n$

2) Divisibility: The polynomial P(x) is divisible by the polynomial $D(x) \neq 0$ iff $P(x) \div D(x)$ is a polynomial.

3) The Division Algorithm: $P(x) = Q(x) \times D(x) + R(x)$.

a) If P(x) is divisible by D(x), Then $R(x) = 0$.

b) P(x) is not divisible by D(x) if $R(x) \neq 0$.

Note: the division process is completed when the degree of R(x) becomes less than the degree of D(x) or when $R(x) = 0$.

4) Remainder Theorem: Given the polynomials P(x), Q(x), and (x - a), if $P(x) = Q(x) \cdot (x - a) + R$ for some number R, then $R = P(a)$.

5) Synthetic Division: note the process and characteristics thereof.
pp. 548 - 550 of text

6) Zero of a polynomial: The number r is called a zero of a polynomial P(x) iff $P(r) = 0$.

7) Factor Theorem: The binomial $x - r$ is a factor of P(x) iff $P(r) = 0$.

8) **Zeros of Integral Polynomials:** An integral polynomial may have the following:

a) a complex number for a zero. eg. $x^2 + 1$

b) an irrational number for a zero: eg. $x^2 - 2$

c) a non - integral rational number for a zero. eg. $2x - 1$

9) **Integral Zero Theorem:** If an integer r is a zero of an integral polynomial P(x), then r is a factor of the constant term of P(x).

10) **Rational Zero theorem:** If a/b [$b \neq 0$, (a) and (b) relatively prime integers] is a zero of the integral polynomial, P(x). Then (a) is a divisor of A_n and (b) is a divisor of A_0.

11) **Fundamental Theorem of Algebra:** If P(x) is a polynomial of degree greater than 0 over the complex numbers, than there is a complex number (r) for which P(r) = 0.

12) **Unique Factorization Theorem:** Every polynomial P(x) of degree $n \geq 1$ over the complex numbers can be factored uniquely into n first - degree factors, not all of which are necessarily distinct, and a constant factor which is the coefficient of the highest degree term of P(x).

- note the phrase "Sum of multiplicities."

13) Graphing Polynomial Functions:

- note this general review of graphs. Observe how the theory of polynomial zeros can assist here.

14) Complex Zeros of Real Polynomials: (Conjugate Zero Theorem)

If a + bi is a zero of a real polynomial P(x), then a - bi is also a zero of P(X).

15) Descartes' Rule of Signs: The sum of multiplicities of positive real zeros of a real polynomial P(x) is at most equal to the number of changes in sign in P(x). If this sum of multiplicities is less than the number of changes in sign, then it differs from it by an even number.

The sum of multiplicities of negative real zeros of P(x) is at most equal to the number of changes in sign in P(-x). Again, if this sum of multiplicities is less than the number of changes in sign, then it differs from it by an even number.

Conics and Mathematical Induction ..... Vance booklet

1) Mathematical Induction: Part (a): Verification for a specific value.

Part (b): Induction property: If the statement is true for n = k, then we wish to prove it true for the next larger value of n, say k + 1.

2) The Circle: Standard Equation: $(x - h)^2 + (y - k)^2 = r^2$, where (h,k) is the center, and r is the radius.

General Equation: $x^2 + y^2 + Dx + Ey + F = 0$ where $\left(-\frac{D}{2}, -\frac{E}{2}\right)$ is the center and $\frac{1}{2}\sqrt{D^2+E^2-4F}$ is the radius.

3) **The Parabola:**

Equations: $y^2 = 4px$, where the focus (p,o) is a point in the x-axis; directrix $x = -p$

$x^2 = 4py$, where the focus (o,p) is a point on the y-axis; directrix $y = -p$

Note: The graphs of both equations above have vertex at the origin.

Latus rectum = $|4p|$

Eccentricity (e) = 1

4) **The Ellipse:** Basic Equation is:

$$\frac{x^2}{(\text{length of semi - axis on x - axis})^2} + \frac{y^2}{(\text{length of semi - axis on y - axis})^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

(a) - length of the semi - major axis

(b) - length of the semi - minor axis

a > b - by definition

(c) - distance of each focus from the origin

Latus rectum: $\frac{2b^2}{a}$

Note that: $a^2 = b^2 + c^2$

Eccentricity (e) = c/a , (0 < e < 1)

5) **The Hyperbola:** Basic Equation

$$\frac{x^2}{(\text{length of semi - axis on x - axis})^2} - \frac{y^2}{(\text{length of semi - axis on y - axis})^2} = \pm 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$$

(a) : length of semi - transverse axis.

(b) : length of semi - conjugate axis.

(c) : distance of each focus from the origin.

Latus rectum: $\frac{2b^2}{a}$

Note that: $c^2 = a^2 + b^2$

Eccentricity (e) = $\frac{c}{a}$, (e>1)

Asymptote equation: $y = \pm\left(\frac{b}{a}\right)x$

Chapter 18: Introduction to Vectors

1) Magnitude: The magnitude of $\vec{XY} = |\vec{XY}|$ For any vector $\vec{AB}$ ← Terminal point; ↖ Initial point

2) Equivalent vectors: $\vec{A} \cong \vec{B}$ iff $|\vec{A}| = |\vec{B}|$ and $\vec{A}$ and $\vec{B}$ have the same direction.

3) Standard position: a vector whose initial point is the origin.

4) Rectangular form of a vector $\vec{AB}$ $[x_2 - x_1, \; y_2 - y_1]$ (x component, y component)

5) Zero Vector: Any vector whose initial point is the same as its terminal point.

6) Polar form of a vector $\vec{AB}$: $[r, u^\circ]$

7) Addition of vectors: $\vec{X} + \vec{Y} = [a + c, b + d]$, $\vec{X} = [a,b]$ and $\vec{Y} = [c,d]$

8) Multiplication of a scalar by a vector: $(s)[a,b] = [s(a), s(b)]$

9) Inner Product of Vectors: $|\vec{X}| \times |\vec{Y}| \times \cos(u^\circ)$. $= \vec{X} \cdot \vec{Y}$

10) Perpendicularity of vectors: $\vec{X} \perp \vec{Y}$ iff $\vec{X} \cdot \vec{Y} = 0$

Mr. Treslan wishes teachers to use the Formulae Chart as they see fit and he would welcome any criticism concerning deletions and/or additions.

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MCATA ANNUAL CONFERENCE

CALGARY INN, OCTOBER 3/4, 1975

Please complete fully. A cheque or money order, payable to the MCATA, must accompany this form.

NAME ____
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Conference registration fee (includes luncheon) $ 10

additional luncheon tickets (___ @ $5) ____

MCATA membership [] regular, $6 [] new

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Do you wish accommodation at the Calgary Inn ($25 per room, single or double)?

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The conference will deal with different aspects of learning and teaching mathematics. The guest speakers are headed by Chuck Allen, a well-known author and public speaker. Others will include Sue Ditchburn, Dick Holmes, Fred Henning, Bev Donaley, Delores Shriner, and Dr. A. Gibb. Registration will start Friday evening, October 3, and will be followed by a presentation from Mr. Allen. A social evening will follow. You may preregister by completing the above form and mailing it, with your cheque, to Pat Beeler, John Ware Junior High School, 10020 - 19 Street, SW, Calgary, T2V 1R2.