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THE TEACHING AND LEARNING OF SECONDARY SCHOOL MATHEMATICS

INTRODUCTION

I am concerned about the state of mathematics teaching and learning at the high school level in Canada today. The majority of students graduating from our high schools have little if any idea of the role which mathematics has played in the development of our culture; they have not been shown the ways in which mathematics is used in our modern society; students in college-bound programs, in particular, view mathematics as an isolated, self-sufficient body of knowledge.

My concern is not of recent origin, although it has grown in degree over the last ten years. When I had just started teaching nine or ten years ago, I remember raising some questions about one of the first experimental programs being introduced in our district in Ontario.* I recognized then - as I still do now - that the attempt to give students a thorough understanding of a concept (rather than using a "cookbook" approach) is a valid one. However, my questions about our goals in teaching mathematics ("Why do we teach what we teach?" "How can we make mathematics relevant for the 90 percent of the students who will use very little mathematics after they graduate from high school?") were brushed aside as "too philosophical." This was the era of reform - and I, along with many others, was swept along by the tide of enthusiasm for the many improvements which the new programs *did* contain.

However, my misgivings increased over the years. It became clear that the development of new programs and courses was not guided by any comprehensive concept of the process of education. Rather than starting with the question *why* we should teach certain topics, the foremost question was *how* we should teach what rather uncritically was accepted as "modern mathematics." Instead of giving positive direction in teaching students to grapple with the problems dealing with the warp and woof of today's culture, today's mathematics curriculum is abstract and removed from the student's everyday experience. Rather than preparing him for *life*, the present mathematics courses by and large prepare him only for even more specialized courses in universities or at technical institutes. My views on the present state of mathematics in high school parallel those of Professor Morris Kline in many ways, although I do feel that in his latest book[1] he has neglected the fact that many programs contained worthwhile improvements over the "traditional" texts they replaced.

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What a person believes about the meaning and purpose of life determines his philosophy of education, his views on the nature of the child and of learning, and

*This was the experimental version of *Mathematics 9* (Coleman et al), used in York county at that time.

his goals for teaching mathematics (whether these be explicit or implicit). In this brief I will show that my basic faith commitment influences *what* mathematics I teach as well as *how* I teach it. At the same time, I recognize that persons with a different philosophy of life may reach other conclusions - such freedom exists in a democratic society. I believe that a school is accountable to the parents in formulating its philosophy of education and its curricular objectives. Parents should then be able to choose the type of school for their children whose philosophy of education agrees with their own sense of values.

For the sake of brevity, I have not substantiated every statement. For a more detailed exposition of my views, I refer you to my chapter on curriculum in *To Prod the Slumbering Giant* [2] and to the mimeographed proceedings of a seminar on *Mathematics in the Christian School* [3].

GOALS OF CHRISTIAN EDUCATION

The curriculum of a school is the plan for learning that translates what one believes about man and his place in our society into a specific program of courses in the school. In our school, the curriculum aims at preparing the student for a Christian life of service in all areas of today's culture. Specifically, our school has the following four objectives:

1. The student's understanding of the unity and rich variety of the created world will be developed. He will systematically learn about matter, plants, animals, and man in society in order to gain knowledge of the structure, function, and interrelationships in creation.
2. The student will be directed to realize that he has a historic place in the world, to live obediently in the god-man covenantal relationship, and to fulfill the God-given task for man. The student will be guided to be a reforming influence in his society to help develop a Christian culture and life style according to the norms of God's Word.
3. The student will be directed to assume responsible discipleship of Jesus Christ in developing skills, knowledge, and insights to live the full Christian Life as a church member, citizen, family member, neighbor, friend, worker, consumer.
4. The student will be led through a general, foundational program of studies. Sufficient competence will be developed so that the student can continue his education in vocational training, a community college, a Christian college, a technical institute, a school of nursing, or a university.

I mention these objectives because I am convinced that a definite philosophy of life, when worked out in a curriculum, will affect a specific course of studies in a substantial way - and this applies to mathematics just as much as to any other subject. Conversely, if one pays lip service to a philosophy of life but does not work it out consistently in a curriculum, one may inadvertantly end up teaching a course based on a philosophy at odds with one's own.

MATHEMATICS CURRICULUM IN TODAY'S SOCIETY

The dominant role of science and technology in our culture has led to material wealth but spiritual poverty. The paradox of modern society is that man has constructed a complex social machine to administer the technical machine he has built, but his whole "creation" stands over and above him and manipulates him. As a result, youth became alienated from our society during the '60s. A reaction against our sterile culture set in - breakdown of authority, skepticism, protest, drugs, astrology, and reactions against scientism and individualism. The youth of 1974 is not as rebellious as that of five years ago, but it shows the continuing alienation in the appalling apathy among a large part of our high school generation.

The "modern" mathematics curricula at best have not counteracted, and at worst have abetted these phenomena. On the whole, the gospel (usually implicit) of modern mathematics texts is this: mathematics should be done for the sake of mathematics; mathematicians do not concern themselves with real life or with the moral problems that society may face; you learn mathematical concepts mainly so that you can use these in developing more advanced mathematical concepts, but whether these are relevant in today's society or are important for historical reasons is immaterial. It is remarkable that the vast majority of even those students disliking mathematics "play the game" to please the teacher or to get their marks; however, when they come to physics class, most of them can't solve an equation because they have only "played the game" with the x's and y's in totally abstract situations. More serious is that most texts (and, therefore, most teachers) seldom show that mathematics *is* relevant, that mathematics *does* have a tremendous impact on our culture, that mathematicians *must* make moral choices in how mathematics is used (it is not without cause that computer centers were prime targets of violence several years ago).

In our school, we try to take the student and make his profession of Christianity a significant one. This means also the mathematics curriculum must lead the student to a deeper understanding of our modern society. He must be made aware of the historical roots of our civilization as well as of the present value systems, the aims and ideals, the ultimate loyalties and allegiances of Western culture. This may not be left to the English and social sciences departments; mathematics also shares in this responsibility.

THE RELATIONSHIP OF MATHEMATICS TO OTHER SUBJECTS IN THE CURRICULUM

One major weakness of the present high school curricula is that students are given almost no insight into the interrelationship of the various disciplines. A worthwhile curriculum is not a hodge-podge of ideas thrown together, but it must provide the student with a sense of unity and purpose, a sense of his responsibility to God, to his community, to society as a whole. It is not the task of the high school to create specialists. Some students will undoubtedly become specialists in their later life, but they must be taught to relate specific knowledge to the overall situation. The curriculum - also in mathematics - must teach them to ask: "How does my specialty affect other areas of knowledge and the whole of life? How can I use my specialty in contributing to the enrichment of human culture? How does my discipline contribute to a meaningful outlook on the future of society?"

The present mathematics curriculum is concerned mainly with techniques and the teaching of facts for the sake of cataloguing knowledge. The danger of such specialization can be seen from the way most texts handle a topic such as linear programming: here is a technique to maximize profit; learn it and apply it to the following seven or eight problems. Unlike most exercises, these problems at least deal with situations the students can visualize occurring in everyday life. On the other hand, the implication of such a section is that the mathematician is concerned *only* with the mathematical technique and not at all with the other aspects of the situation. That a company should maximize its profit by hook or crook is a tacit assumption. The mathematician does not concern himself with the physical and biological aspects of the situation. It is not mentioned that maximizing profit might mean our resources are depleted unnecessarily or that it might upset the ecological balance. Nor does the mathematician concern himself with the psychological or social aspects (might he create unnecessary tensions between workers by putting this into effect?) or the legal aspect (might he not break the spirit, if not the letter, or certain of the government's laws?) or the ethical aspect (is it right to expect workers to have to work overtime or to be laid off at will? By maximizing profit, is the worker being reduced to a semi-robot who cannot do justice to his humanity?).

Our society needs people who know the techniques of mathematics - and these techniques must be taught. But such knowledge may never be taught as an end in itself - our society needs men and women who are aware of the consequences which the use of mathematical and scientific techniques may have, and who are able to make sound dicisions on the basis of such knowledge.

The importance of mathematics lies in its applicability to other fields - not only to the physical sciences and technology, but also to biology, psychology, economics, and political science. A mathematical structure can often serve as a model for many seemingly unrelated problems. Therefore, we must include applications of mathematics to other fields in our curriculum. If this is done at the student's level, it will not only deepen his understanding of the mathematical concepts and techniques, but it will also help to make his studies in mathematics more relevant, leading to better motivation. While a subject such as economics cannot be reduced to mathematics alone, students must become aware of the usefulness of mathematical models in such subjects. Almost all major fields of human endeavor and innumerable situations in everyday life are likely to lead to significant applications of mathematics. We must find problems which are complicated enough to represent a situation honestly, but simple enough so that students have some chance to solve it. I admit that this is not an easy task - as the large number of artificial and insignificant problems in most texts indicate. Pollak discusses this in his *Applications of Mathematics*[4] and states that applications are best chosen from classical analysis, linear algebra, probability and statistics, and computer science, if we keep in mind which fields will be of major importance in the future.

PSYCHOLOGICAL CONSIDERATIONS

Human development is not an automatic, natural process, but requires pedagogical influence and interaction and the exertion of formative power. Education always implies a deliberate attempt on the part of the educator to lead the student in a particular direction according to certain norms. At the same time, education requires a fundamental respect for those we seek to educate. A child does not *develop* into a person: he *is* a person from the start, though an immature one[5].

Mathematics must be taught so that the student is shown how the subject helps him to take on a meaningful calling in life, and it must enable the student to be a full, responsible human being who is actively involved in the educational process: the student must participate, cooperate, and be given opportunities to initiate. Teaching mathematics well requires liveliness, inspiration, stimulation, care and genuine concern for a person's development. A student's opinions and reactions must be respected even if not always approved. To educate means to give direction to the development of a person's life.

However, such development is not a "linear" one (cf. "Relevance in Teaching Mathematics")[6]. The student must be able to explore, to follow up hunches, to start with a problem that's meaningful and try to solve it using his intuition, to go off on a tangent, to choose topics himself from time to time that interest him.

Unfortunately, few mathematics texts are written as if mathematics is exciting, as if it is a fascinating journey with beautiful, useful, and "relevant" results. If Euclid had been introduced to geometry in the same way as my Grade X text[7] approaches it, he would likely never have been excited about mathematics. Pages and pages are spent on definitions and postulates and on seemingly endless similar exercises of proofs - and there are almost no results in the chapters that we "cover" in Grade X which the students didn't already know before they started. This year, after discussing the historical background of geometry before the Greeks, including some of the incorrect results used by the Babylonians, I started deductive geometry with a theorem I'm not "supposed" to take until Grade XI: a dissection-type proof of the Pythagorean theorem. We proved the theorem to the satisfaction of everyone involved, and then I pointed out some of the "gaps" in the proof. After that we worked backward to fill the gaps, taking five weeks to learn the same material that "officially" takes five months. So we now have more than three months to investigate other aspects of geometry.

Textbooks are not written for students; they are written for teachers (with the exception of such books as the SMP series[8]). And they are dull. The typical section has a couple of examples followed by a selection of similar exercises - and sometimes a written description which is usually too difficult to read for all but the best students. There is no attempt to put into practice what well-known educators have held to be psychologically sound, whether this be the use of Ausubel's "advance organizers", the intuitive approach of W. W. Sawyer, or the fact that Dienes has shown that the present sequence for teaching structure in mathematics is backward[9]. Perhaps not all of what these educators have to say is valid, but our textbook writers seem to think that none of it is. And while there are a great many creative teachers, the majority of teachers do not have the time or the ability to design their own curriculum and, therefore, usually depend on their texts.

We must structure the curriculum in such a way that the students are given the emotional freedom to respond to the teacher's guidance in their own unique way: to state their own views, to experiment, to investigate, to search and probe for answers. This means that general mathematics courses cannot and may not be just "more of the same." If students still cannot divide decimals properly when they come to Grade X, don't give them worksheets; give them challenging problems in an everyday context and let them use a calculator to work it out; maybe some of them will be motivated to see the usefulness of dividing decimals, and they may even want to learn the technique themselves. In the college-bound course, don't bind the students in a straitjacket of terminology, symbolism, and axiomatics. Give the students' intuition and imagination free reign; the good students will learn the correct symbolism and correct formal structure in due time, the poor students only get turned off by a too-early introduction to formalism.

Too often we treat students as if they were all mathematicians - with the exception that we don't even allow them the time needed by mathematicians to grasp ideas intuitively and work with them before attempting to become precise and rigorous. Calculus was used for many years before it was put on a rigorous logical basis. Logic and proof are useful tools in mathematics but do not form its essence. In our teaching, we must make clear that our everyday integral experience is the foundation of mathematics: everything in the world is subject to the cosmic law order. An intuitive approach to new topics with many different intuitive considerations is sound both from a philosophical and psychological point of view. We need problems which read: "Here is a situation - think about it - what can you say?"[10] Both teaching and learning in this way will be difficult at first, but it will also prove to be much more rewarding and meaningful than the stereotyped approach usually used now.

THE HIGH SCHOOL MATHEMATICS CURRICULUM

To be effective, a high school curriculum must be unified in purpose and direction. Our staff is working toward a model as described below. This model tries to stress the interdependence of all disciplines, for it is impossible to go very deeply in any one discipline without striking at the roots of another. All courses around the inner "core" are related to this core, and the objectives of each course tie in directly with the objectives of our school.

There are several "mandatory" subjects, including a course in mathematics. We hope to develop such a course during the next few years. Rather than *stressing* technical skills (although some basic ones would be taught!) the main goal would be to have the student come to an understanding of some of the basic structures of mathematics, as well as the place of mathematics within the structure of creation, how it developed through history, and how it is used and misused in today's society. Thus the mandatory course would emphasize the development and place of mathematics in Western culture as well as its relationships to the physical, biological, economic and aesthetic aspects of reality.

This course would be mandatory for all students - whether it be made part of our present courses or taught independently. With the present curricular structure in Alberta, it would probably become part of the present courses we offer, both at the general and matriculation levels. Eventually, however, we hope to teach this as one compulsory "module" in each of Grades X, XI, and XII (such a module could be individualized to a large extent) with a number of optional modules available to the students each year - ranging from algebra to transformations and vectors to calculus to business mathematics. The optional modules would develop topics in much greater depth for students having a special interest in mathematics in general or in one or more of the topics covered by the modules.

With this type of structure, we hope that *all* students will learn to recognize the rightful place of mathematics as a science which describes and investigates in detial the numerical and geometrical aspects of the universe around us. Students will gain comprehension to differing degrees, but it is our aim that all will learn to see mathematics as a functional tool to develop and unfold our society and our world. Specifically, we have set the following goals:

1. The student must gain an understanding of the concepts of number and space and their interrelationship so that he can abstract mathematical properties from concrete situations, so that he can theorize about and develop such properties, and so that he can apply the results of such theorizing to new situations. For example, a mathematical model in physics is a mathematical abstraction that can be worked with in order to clarify and gain more insight into the physical situation.
2. The student must realize that mathematics serves as a functional tool in solving our everyday problems. The student must see that mathematics does not exist independently of other disciplines but contributes to the unity

of all aspects of creation and helps him to analyze the quantitative and spatial situations in other disciplines.

3. The student must realize that mathematics is a developing science, and that throughout history it has influenced and, in turn, has been influenced by cultural forces.

If mathematics is taught to meet these objectives, the student will gain respect for the law structure of our universe and its dependability, and he will discover the order, patterns and relationships that exist in creation. By having a better understanding of these ideas, the student will also appreciate more fully the aesthetic aspects of mathematics. The realization of our objectives may be helped by the development of the student's techniques and skills. The degree of facilitation should be individualized in relation to the student's abilities and direction.

CONCLUSION

It will be difficult to change the direction of mathematics education with such massive curricular reform behind us. Yet, considering (1) the shortcomings of present texts as pointed out in this brief (texts which the majority of teachers depend on) and (2) the dissatisfaction with the present "new mathematics" courses by parents, students and teachers (a change in the elementary curriculum in Edmonton public schools enabled the *Edmonton Journal* to play up the fact that "the new math is dead"), it is clear that fundamental reform in the teaching of mathematics is a necessity.

Mathematics may not be seen in abstraction from the rest of life. An intuitive rather than an axiomatic approach is called for until the student has a thorough grasp of concepts: mathematics starts with *situations*, not with theorems. Of course, the problem is not just one of writing courses with a different philosophy and using sound psychological methods based on an understanding of the nature of learning, but it also involves having such programs implemented at the classroom level.

How can this be achieved? First, new curricular materials must be developed. I hope that *Mathematics Canada* will be able to initiate and perhaps even sponsor workshops where groups of teachers and educators who are in agreement about their philosophy of education and also about the direction of mathematics education will write materials for the classroom, particularly for a "mandatory" course such as suggested in this brief. In order for such material to find its way into the classroom, a concerted effort will have to be made to disseminate such materials widely and help teachers with its implementation. To be useful, the materials should be written *for the student*, with extensive teachers' guides also available (such as are found in the SMP series, Books A to H). Persons writing materials should be thoroughly acquainted with recent significant work that has been done in the psychology of mathematics learning, and make a conscious effort to structure course material accordingly.

Teachers and writers need more input from practising "applied" mathematicians with respect to meaningful applications that can be taught at the high school level. This will help both teachers and writers of curricular materials to show the interrelationship of mathematics with other branches of knowledge. At the same time, writers must be encouraged to show how the study of mathematics helps the school in achieving its objectives and how the subject material integrates with the rest of the curriculum.

Departments of education have given more "breathing room" in the past few years as far as choice of texts is concerned. However, very few teachers make use of the opportunity to depart from the "recommended text" - at least, this is the case in Alberta. Perhaps we have to move away from the all-inclusive texts to individual units, for which teachers' guides and such aids as transparencies are available.

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It is true, of course, that it is far easier to criticize and find shortcomings than to write a good curriculum oneself. The problem of getting students to understand, appreciate, and be able to use mathematics will never be solved completely. However, our present mathematics curricula have the wrong philosophical *and* psychological basis, and we must move in new directions if students are to realize the proper place of mathematics within our universe. This cannot be done overnight, but a start must be made.

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